JOURNAL

OF THE

INSTITUTE OF ACTUARIES.

"I hold every man a debtor to his profession, from the which as men of course do seek to receive countenance and profit, so ought they of duty to endeavour themselves by way of amends to be a help and ornament thereunto."—BACON.

VOL. XLIII.

LONDON:
CHARLES AND EDWIN LAYTON,
FARRINGDON STREET.
PARIS: 8, RUE LAMARTINE, 8.
BERLIN: CARLSTRASSE 11. MELBOURNE: McCARRON, BIRD & CO.
NEW YORK: THE SPECTATOR COMPANY.

1909.

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LONDON:
PRINTED BY CHARLES AND EDWIN LAYTON,
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1903 †Bacon, James, *Liverpool Victoria Insurance Corporation, Ltd., Atlantic House*, 45–49 *Holborn Viaduct*, E.C.

1850 †Bailey, Arthur Hutcheson, F.S.S. (PAST-PRESIDENT, 1878–82), 26 *Mount Ephraim-rd, Streatham*, S.W.

FELLOWS.

Those marked † are Fellows by Examination.

Date of becoming a Fellow.

1896 †Baker, Henry James, *Metropolitan Life Assur. Soc., 13 Moorgate-street*, E.C.

1885 †Barnes, Joseph Howard, F.S.S., *Phœnix Assurance Company, Limited*, 19 *Lombard-street*, E.C.

1895 †Barrand, Arthur Rhys, *Prudential Assurance Company, Holborn-bars*, E.C.

1890 †Bearman, Harry, *Gresham Life Assur. Soc., Ltd., St. Mildred's-house, Poultry*, E.C.

1889 †Bell, Frederick, *Alliance Assurance Co., Limited (Imperial Life Assurance Fund)*, 47 *Chancery-lane*, W.C.

1886 †Berry, Berry Alfred, B.A., *London Life Association, Ltd.*, 81 *King William-street*, E.C.

1895 †Besant, Arthur Digby, B.A., *Clerical, Medical & General Life Assur. Soc.*, 15 *St. James's-square*, S.W.

1879 Besso, Marco, *Besso-Palazzo Corso V.E.* 51, *Rome.*

1894 †Blackadar, Alfred Kimball, M.A., F.A.S., *Government Insur. Department, Ottawa, Canada.*

1883 †Blakey, James, I.S.O., *National Debt Office*, E.C.

1907 †Blanchard, Norman, M.A., *Equity & Law Life Assur. Soc.*, 18 *Lincoln's-inn-fields*, W.C.

1897 †Bradshaw, Thomas, F.A.S., *Imperial Life Assurance Co. of Canada, Toronto, Canada.*

1899 †Brown, Edward Harold, *Prudential Assurance Company, Holborn-bars*, E.C.

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1899 †Bull, Ernest James, 12 *King's Bench-walk, Inner Temple*, E.C.

1866 †Bumsted, David Alexander, *Sparnon, Blyth-road, Bromley, Kent.*

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1887 †Byers, Frederick Timothy Mason, *Clergy Mutual Assurance Soc.*, 2 & 3 *The Sanctuary*, S.W.

1888 †Calderon, Henry Philip, *Law Accident Insurance Society, Limited*, 215 *Strand*, W.C.

1871 †Carment, David, F.F.A., F.A.S., *Australian Mutual Provident Society, Sydney, Australia.*

1907 †Carter, Norman John, *Eagle Insurance Company*, 79 *Pall-mall*, S.W.

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1905 †Chandler, Thomas Richard, 113 *Powis-street, Woolwich*, S.E.

1889 †Chatham, James, F.F.A., F.S.S., *North British and Mercantile Insurance Co.*, 64 *Princes-street, Edinburgh.*

1883 Chisholm, James, F.F.A., F.A.S., *Royal Colonial Institute, Northumberland Avenue*, W.C.

1895 †Clarke, Arthur Harold, *Clerical, Medical and General Life Assurance Society*, 15 *St. James's-square*, S.W.

FELLOWS.

Those marked † are Fellows by Examination.

Date of becoming a Fellow.

1863 Clirehugh, William Palin,
London and Lancashire Life Assurance Company, 66 & 67 Cornhill, E.C.

1879 Cockburn, Henry, F.F.A., F.A.S.,
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1864 †Coles, John, J.P., F.S.S.,
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1903 †Collins, Frank Lakeman,
Clerical, Medical & General Life Assurance Soc., 15 St. James's-square, S.W.

1882 †Colquhoun, Ernest,
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1889 †Cooper, Walter George,
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1903 †Cross, Howard Turner,
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1889 †Cross, Robert,
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Star Life Assurance Society, 32 Moorgate-street, E.C.

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La Mexicana Compañia de Seguros sobre la Vida, Apartado 651, Mexico, D.F., *Mexico.*

Date of becoming a Fellow.

1906 †Curtis, William Allen,
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1886 †Dickinson, Arthur Lowes, M.A., F.C.A.,
54 William-street, New York, U.S.A.

1904 †Diver, Oswald Francis, M.A.,
(TUTOR, Part I),
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1887 Douglas, Gordon, F.F.A.,
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1901 †Dunn, Spencer Græme,
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1901 †Elderton, William Palin,
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1897 †Fraser, Duncan Cumming, M.A.,
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1905 †Gemmill, William, F.S.S.,
Transvaal Chamber of Mines, Johannesburg, South Africa.

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1902 †Gillies, George,
Union Life Branch of the Commercial Union Assurance Co., 1 & 2 Royal Exchange-bldgs, E.C.

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National Mutual Life Association of Australasia, 5 Cheapside, E.C.

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South African Mutual Life Assurance Society, Cape Town, South Africa.

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1882 †Graham, James, F.F.A.,
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1869	†Justican, Edwin, F.S.S., *Gresham Life Assurance Society, Limited, St. Mildred's-house, Poultry*, E.C.
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1894	†McDonald, John, *Prudential Assurance Company. Holborn-bars,* E.C.
1883	†McGowan, James, B.A., *The Treasury, Cape Town, South Africa.*
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1907	†Mackenzie, Michael Alexander, *University of Toronto, Toronto. Canada.*
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1901	†Macphail, Donald, F.F.A., 3 *Hopetoun-place, Gullane, N.B.*
1908	†Maltby, Charles Hugh, *North British and Mercantile Insurance Co.,* 61 *Threadneedle-street,* E.C.
1870	†Manly, Henry William, F.A.S., (PAST-PRESIDENT, 1898-1900), *Glenthorne,* 157 *Highbury New-park,* N.
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1906	†May, Walter Thomas, *Liverpool and London and Globe Insurance Co.,* 1 *Cornhill,* E.C.
1908	†Melville, Henry Edward, *Alliance Assurance Co., Ltd., Bartholomew-lane,* E.C.
1897	†Miller, Neville, *London Assurance Corporation,* 7 *Royal Exchange,* E.C.
1905	†Milligan, Charles Livingstone, *Alliance Ass. Co., Ltd. (Provident Life Fund),* 50 *Regent-street,* W.
1893	†Milner, John William, *North British & Mercantile Insur. Co.,* 61 *Threadneedle-street,* E.C.
1892	†Milton, Henry, M.A., *Law Debenture Corporation, Ltd.,* 41 *Threadneedle-street,* E.C.
1899	†Moir, Henry, F.F.A., F.A.S., *Home Life Insurance Co.,* 256 *Broadway, New York, U.S.A.*
1890	†Molyneux, Arthur Ernest, *Provident Clerks' and General Mutual Life Assurance Assoc.,* 27 & 29 *Moorgate-street,* E.C.
1901	†Moorhouse, Alfred, *Friends' Provident Institution, Bradford, Yorkshire.*
1897	†Moors, Elphinstone McMahon, M.A., *University of Sydney, Australia.*
1896	†Moran, Joseph Flack, *Reversionary Interest Society,* 30 *Coleman-street,* E.C.
1900	†Morgan, Benjamin Charles, M.A., *Commercial Union Assur. Co.,* 24, 25 & 26 *Cornhill,* E.C.
1895	†Muter, Percy, *New Zealand Government Life Insurance Department, Wellington, New Zealand.*
1888	†Nash, Willie Oscar, *Law Reversionary Interest Soc., Limited, Thanet-house,* 231 & 232 *Strand (opposite the Law Courts),* W.C.
1906	†Neill, Samuel Bennett, *China Mutual Life Insurance Co., Shanghai, China.*
1883	Neison, Francis G. P., F.S.S., 19 *Abingdon-st., Westminster,* S.W.

FELLOWS.

Those marked † are Fellows by Examination.

Date of becoming a Fellow.	
1888	†Newman, Philip Lewin, B.A., *Yorkshire Insurance Co., York.*
1865	Newton, Algernon, M.A., *c/o London & Westminster Bank, 94 & 96 High-st., Kensington,* W.
1887	†Nightingale, Harry Ethelston, *Royal Exchange Assurance Corporation, Royal Exchange,* E.C.
1903	†Norris, Charles Arthur, *National Mutual Life Association of Australasia, Limited, Melbourne, Australia.*
1901	†Norton, William Ernest, *National Provident Institution, 48 Gracechurch-street,* E.C.
1905	†Oakley, Henry John Percy, *North British and Mercantile Insurance Company, 61 Threadneedle-street,* E.C.
1899	†Parker, Robert Peter, *2 Langton Villas, Grand Drive, Herne Bay, Kent.*
1864	Pearson, Arthur, *Betchworth-house, The Bank, Highgate,* N.
1905	†Penman, William, Jr., *Atlas Assurance Company, Ltd., 92 Cheapside,* E.C.
1891	†Phelps, William Peyton, M.A., (Hon. Sec.), *Equity and Law Life Assur. Soc., 18 Lincoln's-inn-fields,* W.C.
Under the Charter.	Priestley, John George, *44 St. German's-road, Forest-hill,* S.E.
1891	†Pulley, William Pritchard, *Norwich Union Life Insur. Soc., 71 & 72 King William-st.,* E.C.
1903	†Rae, Joseph, *Finance Department, Town-hall, Upper-street,* N.
1899	†Raisin, Arthur Herbert, *Phœnix Assurance Co., Ltd., 70 Lombard-street,* E.C.
1897	†Rees, Martin, *Law Reversionary Interest Soc., Limited, Thanet-house, 231 & 232 Strand (opposite the Law Courts),* W.C.

Date of becoming a Fellow.	
1901	†Reeve, Charles Ernest, *Royal Exchange Assurance Corporation, Royal Exchange,* E.C.
1902	†Richmond, George William, *Scottish Widows' Fund and Life Assur. Society, 9 St. Andrew-square, Edinburgh.*
1904	†Rietschel, Hermann Julius, *Sun Life Assurance Society, 63 Threadneedle-street,* E.C.
1898	†Robinson, George Frederick, *Legal and General Life Assur. Society, 10 Fleet-street,* E.C.
1905	†Robinson, Hugh Thomas Kay, *Clergy Mutual Assur. Soc., 2 & 3 The Sanctuary,* S.W.
1888	†Rusher, Edward Arthur, F.S.S., *Prudential Assurance Company, Holborn-bars,* E.C.
1882	†Ryan, Gerald Hemmington, F.A.S., *Phœnix Assurance Co., Ltd., 70 Lombard-street,* E.C.
1898	†Salmon, Richard George, F.S.S., *Sun Life Assurance Society, 63 Threadneedle-street,* E.C.
1883	Saunders, Harris Charter Lindon, F.R.A.S., *"Marquise," Twickenham.*
1886	†Schooling, Frederick, F.A.S., *Prudential Assurance Company, Holborn-bars,* E.C.
1901	†Searle, George Morley, *Sun Life Assurance Society, 63 Threadneedle-street,* E.C.
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1905	†Sherriff, Francis Henry, *Provident Clerks' and General Mutual Life Assurance Assoc., 27 & 29 Moorgate-street,* E.C.
1896	†Sim, William Abernethy, F.F.A., *Scottish Union and National Insurance Co., 35 St. Andrew-square, Edinburgh.*
1875	†Smither, Arthur, *Green Bank, Lewes.*

FELLOWS.

Those marked † are Fellows by Examination.

Date of becoming a Fellow.

1881 †Somerville, William Finlay,
Liverpool and London and Globe Insurance Co., 1 *Dale-street, Liverpool.*

1877 †Sorley, James, F.S.S., F.R.S.E.,
82 *Onslow-gardens*, S.W.

1898 †Spencer, John,
English and Scottish Law Life Assurance Assoc., 12 *Waterloo-place*, S.W.

1894 †Sprague, Alfred Ernest, D.Sc., M.A., F.F.A.,
Edinburgh Life Assurance Co., 26 *George-street, Edinburgh.*

1857 Sprague, Thomas Bond, M.A., LL.D., Hon. F.F.A., F.S.S., F.R.S.E. (Past-President, 1882–86),
29 *Buckingham-ter., Edinburgh.*

1906 †Spurgeon, Ernest Frank,
Prudential Assurance Company, Holborn-bars, E.C.

1896 †Stahlschmidt, Louis,
St. John's College, Agra, India.

Under the Charter Stevens, Charles,
Aberdeen Ho., Preston, Brighton.

1888 Stewart, John, F.F.A.,
City of Glasgow Life Assur. Co., 30 *Renfield-street, Glasgow.*

1906 †Stewart, Lionel William,
Alliance Assurance Co., Ltd., Bartholomew-lane, E.C.

1898 Stirling, Robert, F.F.A.,
Rock Life Assurance Company, 15 *New Bridge-street*, E.C.

1892 †Straker, Edward Robert,
Phœnix Assurance Co., Ltd., 70 *Lombard-street*, E.C.

1878 †Straker, Frank Arthur,
Legal and General Life Assur. Society, 10 *Fleet-street*, E.C.

1902 †Strong, William Richard,
London Guarantee & Accident Co., Orient House, New Broad-street, E.C.

1884 †Stuart, John Moody, F.F.A.,
Leeds Permanent Benefit Building Society, Victoria-buildings, Park-lane, Leeds.

1900 †Sutherland, John, M.A.,
Australasian Temperance and General Mutual Life Assurance Society, Swanston-street, Melbourne, Australia.

1906 †Symmons, Frank Percy,
Prudential Assurance Company, Holborn-bars, E.C.

1889 †Tarn, Arthur Wyndham,
Guardian Assurance Company, 28 *King-street, Covent-garden*, W.C.

1887 Tecce, Richard, F.F.A., F.A.S., F.S.S.,
Australian Mutual Provident Society, Sydney, Australia.

1864 †Terry, James,
Hernlee, Lyme Regis, Dorset.

1889 †Thiselton, Herbert Cecil, F.F.A., F.A.S.,
Commercial Union Assurance Co., 24, 25 & 26 *Cornhill*, E.C.

1901 †Thodey, Robert,
Australian Mutual Provident Society, Sydney, Australia.

1893 †Thomas, Ernest Charles,
Gresham Life Assurance Society, Limited, St. Mildred's-house, Poultry, E.C.

1899 †Thomas, Robert Arthur Caradoc,
Phœnix Assurance Co., Ltd., 70 *Lombard-street*, E.C.

1905 †Thompson, Thomas Percy, B.A.,
Phœnix Assurance Co., Ltd., 70 *Lombard-street*, E.C.

1895 †Thomson, Herbert Archer, B.A.,
3 *Kings-bench-walk, Temple*, E.C.

1893 †Thorne, Alfred Charles,
Equity & Law Life Assur. Soc., 18 *Lincoln's-inn-fields*, W.C.

1891 †Tilt, Robert Ruthven,
General Reversionary & Investment Co., Ltd., 26 *Pall-mall*, S.W.

1902 †Tinner, Thomas,
Comptroller's Depart., London County Council, Spring-gardens, S.W.

1881 †Todd, George, M.A., (Vice-President),
Economic Life Assurance Society, 6 *New Bridge-street*, E.C.

FELLOWS.

Those marked † are Fellows by Examination.

Date of becoming a Fellow.

1894 †Todhunter, Ralph, M.A., *University Life Assur. Soc.*, 25 *Pall-mall*, S.W.

1899 †Trouncer, Harold Moltke, M.A., *London Life Association, Ltd.*, 81 *King William-street*, E.C.

1878 Turnbull, Andrew Hugh, F.F.A., F.R.S.E., 18 *Whitehouse-loan, Edinburgh.*

1889 Wallace, Thomas, F.F.A., *North British & Mercantile Insurance Co.*, 64 *Princes-street, Edinburgh.*

1905 †Wandless, John Robert, *Canada Life Assurance Co.*, 14 *King William-street*, E.C.

1906 †Wares, Harold Wallace, *Yorkshire Insurance Co., Ltd.*, *Bank-buildings, Princes-street*, E.C.

1888 †Warner, Samuel George, (VICE-PRESIDENT), *Law Union & Crown Insur. Co.*, 126 *Chancery-lane*, W.C.

1893 †Watson, Alfred William, *Manchester Unity Friendly Soc.*, *Nottingham.*

1895 †Watson, James Douglas, F.A.S., *Star Life Assurance Society*, 32 *Moorgate-street*, E.C.

1904 † Weatherill, Henry, *National Debt Office*, E.C.

1880 †Whittall, Wm. Joseph Hutchings, F.A.S., 18 *Airlie-gardens, Campden-hill*, W.

1905 †Wilson, John Sydney, *Australian Widows' Fund Life Assurance Society, Melbourne, Australia.*

1864 Wilson, Robert, 44 *Talfourd-rd., Camberwell*, S.E.

Date of becoming a Fellow.

1888 †Wilson, Robert, Jr., *General Assurance Company*, 103 *Cannon-street*, E.C.

Under the Charter. Winser, Thomas Boorman, F.R.G.S., F.R.N.S., 81 *Shooter's-hill-road, Blackheath*, S.E.

1899 †Winter, Arthur Thomas, *Phœnix Assurance Co., Ltd.*, 70 *Lombard-street*, E.C.

1897 †Wintle, Lancelot Andrewes, *Economic Life Assurance Soc.*, 6 *New Bridge-street*, E.C.

1904 †Wood, Arthur Barton, B.A., F.A.S., *Sun Life Assurance Co. of Canada, Montreal, Canada.*

1884 †Woods, Ernest, F.A.S., (TREASURER), *Guardian Assurance Company*, 11 *Lombard-street*, E.C.

1902 †Woolmer, Alfred Henry, *Star Life Assurance Society*, 32 *Moorgate-street*, E.C.

1902 †Workman, William Arthur, *Equitable Life Assur. Society*, *Mansion-house-street*, E.C.

1902 †Worthington, William, *Royal Insur. Co., Ltd., Liverpool.*

1875 †Wyatt, Frank Bertrand, F.A.S., (EX-PRESIDENT), *Clergy Mutual Assurance Soc.*, 2 & 3 *The Sanctuary*, S.W.

1906 †Young, Arthur Stanley, *Metropolitan Life Assurance Society*, 13 *Moorgate-street*, E.C.

1874 Young, Thomas Emley, B.A., F.R.A.S., (PAST-PRESIDENT, 1896-8), 108 *Evering-road, Stoke Newington*, N.

ASSOCIATES.

Those marked 2 or 3 have passed two or three of the four Examinations of the Institute.
Those marked (2) have been exempted under the Bye-laws from the Examinations in Parts I and II.

Date of becoming an Associate

1900 [2] Adams, Cecil Francis, *New Zealand Insurance Co., Accident Branch, Palmerston North, New Zealand.*

1908 [2] Addey, Leonard, *Clergy Mutual Assurance Soc., 2 & 3 The Sanctuary,* S.W.

1869 [2] Adey, Theodore Henry, *Scottish Provident Institution, 3 Lombard-street,* E.C.

1908 [2] Alder, Milton Cromwell, *Mutual Life & Citizens' Assurance Co., Limited, Sydney, Australia.*

1908 [2] Anderson, Robert Duncan, *45 Southbrook-road, Lee,* S.E.

1899 [2] Ansell, George Frederic, *National Debt Office,* E.C.

1904 [2] Ashley, Charles Henry, *British Widows' Assurance Co, 1 Old-street,* E.C.

1883 [2] Ashley, John Geo., M.A., *War Office,* S.W.

1901 [3] Ashton, William Richard, *Commercial Union Assur. Co., 24, 25 & 26 Cornhill,* E.C.

1904 [3] Atkins, Leonard George, *Law Union & Crown Insurance Co., 126 Chancery-lane,* W.C.

1881 [2] Ayling, Charles Stephen, *Commercial Union Assur. Co., 26 New Bridge-street,* E.C.

1905 [2] Bain, William Algernon, *Manufacturers Life Insurance Co., Toronto, Canada.*

1903 [2] Ball, Sidney Robertson, *English and Scottish Law Life Assurance Association, 12 Waterloo-place,* S.W.

1905 [2] Barford, Frederick William, M.A., *Australasian Temperance and General Mutual Life Assurance Society, Swanston-street, Melbourne, Australia.*

1904 [2] Barrett, William Goodsman, *United Kingdom Temperance and General Provident Institution, 196 Strand,* W.C.

1885 Barton, Arthur, *Royal Insurance Company, Ltd., Maidstone.*

Date of becoming an Associate.

1894 [3] Barton, Robert Whitchurch, *48 William-street, Montreal, Canada.*

1908 [2] Beatty, Samuel, B.A., *142 Collier-st., Toronto, Canada.*

1901 [2] Benjamin, Stanley O., *Australian Mutual Provident Society, Sydney, Australia.*

1908 [2] Bennett, Samuel, *National Deposit Friendly Soc., 37 Queen-square,* W.C.

1881 Birks, Edmund Alfred, *Yorkshire Insurance Co., York.*

1906 [2] Blake, Francis Seymour, *London County Council, Spring-gardens,* S.W.

1906 [2] Blehl, Ernest M., A.M., A.A.S., *Philadelphia Life Insurance Co., North American Building, Philadelphia, Pa., U.S.A.*

1898 (2) Blount, Edward Thos. Joseph, F.F.A., F.S.S., *Standard Life Assurance Co., 3 George-street, Edinburgh.*

1906 [2] Boag, Harold, *33 Albert-drive, Low Fell, Gateshead.*

1873 [2] Boon, Gerald Inglis, *United Legal Indemnity Insur. Soc., Limited, 222 Strand,* W.C.

1906 [2] Borrajo, Edward Joseph William, *Prudential Assurance Company, Holborn-bars,* E.C.

1908 [2] Bradbury, Algernon Charles, *Australian Mutual Provident Society, Melbourne, Australia.*

1889 (2) Bremner, Thomas William, F.F.A., *Mutual Life of New York Building, Martin-place, Sydney, Australia.*

1905 (2) Brodie, Robert Raynal, F.F.A., *Scottish Provident Institution, 6 St. Andrew-sq., Edinburgh.*

1907 [2] Brown, Arthur Ewart, *Metropolitan Life Assurance Society, 13 Moorgate-street,* E.C.

1896 (2) Brown, George Andrew, *Clerical, Medical & General Life Assurance Society, 1 King William-street,* E.C.

ASSOCIATES.

Those marked 2 or 3 have passed two or three of the four Examinations of the Institute.
Those marked (2) have been exempted under the Bye-laws from the Examinations for Parts I and II.

Date of becoming an Associate.

1899 [2] Brown, Harold, *Scottish Union and National Insurance Co.*, 3 *King William-street*, E.C.

1908 [2] Brown, James, B.A., *Friendly Societies Office, Young-street, Sydney, Australia.*

1886 (2) Buckley, Thomas John Wesley, 9 *St. Andrew-street, Holborn-circus*, E.C.

1882 Burke, David, F.S.S., *Royal Victoria Life Insur. Co. Montreal, Canada.*

1906 [2] Burrows, George Eastoe, *Alliance Assurance Co., Ltd., Bartholomew-lane*, E.C.

1895 [3] Butterfield, William Thos., A.C.A., 9 *Market-street, Bradford, Yorkshire.*

1905 (2) Cameron, Finlay James, F.F.A., *Friends' Provident Institution, Bradford, Yorkshire.*

1908 [2] Carpenter, Thomas B. Boyd, *Clergy Mutual Assur. Society*, 2 *&* 3 *The Sanctuary*, S.W.

1876 Carter, Eric Mackay, 33 *Waterloo-street, Birmingham.*

1906 [2] Carter, George Stanley, *Life Association of Scotland*, 18 *Bishopsgate-street-Within*, E.C.

1904 (2) Cathles, Lawrence Maclagan, F.F.A., *South Western Life Insurance Co., Dallas, Texas, U.S.A.*

1905 [2] Chubb, William, *Sun Life Assurance Company of Canada, Montreal, Canada.*

1908 [2] Clemens, Frederic Broadbent, *Alliance Assurance Co., Ltd., Bartholomew-lane*, E.C.

1898 [2] Coates, Thomas Linnaeus, *Mutual Life Insur. Co. of New York*, 16, 17 *&* 18 *Cornhill*, E.C.

1904 [2] Collier, Charles Aubrey, 6 *Old Palace-yard*, S.W.

1871 Cook, Arthur James, M.J.I., *Victoria Mutual Assur. Society, Farringdon-street*, E.C.

1899 [3] Cook, William Playfair, *Guardian Assurance Company*, 11 *Lombard-street*, E.C.

1897 [2] Coop, Charles Rowland, *United Kingdom Temperance and General Provident Institution*, 28 *High-street, Birmingham.*

1905 [2] Cooper, John James, *Sun Life Assurance Co. of Canada, Montreal, Canada.*

1891 [2] Coote, Ernest Charles, *Alliance Assurance Co., Ltd., Bartholomew-lane*, E.C.

1900 [2] Corbett, Edwin Somerville, *Australasian Temperance and General Mutual Life Assurance Society, Sydney, Australia.*

1871 Coutts, Edwin Arthur, *North British and Mercantile Insurance Company*, 12 *Low-pavement, Nottingham.*

1900 [2] Covington, Oliver Henry, *Prudential Assurance Company, Holborn-bars*, E.C.

1908 [2] Coward, Charles Ernest, B.A., *Estate Duty Office, Somerset House*, W.C.

1907 [2] Cowan, Hugh Francis, F.F.A., *Edinburgh Life Assurance Co.*, 26 *George-street, Edinburgh.*

1884 Craig, Robert Alexander, *Abstainers' and General Assur. Co., Edmund-street, Birmingham.*

1908 [3] Crump, Percy C., *Prudential Assurance Company, Holborn-bars*, E.C.

1908 [2] Dark, Thomas Arthur, *Excelsior Life Insurance Co., Toronto, Canada.*

1906 [2] Davis, Mervyn, B.A., *Connecticut General Life Insur. Co., Hartford, Conn., U.S.A.*

1908 [2] Dawson, Herbert John, B.A., *Royal Military College, Kingston, Ontario, Canada.*

1906 [2] Defries, Frederick, *Union Life Branch of the Commercial Union Assurance Co.*, 1 *&* 2 *Royal Exchange-buildings*, E.C.

1901 [2] Diamond, George Frederick, *City Mutual Life Assur. Society, Hunter-st., Sydney, Australia.*

ASSOCIATES.

Those marked 2 or 3 have passed two or three of the four Examinations of the Institute.
Those marked (2) have been exempted under the Bye-laws from the Examinations in Parts I and II.

Date of becoming an Associate.

1901 (2) Donald, Alexander Graham, M.A., F.F.A., *Scottish Provident Institution, 6 St. Andrew-square, Edinburgh.*

1881 Donaldson, John, *Australian Widows' Fund Life Assurance Society, Collins-street-west, Melbourne, Australia.*

1899 2 Dougharty, Harold, F.S.S., F.C.I.S., (AUDITOR), *London and Lancashire Life Assurance Company, 66 & 67 Cornhill,* E.C.

1902 2 Doust-Smith, Ernest Charles, *Prudential Assurance Company, Holborn-bars,* E.C.

1881 Dovey, William Roadly, F.F.A., F.A.S., 62 *Weston-park, Crouch End,* N.

1905 3 Downes, Edward George, *Legal and General Life Assur. Society,* 10 *Fleet-street,* E.C.

1906 2 Downes, Sidney Cecil, *Prudential Assurance Company, Holborn-bars,* E.C.

1870 Dowson, John, *Royal Insur. Co., Ltd., Liverpool.*

1898 2 Doyle, Arthur James, 54 *Bourke-st., Sydney, Australia.*

1908 2 Duffell, James Henry, *Royal London Mutual Insurance Society, Ltd., Finsbury Square,* E.C.

1901 2 Earle, Arthur Percival, *Columbian National Life Insur. Co.,* 176-180 *Federal-st., Boston, Mass, U.S.A.*

1868 Eaton, Henry William, *Liverpool & London & Globe Insurance Company, William-street, New York, U.S.A.*

1904 2 Ecroyd, Cuthbert W., *Friends' Provident Institution, Ocean Chambers,* 44 *Waterloo-street, Birmingham.*

1905 2 Elderton, Robert Lapidge, *National Provident Institution,* 48 *Gracechurch-street,* E.C.

1907 2 Eldridge, Ernest Edward Booth, *National General Insurance Co., King's House, King-street,* E.C.

1905 3 Ellis, Reginald George Gregson, 12 *Manson-pl., Queen's-gate,* S.W.

1872 2 Evans, William, F.F.A., F.R.S.E., 38 *Morningside-park, Edinburgh.*

1905 2 Farmer, Ernest Chattock, *London, Edinburgh & Glasgow Insurance Co., Ltd., Euston-square,* N.W.

1896 2 Featherstonehaugh, William Irwin, *Commercial Union Assurance Co.,* 24, 25 & 26 *Cornhill,* E.C.

1903 2 Ferguson, Colin C., B.A., *Great West Life Assurance Co., Winnipeg, Manitoba, Canada.*

1906 2 Fielder, William Crowhurst, *Atlas Assurance Company, Ltd.,* 92 *Cheapside,* E.C.

1905 3 File, Lorne K., B.A., F.A.S., *Imperial Life Assurance Co. of Canada, Toronto, Canada.*

1897 2 Findlay, Alexander Wynand, LL.D., *Prudential Assurance Company, Holborn-bars,* E.C.

1902 2 FitzGerald, Charles Reginald, *State Mutual Life Assur. Co., Worcester, Mass., U.S.A.*

1901 2 FitzGerald, William George, B.A., 147 *Macpherson-avenue, Toronto, Canada.*

1890 (2) Fox, Charles Edward, F.F.A., *Standard Life Assurance Co.,* 83 *King William-street,* E.C.

1886 (2) Fox, Morris, F.A.S., *New Zealand Government Life Insurance Dept., Wellington, New Zealand.*

1894 2 Fraser, Thomas John, *Australian Alliance Assurance Company, Melbourne, Australia.*

1907 2 Fulford, William John, *Prudential Assurance Company, Holborn-bars,* E.C.

1901 (2) Gaff, William Robertson, C.A., F.F.A., 54 *New Broad-street,* E.C.

1873 2 Gage, Uriah Woodard, *North British & Mercantile Insur. Co.,* 61 *Threadneedle-st.,* E.C.

ASSOCIATES.

Those marked 2 or 3 have passed two or three of the four Examinations of the Institute.
Those marked (2) have been exempted under the Bye-laws from the Examinations in Parts I and II.

Date of becoming an Associate.

1895 [2] Galwey, Charles Edmund, *New Zealand Government Life Insurance Dept., Wellington, New Zealand.*

1893 [2] Gardiner, Robert Edward, *Sun Life Assurance Society,* 63 *Threadneedle-street,* E.C.

1885 [2] Gayford, Herbert Stannard, *Northern Assurance Company, Ltd.,* 1 *Moorgate-street,* E.C.

1899 [3] Gibb, James Burnett, F.F.A., *Penn Mutual Life Insce. Co. of Philadelphia,* 923 *Chestnut-st., Philadelphia, Pa., U.S.A.*

1871 [2] Glennie, William Gordon, *Scottish Union & National Insur. Co.,* 3 *King William-street,* E.C.

1897 [2] Goggs, Frank Sidney, *Scottish Metropolitan Life Assur. Co., Ltd.,* 25 *St. Andrew-sq., Edinburgh.*

1882 Goldman, Leopold, F.S.S., *North American Life Assurance Co., North American Life Building,* 112-118 *King-street-west, Toronto, Canada.*

1904 [3] Goodman, Gilbert, *Prudential Assurance Company, Holborn-bars,* E.C.

1897 [2] Goodwyn, John, *Ocean Accident and Guarantee Corporation, Ltd.,* 131 *Pitt-st., Sydney, Australia.*

1905 [2] Gould, W. H., M.A., *Volunteer State Life Insurance Co., Chattanooga, Tennessee, U.S.A.*

1908 [(2)] Graham, George, Jr., F.F.A., *Capitol Life Insurance Company, Denver, Colorado, U.S.A.*

1908 [(2)] Granger, Charles Keith, F.F.A., *City of Glasgow Life Assurance Co.,* 30 *Renfield-street, Glasgow.*

1902 [2] Gray, Robert Alexander, B.A., 324 *Markham-street, Toronto, Canada.*

1868 Greig, John Andrew, *Sun Life Assurance Society,* 60 *Charing-cross,* S.W.

1907 [2] Gunningham, Sidney Joseph, B.Sc., *Ecclesiastical Commission, Millbank,* S.W.

1903 [2] Hall, John Bertram, A.A.S., *Dominion Life Assurance Co., Waterloo, Ontario, Canada.*

1905 [2] Hallman, M. S., F.A.S., *Mutual Life Assurance Company of Canada, Waterloo, Ontario, Canada.*

1905 [2] Hammond, Reginald, *British Equitable Assur. Co., Ltd.,* 1, 2 & 3 *Queen-street-place,* E.C.

1869 Hann, Robert George, F.A.S., *Equitable Life Assur. Soc. of the United States,* 120 *Broadway, New York.*

1894 [2] Hardcastle, Edward Edgington, M.A., F.A.S., *Union Central Life Office, Cincinnati, Ohio, U.S.A.*

1900 [2] Harding, Harry Burnard, *Commercial Union Assur. Co.,* 26 *New Bridge-street,* E.C.

1908 [2] Harnack, Frederick William, *Sceptre Life Association, Ltd.,* 40 *Finsbury-pavement,* E.C.

1907 [2] Harris, Ernest Arthur, 40 *Lambert-rd., Brixton-hill,* S.W.

1896 [3] Harris, Frederick Joseph, *Australian Mutual Provident Society, Sydney, Australia.*

1897 [2] Haycraft, William Mellhuish, *Prudential Assurance Company, Holborn-bars,* E.C.

1897 [2] Hazell, James Stanley, (AUDITOR), *National Provident Institution,* 48 *Gracechurch-street,* E.C.

1895 [2] Heness, Leonard Thomas, *Prudential Assurance Company, Holborn-bars,* E.C.

1878 Henry, Alfred, F.C.A., *Throgmorton-house, Copthall-avenue,* E.C.

1894 [2] Hollingworth, Albert Charles, *Australian Mutual Provident Society, Sydney, Australia.*

ASSOCIATES.

Those marked 2 or 3 have passed two or three of the four Examinations of the Institute.
Those marked (2) have been exempted under the Bye-laws from the Examinations in Parts I and II.

Date of becoming an Associate.

1907 [2] Holness, Archibald Stephen, *Phœnix Assurance Co., Ltd., 70 Lombard-street,* E.C.

1883 Holt, Edward Hallett, *Law Life Assurance Society, 187 Fleet-street,* E.C.

1898 [2] Howell, Chas. Edward, M.A., LL.D., *Standard Life Assurance Compy., 59 Dawson-street, Dublin.*

1899 [3] Hudson, Alfred James, *Northern Assurance Company, Ltd., 1 Moorgate-street,* E.C.

1908 [2] Humphreys, John Alfred, *National Mutual Life Assurance Society, 39 King-street, Cheapside,* E C.

1907 [2] Humphry, Edmund William, *Life Association of Scotland, 18 Bishopsgate-st.-Within,* E.C.

1875 Hunt, Richard Aldington, F.S.S., *Wesleyan & General Assur. Soc., Steelhouse-lane, Birmingham.*

1893 (2) Hunter, Arthur, F.F.A., F.A.S., F.S.S., *New York Life Insurance Co., 346 & 348 Broadway, New York, U.S.A.*

1902 [2] Hunter, Robertson G., F.A.S., *161 Devonshire-street, Boston, U.S.A.*

1887 [2] Hunter, Samuel, *66 St. Lawrence-road, Clontarf, Dublin.*

1889 (2) Jacobs, Frederick Job, *Australian Mutual Provident Society, Sydney, Australia.*

1876 [2] James, George Trevelyan, *12 Waterloo-place,* S.W.

1905 (2) Jamieson, Charles William Steele, F.F.A., *Scottish Amicable Life Assur. Society, 1 Threadneedle-st.,* E.C.

1905 [3] Jefferson, John Arthur, *Britannic Assurance Co., Ltd., Broad-st.-corner, Birmingham.*

1871 Jellicoe, George Rogers, *Eagle Insurance Company, 79 Pall-mall,* S.W.

1883 Jerman, Richard, *Commercial Union Assurance Company, Exeter.*

1908 [2] Jerrold, Allan Laman, *64 Rue de la Tour, Paris (XVIe.), France.*

1896 [2] Jobson, Alexander, *Challis House, Martin Place, Sydney, Australia.*

1894 [2] Johnston, Frederick H., F.A.S., *Prudential Life Insurance Co. of America, Newark, N.J., U.S.A.*

1903 [2] Jones, Leonard Alexander Mouat, *Commercial Union Assur. Co., 24, 25 & 26 Cornhill,* E.C.

1903 [2] Jones, Wallace Mouat, *General Reversionary & Investment Company, Limited, 26 Pall-mall,* S.W.

1898 [2] Kaufman, Henry N., A.A.S., *Phœnix Mutual Life Insurance Co., Hartford, Connecticut, U.S.A.*

1876 Kearry, Joseph, *44 Charlwood-street, Belgrave-road,* S.W.

1899 [3] Kelly, John Joseph, *Mutual Life & Citizens' Assur. Co., Ltd., Sydney, Australia.*

1897 [2] Kemp, Julian Ernest Sandford, *Eagle Insurance Company, 79 Pall-mall,* S.W.

1902 [2] Kilgour, David Errett, M.A., F.A.S., *North American Life Assurance Co., Toronto, Canada.*

1874 King, Arthur Thomas, I.S.O., *National Debt Office,* E.C.

1882 [2] King, William Alfred, *Northern Assurance Co., Ltd., 1 Moorgate-street,* E.C.

1908 (2) Kyd, James Gray, F.F.A., *Northern Assurance Co., Ltd., 1 Union-terrace, Aberdeen.*

1907 [2] Laing, John Morrison, *Mutual Life Assurance Co. of Canada, Waterloo, Ontario, Canada.*

1893 [2] Laing, William Claud, *North British and Mercantile Insurance Company, 61 Threadneedle-street,* E.C.

ASSOCIATES.

Those marked 2 or 3 have passed two or three of the four Examinations of the Institute. Those marked (2) have been exempted under the Bye-laws from the Examinations in Parts I and II.

Date of becoming an Associate.

1908 2 Laird, John Melvin, B.A., *London Life Insurance Co., London, Ontario, Canada.*

1897 2 Lane, Arthur Vere, B.A., *Legal & General Life Assurance Society, 217 West George-street, Glasgow.*

1905 3 Langstaff, James Miles, F.A.S., C.A. (Ont.), 666 *Bathurst-street, Toronto, Canada.*

1907 2 Langstaff, Milton Palmer, *Continental Life Insurance Co., Toronto, Canada.*

1905 2 Latham, Bertrand, *Australian Mutual Provident Society, Melbourne, Australia.*

1906 (2) Latta, Alexander, F.F.A., *Guardian Assurance Company, 28 King-st., Covent-garden,* W.C.

1899 2 Lawton, George Herbert, *Clerical, Medical & General Life Assurance Society, 15 St. James's-square,* S.W.

1905 2 Leigh, Samuel George, *Refuge Assurance Co., Oxford-street, Manchester.*

1879 Leitch, Alexander, *Scottish Provident Institution,* 3 *Lombard-street,* E.C.

1897 2 Le Maitre, Frank William, *Sun Life Assurance Society, 63 Threadneedle-street,* E.C.

1885 Leveaux, Arthur Michael, F.S.S., *Registry of Friendly Societies, Central Office, 28 Abingdon-street, Westminster,* S.W.

1907 2 Levey, Ralph, *Prudential Assurance Company, Holborn-bars,* E.C.

1907 2 Ley, James, *Office of the Actuary for Friendly Societies, Melbourne, Australia.*

1868 Litchfield, Edward, *c/o Messrs. Knox & Service, 41 St. Vincent-place, Glasgow.*

1876 2 Lucey, Herbert, *General Assurance Company,* 103 *Cannon-street,* E.C.

Date of becoming an Associate.

1890 (2) Lugton, Hugh, F.F.A., *North British and Mercantile Insurance Co.,* 61 *Threadneedle-street,* E.C.

1900 3 McArthur, Harry de C., *Box 282, Dunedin, New Zealand.*

1867 Macdonald, William Rae, F.F.A., *Scottish Metropolitan Life Assur. Co., Limited, 25 St. Andrew-square, Edinburgh.*

1882 3 McDougald, Alfred, *Phœnix Assurance Co., Ltd., 70 Lombard-street,* E.C.

1905 2 Macfarlane, James Allan, *Monarch Life Assurance Co., Winnipeg, Manitoba, Canada.*

1884 Mackay, Alexander, *Law Union & Crown Insur. Co.,* 126 *Chancery-lane,* W.C.

1905 3 McKechnie, James Baldwin, M.A., F.A.S., *Manufacturers Life Insurance Company, Toronto, Canada.*

1896 2 Macmillan, John Campbell, *Apartado Postal 827, Mexico D.F.*

1905 2 McPhail, Frederick Charles, *Colonial Mutual Life Assurance Soc., Ltd., Melbourne, Australia.*

1883 2 Makeham, William Reed, *Alliance Assurance Co., Ltd. (Imperial Life Assurance Fund), 47 Chancery-lane,* W.C.

1905 2 Makepeace, Francis Lucas, B.A., 229 *Norwood-rd., Herne-hill,* S.E.

1880 Manwaring, Henry, *National Debt Office,* E.C.

1896 2 Martin, Sidney George, *National Mutual Life Assoc. of Australasia, Ltd., 295 Queen-street, Brisbane, Australia.*

1897 2 Mascall, Alfred John, *Standard Life Assurance Co.,* 3 *Pall-mall East,* S.W.

1904 2 Maudling, Reginald G., *c/o. T. G. Ackland, Esq.,* 5 *& 6 Clement's Inn, Strand,* W.C.

1900 2 Maunder, George Harvard, *National Mutual Life Assur. Society, 39 King-st., Cheapside,* E.C.

ASSOCIATES.

Those marked 2 or 3 have passed two or three of the four Examinations of the Institute.
Those marked (2) have been exempted under the Bye-laws from the Examinations in Parts I and II.

Date of becoming an Associate.

1902 (2) Maxwell, Benjamin Bell, F.F.A., *Scottish Equitable Life Assur. Society*, 28 *St. Andrew-square, Edinburgh.*

1899 [2] Meade, Gerald Willoughby, *North British & Mercantile Insurance Company*, 61 *Threadneedle-street*, E.C.

1896 [2] Merfield, Percy Henry, *Law Life Assurance Society*, 187 *Fleet-street*, E.C.

1874 Miller, John W., F.S.S., *Scottish Widows' Fund and Life Assur. Soc.*, 28 *Cornhill*, E.C.

1905 [2] Monilaws, William Barrington, *Scottish Provident Institution*, 3 *Lombard-street*, E.C.

1879 Monilaws, William Macgeorge, *Scottish Provident Institution*, 3 *Lombard-street*, E.C.

1905 [2] Monkhouse, Charles Cosmo, B.A., *Clerical, Medical and General Life Assurance Society*, 15 *St. James's-square*, S.W.

1877 Moon, James, J.P., *Prudential Assurance Company*, 30 *Dale-street, Liverpool.*

1877 Moon, John, *Parkhurst, Didsbury, Manchester.*

1879 Moon, Sidney Norman Laming, *Columbian National Life Insurance Company*, 176-180 *Federal-street, Boston, Mass., U.S.A.*

1903 [2] Moore, George Cecil, *Imperial Life Assurance Co. of Canada, Toronto, Canada.*

1905 [2] Moore, George Edward, *Australian Widows' Fund Life Assurance Society, Melbourne, Australia.*

1905 [2] Moore, Gerald Leslie, A.C.A., 58 *Rosebery-road, Muswell-hill*, N.

1898 [2] Moore, Joseph Patrick, *Mutual Life & Citizens' Assurance Co., Ltd., Sydney, Australia.*

1871 [2] Moore, Roderick Mackenzie, *United Kingdom Temperance and General Provident Institution*, 196 *Strand*, W.C.

Date of becoming an Associate.

1900 [2] Nash, Alfred Charles, *Clerical, Medical and General Life Assurance Society*, 15 *St. James's-square*, S.W.

1897 [2] Newling, Sidney Wallis, B.A., *Woodleigh, South Woodford, Essex.*

1905 [2] Newnham, Ernest Whiffin, *Prudential Assurance Company, Holborn-bars*, E.C.

1907 (2) Nicholl, Charles Carlyon, B.A., F.F.A., *Royal Exchange Assce. Corp., Royal Exchange*, E.C.

1903 [2] Nicholls, Arthur William, *Australian Mutual Provident Society, Brisbane, Australia.*

1884 Nicoll, John, F.F.A., *Life Association of Scotland*, 82 *Princes-street, Edinburgh.*

1883 Orr, Lewis Potter, F.F.A., *Scottish Life Assur. Co., Ltd.*, 19 *St. Andrew-sq., Edinburgh.*

1908 [2] Osborne, William Arthur, *Guardian Assurance Company*, 11 *Lombard-street*, E.C.

1908 [2] Owen, David John, B.A., *Commercial Union Assur. Co.*, 26 *New Bridge-street*, E.C.

1906 (2) Padday, Percy King, F.F.A., *Scottish Metropolitan Life Assurance Co., Ltd.*, 8 *King-st., Cheapside*, E.C.

1895 [2] Pagden, Lionel King, *Union Life Branch of the Commercial Union Assur. Co.*, 1 & 2 *Royal Exchange-buildings*, E.C.

1864 Panton, Edward Henry, 50 *Wood-vale, Forest Hill*, S.E.

1901 [3] Papps, Percy Charles Herbert, F.A.S., *Mutual Benefit Life Insur. Co., Newark, New Jersey, U.S.A.*

1895 [2] Paradice, William Henry, *Australian Mutual Provident Society, Sydney, Australia.*

ASSOCIATES.

Those marked 2 or 3 have passed two or three of the four Examinations of the Institute. Those marked (2) have been exempted under the Bye-laws from the Examinations in Parts I and II.

Date of becoming an Associate.

1869 Park, David Francis, C.A., F.F.A., *Crédit Foncier of Mauritius, Ltd.*, 12 *King William-st.*, E.C.

1907 2 Parker, John Gowans, *Imperial Life Assurance Co. of Canada, Toronto, Canada.*

1905 2 Paton, Albert George, *London Assurance Corporation*, 7 *Royal Exchange*, E.C.

1898 (2) Pearce, Henry John, F.F.A., *Scottish Amicable Life Assurance Society*, 1 *Threadneedle-st.*, E.C.

1899 2 Peele, Thomas, *Universal Insur. Loan & Investment Co., Ltd., New Briggate, Leeds.*

1900 2 Peters, Charles Furness, *L'pool. Victoria Legal Friendly Society, St. Andrew-street*, E.C.

1907 2 Phillips, Thomas Ashley, *New York Life Insurance Co.*, 346 & 348 *Broadway, New York, U.S.A.*

1908 2 Pickup, John Richardson, *National Provident Institution*, 48 *Gracechurch-street*, E.C.

1902 2 Pigrome, George Davey, *Prudential Assurance Company, Holborn-bars*, E.C.

1899 2 Pipe, Sidney Herbert, 808 *Temple Building, Toronto, Canada.*

1906 2 Portch, Albert Garfield, F.A.S., *Canada Life Assurance Co., Toronto, Canada.*

1890 3 Powell, Alfred, *Alliance Assurance Company, Limited, Bartholomew-lane*, E.C.

1869 Pringle, James, C.A., F.F.A., 42 *Drumsheugh-gardens, Edinburgh.*

1884 Pullar, James, F.F.A., *Colonial Mutual Life Assurance Society, Melbourne, Australia.*

1881 Purves, Thomas Peter, *New York Life Insurance Company, Sydney, Australia.*

Date of becoming an Associate.

1904 (2) Rankin, John Adam, F.F.A., *Edinburgh Life Assurance Co.*, 26 *George-street, Edinburgh.*

1867 Rattray, Patrick, C.A., *Kanishee, Dunblane.*

1874 2 Ray, Charles Richard, *Commercial Union Assur. Co.*, 26 *New Bridge-street*, E.C.

1905 3 Raynes, Harold Ernest, *Legal and General Life Assurance Society*, 10 *Fleet-street*, E.C.

1885 Rea, Charles Herbert Edmund, F.R.A.S., F.S.S., *National Standard Assurance Corporation*, 149 *Leadenhall-st.*, E.C.

1898 2 Reid, Edward E., B.A., *London Life Insurance Co., London, Ontario, Canada.*

1907 2 Reynolds, William Daniel, *Prudential Assurance Company, Holborn-bars*, E.C.

1901 2 Rhodes, Francis, B.A., *Royal Insurance Company, Ltd., Liverpool.*

1887 (2) Richardson, Josephus Hargreaves, F.F.A., F.A.S., *New Zealand Government Life Insurance Department, Wellington, New Zealand.*

1879 Roberts, Thomas B., *Australian Alliance Assurance Company, Collins-street, Melbourne, Australia.*

1908 2 Robertson, Bernard, *Prudential Assurance Company, Holborn-bars*, E.C.

1904 3 Robertson, Frederick William, F.F.A., *Yorkshire Insurance Co., Ltd., York.*

1904 3 Robertson, James Leask, F.F.A., *Edinburgh Life Assurance Co.*, 26 *George-street, Edinburgh.*

1878 Robertson, William, F.F.A., 29 *Stafford-street, Edinburgh.*

1876 Robinson, Andrew, *Sunningdale-park, Sunningdale, Berks.*

ASSOCIATES.

Those marked 2 or 3 have passed two or three of the four Examinations of the Institute.
Those marked (2) have been exempted under the Bye-laws from the Examinations in Parts I and II.

Date of becoming an Associate.

1885 Ronald, Thomas Robert, *Law Guarantee and Trust Soc., Ltd.*, 49 *Chancery-lane*, W.C.

1904 [2] Rudd, Alfred James, *Australian Widows' Fund Life Assurance Society, Grenfell-street, Adelaide, South Australia.*

1897 [2] Ryley, Edmund, *Prudential Assurance Company, Holborn-bars*, E.C.

1896 [2] Sanderson, Frank, M.A., F.F.A., F.A.S., F.S.S., *Canada Life Assurance Company, Toronto, Canada.*

1904 [2] Sare, Thomas Henry, *Commercial Union Assur. Co.*, 24, 25 & 26 *Cornhill*, E.C.

1905 [3] Savery, Robert S. B., *Gresham Life Assurance Society, Ltd., Giselastrasse* 1, *Vienna.*

1884 Schooling, John Holt, *Fotheringay-house, Montpelier-row, Twickenham.*

1899 [2] Schouten, Pieter, *Verzekering Maatschappij, "Arnhem," Stations-plein*, 17, *Arnhem, Holland.*

1906 (2) Scott, Albert George, (AUDITOR), *English and Scottish Law Life Assur. Association*, 12 *Waterloo-place*, S.W.

1873 Scott, Ernest Willem, F.A.S., *Algemeene Maatschappij van Levensverzekering en Lijfrente, Damrak*, 74, *Amsterdam.*

1904 [3] Searle, Arthur Joseph, *English & Scottish Law Life Assurance Association*, 12 *Waterloo-place*, S.W.

1861 [2] Searle, Thomas John, *Mansion - house - chambers, Bucklersbury*, E.C.

1900 [2] Searls, Edwin Richard, *Northern Assurance Company, Ltd.*, 1 *Moorgate-street*, E.C.

1900 [2] Sharpe, Edgar Cecil Engledue, *London Life Association, Ltd.*, 81 *King William-street*, E.C.

1907 (2) Shearer, Gilbert Edward, F.F.A., *Scottish Provident Institution*, 3 *Lombard-street*, E.C.

1894 [3] Sheppard, Herbert Norman, B.A., F.A.S., *Home Life Insurance Company*, 256 *Broadway, New York, U.S.A.*

1897 [2] Shimmell, James Edward, F.S.S., *United Provident Assurance Co., Ltd.*, 96 *Oxford-rd., Manchester.*

1896 [2] Shlager, Joseph, *Equitable Life Assurance Society of the United States, Mansion-house-chambers, Adderley-street, Cape Town, South Africa.*

1903 [2] Shovelton, Sydney Taverner, M.A., *Polruan, Stanhope-av., Finchley*, N.

1905 [2] Shute, Oxenham Bent, *National Provincial Bank of England*, 53 *Baker-street*, W.

1864 Smith, Howard Samuel, F.F.A., F.C.A., F.S.S., *Bank-chambers*, 11 *Waterloo-street, Birmingham.*

1898 [2] Smith, Robert Parker, *Royal Insurance Company, Ltd., Liverpool.*

1907 (2) Smith, John Tasker, F.F.A., *Britannic Assurance, Co., Ltd.*, 178 *Avenue-parade, Accrington.*

1906 [2] Smither, Herbert Buxton, *University Life Assurance Soc.*, 25 *Pall-mall*, S.W.

1905 [2] Somerville, Walter Harold, *Mutual Life Assur. Co. of Canada, Waterloo, Ontario, Canada.*

1871 Spencer, Robert James, F.S.S., 75 *King's-road, Southsea.*

1868 Spens, William George, 44 *Albany-street, Edinburgh.*

1866 Stark, William Emery, *Chapel-walks, Manchester.*

1878 Stevenson, Charles, 9 *Albert-square, Manchester.*

1880 Stock, Edward James, *National Mutual Life Assoc. of Australasia, Melbourne, Australia.*

ASSOCIATES.

Those marked 2 or 3 have passed two or three of the four Examinations of the Institute.
Those marked (2) have been exempted under the Bye-laws from the Examinations in Parts I and II.

Date of becoming an Associate.

1906 2 Story, Cyril Lionel William Steane, *Norwich Union Life Insurance Society, 71 & 72 King William-street,* E.C.

1905 2 Strong, Allan Wilmot, *Sun Life Assurance Co. of Canada, Montreal, Canada.*

1896 2 Stuckey, Jos. James, M.A., *Salisbury Chambers, 49a King William-street, Adelaide, South Australia.*

1905 2 Stuckey, Reginald Robert, *Australian Mutual Provident Society, Adelaide, South Australia.*

1905 2 Sturt, Herbert Rothsay, *Phœnix Assurance Co., Ltd., 70 Lombard-street,* E.C.

1904 (2) Tatlock, John, M.A., F.R.A.S., F.A.S., 376 *West End Avenue, New York, U.S.A.*

1893 2 Taylor, Arthur, *Guardian Assurance Company, 28 King-street, Covent-garden,* W.C.

1908 2 Thompson, John Henry Reginald, *Prudential Assurance Company, Holborn-bars,* E.C.

1906 3 Thompson, John Spencer, *Mutual Life Insurance Co. of New York, New York, U.S.A.*

1906 2 Thomson, Frederick Robert T., *Law Union & Crown Insur. Co.*, 126 *Chancery-lane,* W.C.

1904 (2) Thomson, John Walter, F.F.A., *Scottish Life Assur. Co., 19 St. Andrew-square, Edinburgh.*

1883 2 Titmuss, Walter George, *Alliance Assur. Co., Ltd. (Provident Life Fund), 50 Regent-street,* W.

1905 2 Touzel, Philip Duncan, *Australian Mutual Provident Society, Melbourne, Australia.*

1905 3 Townley, Ebenezer William, *National Mutual Life Assurance Soc., 39 King-st., Cheapside,* E.C.

Date of becoming an Associate.

1902 3 Traversi, Antonio Thomas, *Friendly Societies' Department, Wellington, New Zealand.*

1883 Tregaskis, George Alfred, *Hand-in-Hand Insurance Office, 26 New Bridge-street,* E.C.

1894 2 Trenerry, Charles Farley, B.A. D.Sc., *University of London, South Kensington,* S.W.

1905 2 Tully, Arthur Patrick Thomas, *Gresham Life Assurance Society, Ltd., Sharia Suleiman Pacha, Cairo, Egypt.*

1891 2 Turnbull, A. D. Lindsay, C.A., F.F.A., F.C.I.S., 4 *Bishopsgate-street-Within,* E.C.

1907 3 Turner, Sidney, B.A., 20 *Minster-road, Cricklewood,* N.W.

1907 2 Underwood, Reginald Edward, *Clerical, Medical and General Life Assurance Society, 15 St. James's-square,* S.W.

1884 Vian, William Collett, *Railway Passengers' Assurance Company, 64 Cornhill,* E.C.

1884 Vincent, Frederick James, F.S.S., *London, Edinburgh & Glasgow Assurance Co., Ltd., Euston-square,* N.W.

1899 2 Vokins, George Alfred, *Prudential Assurance Company, Holborn-bars,* E.C.

1908 2 Walker, Dwight A., *Equitable Life Assurance Society of the United States, 120 Broadway, New York, U.S.A.*

1879 Wall, Walter George, 34 *Kingsland-road,* B'head.

1905 2 Wansbrough, Thomas Percival, *English and Scottish Law Life Assurance Assoc., and British Law Fire Insurance Co., 37 Queen Victoria-street,* E.C.

1906 (2) Wardrop, James Charles, *Life Association of Scotland, 18 Bishopsgate-st.-Within,* E.C.

ASSOCIATES.

Those marked 2 or 3 have passed two or three of the four Examinations of the Institute.
Those marked (2) have been exempted under the Bye-laws from the Examinations in Parts I and II.

Date of becoming an Associate.

1907 [2] Warren, Cyril Ferdinand,
Prudential Assurance Company, Holborn-bars, E.C.

1903 [2] Watherston, Charles F., B.A.,
War Office, S.W.

1883 [2] Watson, John Robertson,
British Law Fire Insurance Co., 105 *West George-st.*, *Glasgow.*

1908 [2] Watt, Arthur William,
Sun Life Assurance Co. of Canada, Montreal, Canada.

1894 [2] Watt, George,
Royal Insurance Company, Ltd., Liverpool.

1900 (2) Watt, James, W.S., F.F.A.,
24 *Rothesay-terrace, Edinburgh.*

1902 [2] Weatherill, Charles,
Scottish Office, S.W.

1894 (2) Weeks, Rufus Wells, F.A.S.,
New York Life Insurance Co., 346 & 348 *Broadway, New York, U.S.A.*

1898 [3] Whigham, Charles Frederick, F.F.A., C.A.,
Messrs. Moncreiff & Horsbrugh, 46 *Castle-street, Edinburgh.*

1908 [2] White, Osborn Denyer,
Commercial Union Assurance Co., 24, 25 & 26 *Cornhill*, E.C.

1897 [2] Wickens, Charles H.,
Commonwealth Bureau of Census and Statistics, Melbourne, Victoria, Australia.

1896 [2] Wilkinson, Edward Berkeley,
24 *Maxilla-gardens, N. Kensington*, W.

1903 [2] Wilkinson, William Magnay,
Mutual Life & Citizens' Assurance Co., Ltd., Sydney, Australia.

1904 [2] Williams, Frederick Alfred, F.S.S., A.A.S.,
"La Nacional" Compania de Seguros sobre la Vida, Apartado 1420, *Mexico D.F.*

1904 [2] Wilson, Arthur Benjamin,
Australian Mutual Provident Soc., Wellington, New Zealand.

1900 [2] Wilson, George,
Standard Life Assurance Company, 3 *George-st., Edinburgh.*

1870 [2] Wilson, Henry Edward,
Northern Assurance Co., Ltd., 1 *Moorgate-street*, E.C.

1873 [2] Windett, Charles,
Legal & General Life Assurance Society, 10 *Fleet-street*, E.C.

1905 [2] Winstanley, Charles William,
North British & Mercantile Insurance Co., 61 *Threadneedle-street*, E.C.

1903 [2] Wood, William Archibald Porter, B.A.,
Canada Life Assurance Co., Toronto, Canada.

1883 Woodhouse, Lister, A.C.A., F.S.S.,
City Comptroller, Westminster City-hall, Charing Cross-road, W.C.

1877 [2] Woods, Arthur Biddle,
Rock Life Assurance Company, 15 *New Bridge-street*, E.C.

1866 Woods, Bernard,
Metropolitan Life Assurance Society, 13 *Moorgate-street*, E.C.

1879 Wornum, Thornton Selden,
Rock Life Assurance Company, 15 *New Bridge-street*, E.C.

1903 [2] Worth, Bertram Oliver,
Clerical, Medical & General Life Assurance Society, 15 *St. James's-square*, S.W.

1873 Young, Alexander Hunter,
60 *Market-street, Melbourne, Australia.*

STUDENTS.

Those marked 1, 2, or 3 have passed one, two, or three of the four Examinations of the Institute. Those marked (1) have been exempted under the Bye-laws from the Examination in Part I.

Date of becoming a Student.

1892 [1] Aaron, David Hyam, *Sun Life Assurance Society*, 63 *Threadneedle-street*, E.C.

1903 [1] Aeum, Wilfred Harry, 15 *Lordship-lane, Wood Green*, N.

1905 [1] Agutter, William John, *Prudential Assurance Company, Holborn-bars*, E.C.

1908 [1] Aldridge, Wilfred Henry, 24 *St. John's-road, East Ham*, E.

1906 [2] Allen, Arthur Ormiston, M.A., B.Sc., 2 *Norwood-grove, Leeds*.

1907 [1] Allen, Sidney, A.C.A., 1 *Walbrook*, E.C.

1904 [1] Allison, Sinclair E., A.A.S., *New York Life Insurance Co.*, 346 & 348 *Broadway, New York, U.S.A.*

1908 [1] Armon, Thomas, *Pearl Life Assurance Company, London-bridge*, E.C.

1904 [1] Armstrong, Charles Henry, *Imperial Life Assurance Co. of Canada, Toronto, Canada.*

1886 Arnold, Thomas, Jr., *British Equitable Assur. Co., Ltd.*, 1, 2 & 3 *Queen-street-place*, E.C.

1902 [1] Askwith, Thomas Nowell, *London Life Association, Ltd.*, 81 *King William-street*, E.C.

1905 [1] Atkins, Francis Cuthbert, *Prudential Assurance Company, Holborn-bars*, E.C.

1904 [1] Ayscough, Ivan, *Equity and Law Life Assurance Soc.*, 18 *Lincoln's-inn-fields*, W.C.

1899 [1] Baber, Walter Crosbie, A.A.S., *Royal Victoria Life Insur. Co. of Canada, Montreal, Canada.*

1903 [1] Baggs, Henry Ernest, *English and Scottish Law Life Assurance Association*, 12 *Waterloo-place*, S.W.

1907 [1] Bailey, Frank Arthur, *General Reversionary & Investment Co., Ltd.*, 26 *Pall Mall*, S.W.

1907 [1] Baker, Sydney Harry, *Sun Life Assurance Society*, 63 *Threadneedle-street*, E.C.

1907 (1) Bannatyne, Arthur Gordon, B.A., 19 *Westbourne-square*, W.

1899 [2] Barnett, Isaac, *North British and Mercantile Insurance Co.*, 61 *Threadneedle-street*, E.C.

1896 [1] Barry, David, *Acting Actuary for Friendly Societies, Melbourne, Australia.*

1908 [1] Bazell, Harry, *Legal Insurance Co., Thanet-house*, 231 & 232 *Strand*, W.C. (*opposite the Law Courts*).

1907 (1) Beaven, Cecil Livingstone, B.A., *Royal Military Academy, Woolwich*, S.E.

1907 [1] Beeston, Harold Lewis, *Prudential Assurance Company, Holborn-bars*, E.C.

1908 [1] Bell, William Francis, *Universal Insurance Loan and Investment Co., Ltd., New Briggate, Leeds.*

1898 [1] Bennell, Samuel Thomas, 25 *Meath-road, Ilford.*

1906 [1] Bennett, Henry Gordon, *Australian Mutual Provident Society, Melbourne, Australia.*

1902 [1] Biden, Norman Frederick, 11 *Lower Wycombe-rd., Neutral Bay, Sydney, Australia.*

1895 [1] Bigby, Robert Frederick Mitchell, *General Assurance Company*, 103 *Cannon-street*, E.C.

STUDENTS.

Those marked 1, 2, or 3 have passed one, two, or three of the four Examinations of the Institute. Those marked (1) have been exempted under the Bye-laws from the Examination in Part I.

Date of becoming a Student.	
1900	[1] Bingeman, Milton H., *Great West Life Assurance Co., Winnipeg, Manitoba, Canada.*
1903	[1] Binney, Charles Eardley Wilmot, *Royal Exchange Assurance Corporation, Royal Exchange,* E.C.
1905	[1] Blackadar, E. Gordon, B.A., A.A.S., *Canada Life Assurance Co., Toronto, Canada.*
1908	[1] Blake, Leslie Sarjant, 10 *Onslow-road, Richmond, Surrey.*
1887	Blossom, James, 24 *Grange-crescent, Sheffield.*
1892	[1] Boddy, Henry Mitchell, F.S.S., *Manufacturers Life Insurance Co., Cape Town, South Africa.*
1906	[1] Bolt, Jan Cornelis, *Middelland-straat* 102, *Rotterdam, Holland.*
1902	[1] Bowerman, Judah Philip, *Southern States Mutual Life Insur. Co., Charleston, Kanawha County, West Virginia, U.S.A.*
1897	[1] Bowles, Francis Marsh, *Pearl Life Assurance Company, London-bridge,* E.C.
1891	[1] Boyd, Henry Norris, F.F.A., *City of Glasgow Life Assurance Company,* 21 *St. Andrew-square, Edinburgh.*
1905	[2] Bradshaw, Frank Law, *Law Guarantee and Trust Soc., Ltd.,* 49 *Chancery-lane,* W.C.
1899	[1] Brady, John Francis, *Mutual Life & Citizens' Assurance Co., Ltd., Sydney, Australia.*
1906	[1] Breeds, Arthur Heywood, *Prudential Assurance Company, Holborn-bars,* E.C.
1908	[1] Brenton, William Percy, *Radlett, Herts.*
1894	[1] Brough, Frank, *Federal Life Assurance Company, Hamilton, Ontario, Canada.*
1906	[1] Brown, B. G. H., *Royal Exchange Assurance Corporation, Royal Exchange,* E.C.
1906	[1] Brown, Peter Gordon, *Ecclesiastical Commission, Millbank,* S.W.
1891	[1] Brown, William Heron, *Gresham Life Assur. Soc., Ltd., St. Mildred's-house, Poultry,* E.C.
1907	[1] Bullwinkle, Leonard Albert, *c/o* T. G. Ackland, Esq., 5 & 6 *Clement's Inn, Strand,* W.C.
1905	[2] Burrows, Victor Albert, *Sun Life Assurance Society,* 63 *Threadneedle-street,* E.C.
1908	[1] Cammack, Edmund Ernest, A.A.S., *P.O. Box* 1114, *Johannesburg, South Africa.*
1904	[1] Canter, Harold, *National Provident Institution,* 48 *Gracechurch-street,* E.C.
1903	[1] Capon, Frank Christopher, *Prudential Assurance Company, Holborn-bars,* E.C.
1902	[1] Capon, Geoffrey William, *Norwich Union Life Insurance Society, Norwich.*
1907	[1] Carey, Norman Lewis, *Clerical, Medical and General Life Assurance Society* 15 *St. James's-square,* S.W.
1908	[1] Carpmael, Charlton, *The Limes, Ingatestone, Essex.*
1907	[1] Casebow, Percival Clear, *General Assurance Company,* 103 *Cannon-street,* E.C.
1907	[1] Cashman, Thomas, *North British and Mercantile Insurance Co.,* 61 *Threadneedle-street,* E.C.
1900	[1] Chambers, John Joseph, *New Bank, Pool, near Leeds.*
1907	[1] Chandler, Francis Philip, *London Assurance Corporation,* 7 *Royal Exchange,* E.C.
1902	[1] Chandler, Frederick Joseph, *Eagle Insurance Co.,* 79 *Pall-mall,* S.W.
1907	[1] Charles, Ashley Hyde, 13 *South Moulton-street,* W.

STUDENTS.

Those marked 1, 2, or 3 have passed one, two, or three of the four Examinations of the Institute. Those marked (1) have been exempted under the Bye-laws from the Examination in Part I

Date of becoming a Student.	
1908	[1] Chase, Harold Philip, *Liverpool and London and Globe Insurance Company*, 1 *Cornhill*, E.C.
1897	[1] Cherry, John Arnold, *Chamber of London, Guildhall*, E.C. (*Reinstated*, 1905.)
1903	[1] Cheshire, Harold Frank, 9 *Wellington-place, Hastings.*
1905	[1] Clarke, Herbert George, *Australian Widows' Fund Life Assurance Society, Melbourne, Australia.*
1907	(1) Clarke, Harold Thomas, B.A., *Clerical, Medical and General Life Assurance Society*, 15 *St. James's-square*, S.W.
1897	[1] Clinton, George, *Prudential Assurance Company, Holborn-bars*, E.C.
1902	[3] Clinton, Louis Ernest, *Alliance Assurance Company, Ltd., Bartholomew-lane*, E.C.
1902	[2] Coates, Frederick George, *Commercial Union Assur. Co.*, 26 *New Bridge-street*, E.C.
1901	[1] Cockerton, John Leonard, *Pioneer Life Assurance Co., Ltd.*, 67 *Dale-street, Liverpool.*
1895	[1] Cogar, William Edward, *New York Life Insurance Co., Trafalgar-square*, W.C.
1899	[1] Collins, Patrick A., *Mutual Life & Citizens' Assur. Co., Ltd., Sydney, Australia.*
1902	[1] Collins, William Ernest, *Fell. Inst. Accts. S.A.*, 49*a King William-street, Adelaide, South Australia.*
1896	[1] Cook, Henry Milton, *Standard Life Assurance Co., Dalhousie-sq., Calcutta, India.*
1900	[1] Cooper, Bernard Hugh, *Prudential Assurance Company, Holborn-bars*, E.C.

Date becoming a Student.	
1906	[1] Cooper, John Lewis, *Liverpool and London and Globe Insur. Co.*, 1 *Dale-st., Liverpool.*
1899	[1] Cotterill, William Ernest, *Mutual Life Association of Australasia, Ltd., Sydney, Australia.* (*Reinstated*, 1905.)
1903	[1] Cotton, Arthur Sparkes, *Scottish Office*, S.W.
1905	[1] Coutts, Kenneth Vawdrey, *Clergy Mutual Assurance Soc.*, 2 & 3 *The Sanctuary*, S.W.
1904	[1] Cowdy, Henry Leslie, *Scottish Union & National Insur. Co.*, 3 *King William-street*, E.C.
1894	Cox, Edward William, *Canada Life Assurance Co., Toronto, Canada.*
1907	[1] Cox, Harry, *London, Edinburgh and Glasgow Assurance Company, Euston-square*, N.W.
1894	Cox, Herbert Coplin, *Canada Life Assurance Co., Toronto, Canada.*
1905	[1] Cox, Stanley Nelson, *Prudential Assurance Company, Holborn-bars*, E.C.
1907	[1] Crang, James Simon, 77 *Blenheim-road, Walthamstow*, E.
1887	[1] Cross, Henry John, 240 *Trinity-rd., Wandsworth-common*, S.W.
1907	[1] Currie, James Thorn, *Australian Mutual Provident Society, Melbourne, Australia.*
1907	[1] Curtis, Augustus Thomas George, *London, Edinburgh and Glasgow Assurance Company, Euston-square*, N.W.
1904	[1] Cushing, Robertson Macaulay, *Sun Life Assurance Company of Canada, Montreal, Canada.*
1904	[1] Dalrymple, Alfred George, *Canada Life Assurance Company, Toronto, Canada.*

STUDENTS.

Those marked 1, 2, or 3 have passed one, two, or three of the four Examinations of the Institute. Those marked (1) have been exempted under the Bye-laws from the Examination in Part I.

Date of becoming a Student.	
1897	[1] Dalton, John, *London Life Association, Ltd., 81 King William-street,* E.C.
1889	[1] Davies, Hugh Myddleton, *Royal Insur. Co., Ltd., Liverpool.*
1900	[1] Davies, William Allison, *Assistant Borough Treasurer, Town Hall, Birkenhead.*
1906	[1] Davis, Archibald Percy, *Sydenham-road, Marrickville, Sydney, Australia.*
1891	[1] Dawson, Frank Aubrey, *Ecclesiastical Insurance Office, Limited, 11 Norfolk-street, Strand,* W.C.
1902	[2] Deck, James Gilbert, *National Provident Institution, 48 Gracechurch-street,* E.C.
1902	[1] Denmark, Robert John, *Norwich Union Life Insurance Society, Norwich.*
1901	[1] Dent, Ernest Edward, *London and Lancashire Life Assurance Company, 66 & 67 Cornhill,* E.C.
1905	[2] Derrick, Victor Percival Augustine, *Alliance Assurance Co., Ltd., Bartholomew-lane,* E.C.
1896	[1] de Ville, Francis, *Clergy Pensions Institution, 11 Norfolk-street, Strand,* W.C.
1906	[1] Dobbie, John Albert, *Provincial Normal School, Ottawa, Canada.*
1890	[1] Docker, Leslie, *North British and Mercantile Insurance Co., 61 Threadneedle-street,* E.C.
1906	[2] Doucet, Gerald Danby, *Rock Life Assurance Company, 15 New Bridge-street,* E.C.
1906	[1] Doyle, Joseph Patrick, *Mutual Life & Citizens' Assurance Co., Ltd., Sydney, Australia.*
1904	[1] Drake, Charles Clifford Hall, *Prudential Assurance Company, Holborn-bars,* E.C.
1905	[1] Dulley, John Francis, *Prudential Assurance Company, Holborn-bars,* E.C.
1908	[1] Eames, George Stanley, *Prudential Assurance Company, Holborn-bars,* E.C.
1905	[1] Eastcott, William Merrill, *Sun Life Assur. Co. of Canada, Montreal, Canada.*
1892	[1] Edwards, Edward Samuel, M.A., *Australian Mutual Provident Society, Sydney, Australia.*
1905	[1] Edwards, Herbert Alfred, 28 *Plashet-rd., Upton Manor,* E.
1905	[2] Edwards, Herbert Horace, 24 *Chetwynd-road, Highgate-road,* N.W.
1902	[1] Edwards, Thomas Baker, *Comptroller's Dept., London County Council, Spring-gardens,* S.W.
1892	[1] Eedy, Arthur Malcolm, *Mutual Life & Citizens' Assurance Co., Ltd., Sydney, Australia.*
1901	[1] Egleton, Harold Edward, *Prudential Assurance Company, Holborn-bars,* E.C.
1907	[1] Emery, Charles Grover, *Australian Mutual Provident Society, Melbourne, Australia.*
1906	[2] Emery, Walter Sydney, *Australian Widows' Fund Life Assurance Society, Melbourne, Australia.*
1906	[1] Emmerson, Walter Hector Ross, *London and Lancashire Life Assurance Company, 66 & 67 Cornhill,* E.C.
1907	(1) Epps, George Selby Washington, B.A., 55 *Queen Anne-street,* W.
1908	[1] Evans, Cyril Ormond, *Australian Mutual Provident Society, Melbourne, Australia.*

STUDENTS.

Those marked 1, 2, or 3 have passed one, two, or three of the four Examinations of the Institute. Those marked (1) have been exempted under the Bye-laws from the Examination in Part I.

Date of becoming a Student.

1892 [1] Farrell, John, *Mutual Life & Citizens' Assurance Co., Ltd., Sydney, Australia.*

1906 [1] Fender, William Martin, *Australian Mutual Provident Society, Melbourne, Australia.*

1907 [1] Fidler, William Edward, *Standard Life Assurance Co., 83 King William-street,* E.C.

1908 [1] Fielder, Tom Lionel, *Standard Life Assurance Co., 83 King William-street,* E.C.

1908 [1] Finlayson, G. D., *Government Insurance Department, Ottawa, Canada.*

1904 [3] Fippard, Richard Clift, *Prudential Assurance Company, Holborn-bars,* E.C.

1901 [1] Fisher, John William, B.A., A.A.S., *Crown Life Insurance Co., Toronto, Canada.*

1896 [1] Fisk, George William Victor, *Prudential Assurance Company, Holborn-bars,* E.C.

1904 [1] Fletcher, Andrew W. A. C., *Standard Life Assurance Co., 3 George-street, Edinburgh.*

1905 [1] Flynn, Benedict Devine, F.A.S., *Travelers Insurance Company, Hartford, Conn., U.S.A.*

1905 [1] Forbes, James, *Great West Life Assurance Co., Winnipeg, Manitoba, Canada.*

1906 [1] Foster, Joseph, *33 Westwood-street, Moss Side, Manchester.*

1906 [1] Foster, Wilfred Justus, *Prudential Assurance Company, Holborn-bars,* E.C.

1901 [1] Franklin, Herbert Dare, *Australian Mutual Provident Society, Melbourne, Australia.*

1908 [1] Frisby, Herbert Rowell, *Alliance Assurance Co., Ltd., Bartholomew-lane,* E.C.

1906 [1] Frost, Charles Frederick, *Prudential Assurance Company, Holborn-bars,* E.C.

1900 [1] Garner, James, 138 *Chiswick-high-road,* W.

1908 [1] Gawler, Oswald, *Australian Widows' Fund Life Assurance Society, Melbourne, Australia.*

1901 (1) Gerrish, Frank Wilfred, B.A., *Minerva-villa, Albert-rd.-south, Buckhurst-hill, Essex.*

1901 [1] Glassford, David Murray, *Mutual Life & Citizen's Assur. Co., Ltd., Sydney, Australia.*

1893 Glasson, George Cornish, *Economic Life Assurance Soc., 4 St. Stephen's-chbrs., Baldwin-street, Bristol.*

1902 [1] Gleave, Charles Sheldon, *Refuge Assurance Co., Oxford-street, Manchester.*

1899 [1] Goddard, Egbert, *Coombe Brook, Coombe, Surrey. (Reinstated,* 1908).

1894 [1] Golding, Arthur, 40 *Allerton-road, Stoke Newington,* N.

1905 [1] Goodall, Ernest Victor, *Commercial Union Assur. Co., 24, 25 & 26 Cornhill,* E.C.

1903 [1] Gopp, John Ive, 14 *Church-hill-road, Walthamstow,* E.

1886 Gover, Frederick Field, F.S.S., 10 *Lee-park, Blackheath,* S.E.

1907 [1] Grant, Frederick John, *Edinburgh Life Assurance Co., 12 King-street, Manchester.*

1907 [1] Green, John Spencer, *British Widows' Assurance Co., 1 Old-street,* E.C.

1907 [1] Green, William James, *Australian Metropolitan Life Assurance Company, Sydney, Australia.*

STUDENTS.

Those marked 1, 2, or 3 have passed one, two, or three of the four Examinations of the Institute. Those marked (1) have been exempted under the Bye-laws from the Examination in Part I.

Date of becoming a Student.

1886 Greening, Herbert Joseph, *Abstainers' & General Insur. Co., Edmund-street, Birmingham.*

1907 [1] Guthrie, Isles Hampden, *Scottish Provident Institution, 3 Lombard-street,* E.C.

1901 [1] Hall, Arthur F., *North American Life Assurance Co., Toronto, Canada.*

1902 [3] Hallett, William Sebastian, M.A., *Equitable Life Assurance Soc., Mansion-house-street,* E.C.

1901 [1] Hamilton, George Powell, *North American Life Assurance Co., McLean Block,* 10 *Douglas-street, Guelph, Ontario, Canada.*

1905 [1] Hamley, Ernest Fountain, *Australasian Temperance and General Mutual Life Assurance Society, Melbourne, Australia.*

1902 [1] Hammant, Francis Clive, *Prudential Assurance Company, Holborn-bars,* E.C.

1905 [1] Hammond, Harry Pierson, B.A., A.A.S., *Insurance Dept. of the State of Connecticut, Hartford, Conn., U.S.A.*

1892 Hancock, Arthur Tom, *Clerical, Medical & General Life Assurance Society,* 15 *St. James's-square,* S.W.

1903 [3] Hancock, Edwin J., *Sun Life Assurance Society,* 63 *Threadneedle-street,* E.C.

1906 [1] Handford, John James William, *Scottish Office,* S.W.

1907 [1] Harding, Denys Aubrey, *Railway Passengers' Assurance Company,* 64 *Cornhill,* E.C.

1902 [1] Hardy, Reginald Herbert, 32 *Highfield-street, Leicester.*

1903 [1] Harley, Brian, *National Provident Institution,* 48 *Gracechurch-street,* E.C.

1901 [1] Harpell, James John, *Policyholders' Association,* 14 *Classic-avenue, Toronto, Canada.* (*Reinstated,* 1908.)

1901 [1] Harper, Henry, 83 *Waverley-road, Small Heath, Birmingham.*

1905 [1] Harrington, Eustace Woods, *Northern Assurance Company, Ltd.,* 1 *Moorgate-street,* E.C.

1889 [1] Harris, Henry, *Friends' Provident Institution,* 17 *Gracechurch-street,* E.C.

1908 [1] Harris, Sydney Ewart, 41 *Houston-road, Forest-hill,* S.E.

1908 [1] Harrison, Alfred Lowther, *Northern Assurance Company, Ltd.,* 1 *Moorgate-street,* E.C.

1905 [1] Harrison, Launcelot, *Mutual Life & Citizens' Assurance Co., Ltd., Sydney, Australia.*

1908 [1] Harrison, Wilfrid, *Eastburn, Hexham-on-Tyne.*

1907 [1] Harvey, Percy Norman, *Atlas Assurance Company, Ltd.,* 92 *Cheapside,* E.C.

1896 Haskins, George Frederick, A.C.A., 18 *Walbrook,* E.C.

1894 [1] Hatten, David Leslie, *Standard Life Assurance Co.,* 83 *King William-street,* E.C.

1908 [1] Hawes, Ernest Edward, *North British & Mercantile Insurance Co.,* 61 *Threadneedle-street,* E.C.

1907 [2] Henry, Alfred, 5 *Branstone-road, Kew-gardens.*

1905 (1) Heron, David, M.A., *Viewbank, New Scone, Perth, N.B.*

1906 [1] Hilbery, Reginald William, *Clerical, Medical & General Life Assurance Society,* 15 *St. James's-square,* S.W.

1896 [2] Hines, Walter Robert, *Norwich Union Life Insurance Society, Norwich.*

STUDENTS.

Those marked 1, 2, or 3 have passed one, two, or three of the four Examinations of the Institute. Those marked (1) have been exempted under the Bye-laws from the Examination in Part I.

Date of becoming a Student.	
1902	[1] Hodge, Cecil Wilfred, *Star Life Assurance Society, 32 Moorgate-street,* E.C.
1896	[1] Hogg, Charles, *Ecclesiastical Commission, Millbank,* S.W.
1907	[1] Holgate, Benjamin, *Refuge Assurance Co., Oxford-street, Manchester.*
1908	[1] Holgate, Thomas, *Refuge Assurance Co., Oxford-street, Manchester.*
1905	[1] Homan, Russell Charles, *3 The Terrace, Camden-sq.,* N.W.
1898	[2] Hooper, George Duncan, *Prudential Assurance Company, Holborn-bars,* E.C.
1895	[2] Horn, Ernest Frederick, *Equity & Law Life Assur. Soc., 18 Lincoln's Inn Fields,* W.C.
1902	[1] Houston, Charles Cornelius, *Metropolitan Asylums Board, Victoria-embankment,* E.C.
1901	[1] Howell, Archibald Rennie, B.A., *Royal Insurance Company, Ltd., Montreal, Canada.*
1907	[1] Howell, Percy, *46 Temple Flats, Old Bethnal-green-road,* N.E.
1907	[1] Hudson, Claude Hamilton, *Australasian Temperance and General Mutual Life Assurance Society, Box 505, G.P.O., Wellington, New Zealand.*
1898	Hughes, Arthur J., *China Mutual Life Insur. Co., Shanghai, China.*
1902	[1] Hughes, Charles, A.A.S., *State of New York Insur. Dept., 11 Broadway, New York, U.S.A.*
1902	[1] Hugill, Herbert, *"Briarfield," Keighley.*
1908	[1] Hull, Edgar Penrose, *67 Beauval-road, Dulwich,* S.E.
1904	[1] Humphreys, Harry Lewis, *Phœnix Assurance Company, Limited, 70 Lombard-street,* E.C.
1891	Hunt, Arthur Leonard, *Wesleyan and General Assur. Soc., 101 Finsbury-pavement,* E.C.
1906	[1] Hustwitt, William Edmund, *Prudential Assurance Company, Holborn-bars,* E.C.
1907	[1] Hutchings, Leonard Hollinworth, *Phœnix Assurance Company, Limited, 70 Lombard-street,* E.C.
1902	(1) Jackson, Charles William, M.A., *Greenboro Life Insurance Co., Greenboro, N.C., U.S.A.*
1902	[2] Jackson, Herbert Moore, *Australian Mutual Provident Society, Sydney, Australia.*
1890	[2] Jackson, Samuel, F.F.A., *Scottish Widows' Fund and Life Assurance Society, Liverpool.*
1907	[1] James, Reginald William, *151 Praed-st., Paddington,* W.
1896	[1] Jepps, John Blacklee, *Star Life Assurance Society, 32 Moorgate-street,* E.C.
1905	[1] Johns, Arthur Humphreys, *Colonial Mutual Life Assurance Society, Melbourne, Australia.*
1904	[1] Johnson, Frank Henry, *Law Life Assurance Society, 187 Fleet-street,* E.C.
1898	[1] Johnston, Arthur Edward, *3 Cumnor-road, Sutton.*
1908	[1] Johnston, Henry Frieborn, *University of Toronto, Toronto, Canada.*
1908	[1] Johnstone, William Darnley, *Phœnix Assurance Co., Ltd., 70 Lombard-street,* E.C.
1903	[2] Jones, Ernest Stephens, *National Debt Office,* E.C.
1908	[1] Jones, Ernest Washington, *30 Bertram-road, Hendon,* N.W.
1896	[1] Jones, Richard Foxley, A.C.I.S., *Refuge Assurance Co., Oxford-street, Manchester.*
1907	[1] Keable, Henry Batten, *Prudential Assurance Company, Holborn-bars,* E.C.

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Those marked 1, 2, or 3 have passed one, two, or three of the four Examinations of the Institute. Those marked (1) have been exempted under the Bye-laws from the Examination in Part I.

Date of becoming a Student.	
1907	[1] Keachie, Morton M., *Canada Life Assurance Co., Toronto, Canada.*
1906	[1] Kearns, William Norman, *Royal Insur. Co., Ltd., Liverpool.*
1905	[1] Keevil, Norman Alexander Clement, *Blagdon, Park-road, Watford, Herts.*
1905	[1] Kenchington, Frank, *North British and Mercantile Insurance Company*, 61 *Threadneedle-street*, E.C.
1906	[1] Kidd, Alan Bruce, *North British and Mercantile Insurance Company*, 1 *Dawson-street, Dublin.*
1906	[1] Kime, Virgil Morrison, *University of Michigan, Ann Arbor, Michigan, U.S.A.*
1905	[1] King, Albert Edward, *Provident Clerks' and General Mutual Life Assur. Association*, 27 *&* 29 *Moorgate-street*, E.C.
1894	[2] Kingsbury, James William, *Australian Mutual Provident Society, South Sea House*, 37 *Threadneedle-street*, E.C.
1903	[1] Kirsopp, Frederick, *Liverpool Victoria Legal Friendly Society, St. Andrew-street*, E.C.
1895	[1] Knight, Alfred Murray, *Bank-house, Chapel-st., Devonport.*
1908	[1] Kubota, Takajiro, 82*b* *Portsdown-rd., Maida-vale*, W.
1905	Lafford, Harry George, *Legal and General Life Assur. Society*, 10 *Fleet-street*, E.C.
1902	Lang, Frederick John, *Royal London Mutual Insur. Soc., Ltd., Finsbury-square*, E.C.
1907	[1] Latham, Fergus Norman Wilkinson, 3 *Wyresdale-rd., Bolton, Lancs.*
1907	[1] Ledger, Robert John, *Grove-lodge, Grove-rd., Epsom.*
1904	[1] Lee, Frank Sidney, *Ocean Accident and Guarantee Corporation*, 36-44 *Moorgate-street*, E.C.
1906	[1] Leigh, Walter Lewis, 58 *Lichfield-road, Bow*, E.
1894	Leonard, Maurice, *Frith Hill Cottage, Great Missenden, Bucks.*
1906	[1] Le Rossignol, Leonard F., *English and Scottish Law Life Assur. Association*, 12 *Waterloo-place*, S.W.
1908	[1] Lever, Ernest H., *Prudential Assurance Company, Holborn-bars*, E.C.
1906	[1] Lewis, David Hugh, *Refuge Assurance Company, Oxford-street, Manchester.*
1904	[2] Lewty, Francis Arthur, *Equity and Law Life Ass. Soc.*, 18 *Lincoln's-inn-fields*, W.C.
1889	[1] Lighton, Harold John, *Law Union & Crown Insurance Co.*, 126 *Chancery-lane*, W.C.
1904	[1] Linzmeyer, Louis, F.A.S., *Manhattan Life Insurance Co.*, 64-70 *Broadway, New York, U.S.A.*
1908	[1] Lithgow, James Hector Farncombe, *Trinity College School, Port Hope, Ontario, Canada.*
1895	[1] Littell, Lewis Lloyd, *Standard Life Assurance Co.*, 83 *King William-street*, E.C.
1904	[1] Littlefair, James Taylor, *Refuge Assurance Co., Oxford-street, Manchester.*

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Those marked 1, 2, or 3 have passed one, two, or three of the four Examinations of the Institute.
Those marked (1) have been exempted under the Bye-laws from the Examination in Part I.

Date of becoming a Student.	
1906	[1] Lohan, John Joseph. *National Mutual Life Association of Australasia, Melbourne, Australia.*
1906	[1] Lolley, Clement Francis, *Universal Insurance Loan and Investment Co., Ltd., New Briggate, Leeds.*
1908	[1] Long, Walter Meriton, *Sun Life Assurance Society, 63 Threadneedle-street,* E.C.
1890	Love, Robert, *Ecclesiastical Insur. Office, Ltd., 11 Norfolk-street, Strand,* W.C.
1906	[1] McCall, Robert, *58 Lichfield-road, Bow,* E.
1888	[1] McConway, James Robert, *15 Henthorn-road, New Ferry, Cheshire.*
1906	[1] McCulloch, James Arthur, *Ecclesiastical Commission, Millbank,* S.W.
1903	[1] Macdonald, Charles Strange, M.A., *Confederation Life Association, Toronto, Canada.*
1907	[1] Mace, Douglas William, *Marine & General Mutual Life Assur. Society, 14 Leadenhall-street,* E.C.
1904	[1] Macfarlane, Edmond Scales, *Manufacturers Life Insurance Company, 23 Water Street, Yokohama, Japan.*
1907	[1] Macleod, John, *Yorkshire Insurance Co., Ltd., 2 Bank-buildings, Princes-st.,* E.C.
1908	[1] McMullen, William Albert, *32 Woodberry-grove, Finsbury-park,* N.
1907	[1] Macorquodale, F. D., *Manufacturers Life Ins. Co., Toronto, Canada.*
1908	(1) MacTavish, Archie Neil, B.A., *Government Insurance Department, Ottawa, Canada.*
1903	[1] Manly, George William, B.A., *Clerical, Medical & General Life Assurance Society, 15 St. James's-square,* S.W.

Date of becoming a Student	
1908	[1] Mann, Frederick Christmas, *"Glaisdale," Woodstock-road, Golder's-green,* N.W.
1904	[1] Marlin, James Harold, *Ocean Accident and Guarantee Corporation, 36-44 Moorgate-street,* E.C.
1905	[1] Marshall, Arthur William, *Consolidated Assur. Co., Ltd., Temple-bar-house, 23 Fleet-street,* E.C.
1908	[1] Marshall, Cecil George, *Prudential Assurance Company, Holborn-bars,* E.C.
1905	[1] Marshall, John Edwin, *Prudential Assurance Company, 47 Earl-street, Coventry.*
1903	[1] Martin, Frederick Charles, *Prudential Assurance Company, Holborn-bars,* E.C.
1906	[1] Martin, William Alexander, *National Mutual Life Association of Australasia, Melbourne, Australia.*
1904	[1] Matheson, Donald, *Imperial Life Assurance Co. of Canada, Toronto, Canada.*
1906	[1] Maunder, Henry Ernest, *69 Tyrwhitt-road, St. John's,* S.E.
1895	[1] Mayhew, Percy Craske, *Wannock, The Drive, Coulsdon, Surrey.*
1908	[1] Meakin, William Lionel, *National General Ins. Co., King's House, King Street, Cheapside,* E.C.
1890	[1] Meikle, Henry George Watson, F.F.A., *Oriental Government Security Life Assurance Co., Limited, Bombay, India.*
1908	[1] Meredith, Charles Edward Buchanan, *Commercial Union Assurance Co., 24, 25 & 26 Cornhill,* E.C.
1892	[1] Meyers, Henry Wilson, *National Mutual Life Association of Australasia, 5 Cheapside,* E.C.

STUDENTS.

Those marked 1, 2, or 3 have passed one, two, or three of the four Examinations of the Institute
Those marked (1) have been exempted under the Bye-laws from the Examination in Part I.

Date of becoming a Student.	
1907	[1] Miller, Arthur Axel, *32 Kyverdale-road,* N.
1899	[2] Minns, Ernest Edwin, *Norwich Union Life Insurance Society, Norwich.*
1907	[1] Mol, Wilhelmus Johannes Bartholomeus, *Zuid-Holl Maatschappij van Verzekering op het leven en Cij Ziekte, Heerengracht* 12, *The Hague, Holland.*
1907	[1] Monilaws, Stanley Hope, *Scottish Provident Institution,* 3 *Lombard-street,* E.C.
1902	[1] Moore, Hubert Fred, *London Assurance Corporation,* 7 *Royal Exchange,* E.C.
1904	[1] Moran, Albert James, *Heathfield, St. Nicholas-road, Upper Tooting,* S.W.
1902	[1] Morton, Francis, *Commercial Union Assur. Co.,* 24, 25 & 26 *Cornhill,* E.C.
1907	[1] Morton, Frederick William, *British Widows' Assurance Co.,* 1 *Old-street,* E.C.
1902	[1] Muckle, Charles P., *Union Life Assur. Co., Toronto, Canada.* (*Reinstated,* 1907.)
1904	[1] Mulcahy, Francis Benedict, *Mutual Life & Citizens' Assur. Co., Ltd., Sydney, Australia.*
1903	[1] Myers, Harry Duxbury, A.S.A.A., *Burlington-chambers, North-st., Keighley.*
1906	[1] Naismith, Keith Errol, *Refuge Assurance Co., Oxford-street, Manchester.*
1907	[1] Nash, Kenneth Oscar, *London Life Association, Ltd.,* 81 *King William-street,* E.C.
1906	[1] Nathan, Eric Burnett, *Norwich Union Life Insur. Soc.,* 168 *Whitechapel-road,* E.
1906	[1] Needell, Brian, *Alliance Ass.Co.,Ltd.* (*Provident Life Fund*), 50 *Regent-street,* W.
1903	[1] Neill, William Adam Hoyes, *Scottish Widows' Fund & Life Assur. Soc.,* 28 *Cornhill,* E.C.
1907	[1] Newland, Edward Albert, *Consolidated Assurance Company,* 20 *Birchin-lane,* E.C.
1902	(1) O'Connor, William, M.A., M.D., *Mutual Life Insurance Co. of New York, Toronto, Canada.*
1892	[1] O'Reilly, Anthony James, *Government Insurance Department, Ottawa, Canada.*
1897	[1] Osborn, Nathaniel Banner Francis, 11 *Bruce-grove, Tottenham,* N.
1893	[1] Owen, Edgar Theodore, F.S.S., *Registrar of Friendly Societies and Government Actuary, Perth, West Australia.*
1901	[1] Papworth, Frederick William, A.S.A.A., 24 *Modena-road, Hove, Sussex.*
1904	[1] Parker, Walter Montgomery, *Prudential Assurance Company, Holborn-bars,* E.C.
1895	[1] Pascoe, William Yeoman Bennett, *Prudential Assurance Company, Holborn-bars,* E.C.
1897	[1] Patrick, James, *Audit Office, Town-hall, Birkenhead.* (*Reinstated,* 1905.)
1906	[1] Patrick, Walter S., *Sydnell, Greenhill-road, Moseley, Birmingham.*
1907	[1] Pattison, George Benjamin, *Manufacturers Life Assurance Company, Toronto, Canada.*
1896	[2] Penny, Charles Augustus, *Prudential Assurance Company, Holborn-bars,* E.C.
1905	[2] Perry, Sidney James, *Northern Assurance Company, Ltd.,* 1 *Moorgate-st.,* E.C.
1906	[2] Peter, James Calthorpe, *London and Lancashire Life Assurance Company,* 66 & 67 *Cornhill,* E.C.
1901	[1] Petter, Herbert, *Britannic Assurance Co., Ltd., Broad-st.-corner, Birmingham.*

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Those marked 1, 2, or 3 have passed one, two, or three of the four Examinations of the Institute. Those marked (1) have been exempted under the Bye-laws from the Examination in Part I.

Date of becoming a Student.		
1908	1	Phillips, Ernest William, *Roe Green, Hatfield, Herts.*
1904	1	Phillips, Walter, A.C.I.S., A.S.A.A., *c/o Messrs. Arthur Guinness, Son & Co., Ltd., St. James's-gate, Dublin.*
1908	1	Pickworth, Edgar Broughton, 65 *Hindes-road, Harrow.*
1907	1	Pocock, Horace George Grooby, *Alliance Assurance Company, Ltd.,* 47 *Chancery-lane,* W.C.
1905	1	Pollard, Edward Cecil, *Law Fidelity and General Ins. Corporation,* 59–62 *Chancery-lane,* W.C. (*Reinstated,* 1908).
1898		Poort, Willem Anthonie, Phil. Nat. Doct., *Algemeene Friesche Levensverzekerings Maatschappij Leeuwarden, Leeuwarden, Holland.*
1903	(1)	Porter, Frank, M.A., *Mansfield House, Canning Town,* E.
1907	(1)	Preston, John Edwin, B.A., *Yorkshire Insur. Co., Ltd., York.*
1906	1	Priestman, Basil, 23 *Highfield-road, Edgbaston, Birmingham.*
1908	1	Proddow, William Norman, *Pearl Life Assurance Company, London-bridge,* E.C.
1907	1	Prout, Herbert John, *Prudential Assurance Company, Holborn-bars,* E.C.
1908	1	Purry, William Baldwin, *Prudential Assurance Company, Holborn-bars,* E.C.
1901	1	Ramsay, Cecil Byron, *Mutual Life Insur. Co. of New York,* 16, 17 *&* 18 *Cornhill,* E.C.
1905	2	Reeve, Gilfrid Montier, *Guardian Assurance Company,* 11 *Lombard-street,* E.C.
1898	1	Reynell, Guy Courtenay, *Eagle Insurance Company,* 79 *Pall-mall,* S.W.
1904	1	Reyner, Harry Fane, *Refuge Assurance Company, Oxford-street, Manchester.*
1894	1	Richards, Gilbert P. A., *Oak Cottage, Bulwer-road, New Barnet.*
1908	1	Richardson, George Rowley, Jr., "*Dawunkuttie,*" *Adelaide-road, Surbiton.*
1904	1	Ridgway, Wulfric, *Sun Life Assurance Society,* 63 *Threadneedle-street,* E.C.
1902	1	Robertson, Aubrey Charles, *London Assurance Corporation,* 7 *Royal Exchange,* E.C.
1908	1	Robinson, Archie, *Standard Life Assurance Co.,* 83 *King William-street,* E.C.
1903	1	Robinson, Ernest William, *Standard Life Association, Ltd.,* 28 *Elizabeth-street, Sydney, Australia.*
1896	1	Robinson, Frederick Charles, *Royal Exchange Assur. Corporation, Royal Exchange,* E.C.
1893	1	Roll, Frederick James, *Pearl Life Assurance Company, London-bridge,* E.C.
1893	1	Roodenburch, Bartholomeus Adrianus, *Verzekeringsbank Victoria,* 126 *Keizersgracht, Amsterdam.*
1895	1	Ross, Christopher Watson, *c/o Messrs. M. Moss & Co., Flinders-lane, Melbourne, Australia.*
1901	1	Rountree, Arthur FitzGerald, *The Rectory, Stretford, near Manchester.*
1905	1	Rowland, Stanley Jackson, *Clerical, Medical and General Life Assurance Society,* 15 *St. James's-square,* S.W.
1895		Rowley, James Edward, A.C.A., 7 *Waterloo-street, Birmingham.*
1906	1	Ruddle, Francis, *Consolidated Assur. Co., Temple-bar-house,* 23 *Fleet-street,* E.C.
1907	1	Rushton, Thomas Arthur, 85 *Fleet-street,* E.C.

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Those marked 1, 2, or 3 have passed one, two, or three of the four Examinations of the Institute.
Those marked (1) have been exempted under the Bye-laws from the Examination in Part I.

Date of becoming a Student.

1899 [1] Rutter, Edward Valentine, *Phœnix Assurance Company, Limited*, 70 *Lombard-street*, E.C.

1904 [1] Sadler, Percy, *Prudential Assurance Company, Holborn-bars*, E.C.

1894 Salter, George Ferry, M.S., F.A.S., *Prudential Insurance Co. of America, Newark, N.J., U.S.A.*

1907 [1] Sanders, Bertram G. T., *Standard Life Assurance Co.*, 83 *King William-street*, E.C.

1905 [1] Schooling, Terence Holt, *London and Lancashire Fire Insurance Company*, 76 *King William-street*, E.C.

1897 [1] Scott, Alexander Lewis, *Australian Mutual Provident Society, Melbourne, Australia.*

1907 (1) Sen, Jogesh Chandra, M.A., B.L., 15 *Sitaram Ghose's-st., Calcutta, India.*

1905 [1] Sharp, Harold Gregory, *Friends' Provident Institution, Bradford.*

1908 (1) Shepherdson, Herbert Jepson, B.A., B.Sc., *Royal Insurance Company, Limited, Liverpool.*

1907 [1] Shine, John Nugent, *Prudential Assurance Company, Holborn-bars*, E.C.

1907 [1] Shinmi, Shoji, *Manufacturers Life Insurance Company, Toronto, Canada.*

1908 [1] Shurrock, Christopher William, *London, Edinburgh & Glasgow Assurance Co., Euston-square*, N.W.

1906 [2] Simmonds, Reginald Claud, *Alliance Assurance Co., Ltd., Bartholomew-lane*, E.C.

1892 [1] Simpson, William Murray, *North British and Mercantile Insurance Company*, 61 *Threadneedle-street*, E.C.

1905 [1] Sinclair, Coll Claude, B.A., *Great West Life Assurance Co., Winnipeg, Manitoba, Canada.*

1891 [1] Sindall, Alfred John, *London and Lancashire Life Assur. Co.*, 66 & 67 *Cornhill*, E.C.

1907 [1] Singer, Charles Paul, 84 *Grove-park-ter., Chiswick*, W.

1888 [2] Slimon, William James, F.F.A., 10 *Mayfield-terrace, Edinburgh.*

1905 [1] Sloan, Joseph James Eastwood, *Royal Insur. Co., Ltd., Liverpool.*

1907 [1] Smith, Frederick James, *Refuge Assurance Company, Oxford-street, Manchester.*

1907 [1] Smith, Reginald Thomas, 7 *Ashmount-rd., Hornsey-lane*, N.

1907 [1] Smith, Sydney Arthur George, *Fire Offices Committee, Appliances Department*, 3 *York-street, Manchester.*

1903 [1] Smith, William, B.A., *Standard Life Association, Ltd.*, 28 *Elizabeth-street, Sydney, Australia.*

1903 [2] Sneddon, Andrew William, *Australian Mutual Provident Society, South Sea House*, 37 *Threadneedle-st.*, E.C.

1907 [1] Spiegel, Ellis William Ralfs, 70 *Coniston-rd., Muswell-hill*, N.

1904 [1] Spring, Stanley Harold, *London Guarantee and Accident Company, Orient House, New Broad-street*, E.C.

1901 [1] Steffensen, Johan F., *Forsikringsraadet*, 1 *Christiansgade, Copenhagen.*

1906 [1] Stephenson, Herbert Roy, *Manufacturers Life Insurance Company, Toronto, Canada.*

1886 [2] Stirling, James, F.F.A., *Law Union and Crown Insur. Co.*, 126 *Chancery-lane*, W.C.

STUDENTS.

Those marked 1, 2, or 3 have passed one, two, or three of the four Examinations of the Institute.
Those marked (1) have been exempted under the Bye-laws from the Examination in Part I.

Date of becoming a Student.

1908 [1] Stiver, Claud Frank,
Manufacturers Life Insurance Co., Toronto, Canada.

1903 [1] Stocks, Joseph,
Norwich Union Life Insurance Society, 10 *Southwark st.,* S.E.

1888 [1] Stott, Walter,
Royal Insur. Co., Ltd., Liverpool.

1893 [1] Streeter, Revd. Theodore Edward,
St. John's College, Winnipeg, Manitoba, Canada.

1904 [2] Strong, Gordon Gilbert,
Sun Life Assurance Society, 63 *Threadneedle-street,* E.C.

1902 [1] Strong, William Boughton,
Prudential Assurance Company, Holborn-bars, E.C.

1904 [1] Stuart, Arthur William,
National Provident Institution, 48 *Gracechurch-street,* E.C.

1907 [1] Sturgeon, Robert W.,
Royal Insur. Co., Ltd., Liverpool.

1904 [1] Sturt, Arthur James,
Phœnix Assurance Company, Limited, 70 *Lombard-street,* E.C.

1907 [1] Stutfield, Martin,
Consolidated Assur. Company, Temple-bar-hse., 23 *Fleet-st.,* E.C.

1906 [1] Sutton, Maurice William,
93 *Grove-end-road, St. John's-wood,* N.W.

1904 [1] Tamkin, Walter Ellis,
Prudential Assurance Company, Holborn-bars, E.C.

1907 [1] Tayler, Harold H.,
London, Edinburgh & Glasgow Assurance Company, Euston-square, N.W.

1907 [1] Taylor, Frederick George,
Prudential Assurance Company, Holborn-bars, E.C.

1907 [1] Taylor, Frederick Rowland Stallard,
Commercial Union Assurance Co., 24, 25 & 26 *Cornhill,* E.C.

1907 [1] Taylor, Herbert George Brooks,
London, Edinburgh and Glasgow Assurance Company, Euston-square, N.W.

Date of becoming a Student.

1908 [1] Thomlinson, Harry,
Yorkshire Insur. Co., Ltd., York.

1905 [1] Thompson, Joseph William,
Norwich Union Life Insurance Society, Norwich.

1904 [1] Thompson, William George,
Commercial Union Assurance Co., 24, 25 & 26 *Cornhill,* E.C.

1906 [1] Thomson, Ernest H. W.,
London and Lancashire Fire Insrce. Co., Dale-st., Liverpool.

1902 [1] Thwaites, Frederick George,
Norwich Union Life Insurance Society, Norwich.

1907 [1] Tomlinson, Benjamin,
London, Edinburgh and Glasgow Assurance Company, Euston-square, N.W.

1897 [1] Townshend, Edward Villiers,
Scottish Widows' Fund and Life Assurance Society, 28 *Baldwin-street, Bristol.*

1907 [1] Trembath, Allan Edward,
Prudential Assurance Company, Holborn-bars, E.C.

1907 [1] Turner, John Gilmour,
Australian Mutual Provident Society, Melbourne, Australia.

1905 [1] Tutill, Hubert Linzee,
English & Scottish Law Life Assur. Association, 12 *Waterloo-place,* S.W.

1908 [1] Tyler, David Kinnear,
Australian Mutual Provident Society, Melbourne, Australia.

1891 Tyler, Edgar Alfred, F.S.A.A., F.C.I.S., F.S.S.,
9 *Old Jewry-chambers, Bank,* E.C.

1906 [1] Tyler, Victor William,
Alliance Assurance Company, Ltd., Bartholomew-lane, E.C.

1906 [2] Vaughan, Hubert,
Mutual Life & Citizens' Assurance Co., Ltd., Sydney, Australia.

STUDENTS.

Those marked 1, 2, or 3, have passed one, two, or three of the four Examinations of the Institute. Those marked (1) have been exempted under the Bye-laws from the Examination in Part I.

Date of becoming a Student.

1907 [1] Vineberg, Harris Elias, *Provident Savings Life Assur. Society*, 35 *Nassau-street, New York, U.S.A.*

1908 [1] Walters, Arthur Hawksley, A.C.A., 15 *George-street, Mansion-house*, E.C.

1906 [1] Warhurst, James, *Alliance Assur. Co., Ltd. (Provident Life Fund)*, 68 *Fountain-street, Manchester.*

1908 [1] Warner, Arthur Joseph, 2 *Shardcroft-avenue, Herne-hill*, S.E.

1904 [1] Warnock-Fielden, Francis Hugh, *Prudential Assurance Company, Holborn-bars*, E.C.

1903 [1] Watson, Alexander R. D., 89 *Queen-street, Auckland, New Zealand.*

1906 [1] Watson, Andrew Daniel, *Government Insurance Department, Ottawa, Canada.*

1906 [1] Watson, John A., *Law Guarantee and Trust Soc., Ltd.*, 49 *Chancery-lane*, W.C.

1906 (1) Webb, Herbert Anthony, M.A., *Trinity College, Cambridge.*

1898 [1] Webb, Lloyd, *Commercial Union Assur. Co.*, 24, 25 & 26 *Cornhill*, E.C.

1907 [1] Welch, Leslie Gordon, *Phœnix Assurance Company, Limited*, 70 *Lombard-street*, E.C.

1905 [1] Wellington, Frank, *Australian Mutual Provident Society, Melbourne, Australia.*

1893 [1] Welman, Arthur Joseph, *Legal & General Life Assurance Soc.*, 15 *Tithebarn-st., Liverpool.*

1905 [1] Welsh, Willis, *Prudential Assurance Company, Holborn-bars*, E.C.

Date of becoming a Student.

1904 [1] Wenn, Albert Edward *Prudential Assurance Company, Holborn-bars*, E.C.

1907 [1] Wenyon, Herbert John, *Sun Life Assurance Society*, 63 *Threadneedle-street*, E.C.

1902 [1] White, Wilfred Clare, *Federal Life Assurance Co., Hamilton, Ontario, Canada.*

1907 [1] Wilkinson, Cecil S., *Alliance Assurance Co., Ltd.*, 3 *Mincing-lane*, E.C.

1908 [1] Williams, Caradoc, *Alliance Assurance Co., Ltd., St. James's-street*, S.W.

1886 [1] Williams, David, F.S.S., 181 *Queen Victoria-street*, E.C.

1905 [1] Williams, Hugh Corden, *c/o Professor Williams, M.A., The University, Hobart, Tasmania.*

1895 [1] Williams, Henry Samuel Walter, *Liverpool & London & Globe Insur. Co., Melbourne, Australia.*

1900 (1) Williams, Lewis, B.A., *Commercial Union Assur. Co.*, 24, 25 & 26 *Cornhill*, E.C.

1907 [1] Williams, Thomas Walter, *London, Edinburgh and Glasgow Assurance Company, Euston-square*, N.W.

1906 [1] Williamson, Wallace White, *Norwich Union Life Insurance Society, Norwich.*

1907 [1] Wilson, William Clement, *Alliance Assurance Co., Ltd., Bartholomew-lane*, E.C.

1901 [2] Wilton, Herbert George, *Norwich Union Life Insurance Society, Norwich.*

1894 [1] Windett, Sydney V., *Eagle Ins. Co.*, 79 *Pall-mall*, S.W.

STUDENTS.

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Those marked (1) have been exempted under the Bye-laws from the Examination in Part I.

Date of becoming a Student.

1905 [1] Wisdom, Sidney Herbert, *Estate Duty Office, Somerset-house,* W.C.

1903 [2] Wolfenden, Edgar Sydney, *Australian Mutual Provident Society, Sydney, Australia.*

1908 [1] Wolfenden, Hugh Herbert, *The Grange, Sidcup, Kent.*

1895 [1] Wood, David James, *Commercial Union Assurance Co., 24, 25 & 26 Cornhill,* E.C.

1901 [1] Wood, Roland Stuart, *Liverpool & London & Globe Insurance Co., 1 Cornhill,* E.C.

1906 [1] Woodall, Edward Arthur, *National Mutual Life Assurance Soc., 39 King-st., Cheapside,* E.C.

1902 [1] Woodhouse, David Alfred, *Refuge Assurance Co., Oxford-street, Manchester.*

1905 [1] Woodward, James Howard, 137 *Gillott-road, Birmingham.*

Date of becoming a Student.

1900 [1] Woolston, Paul Livingston, B.S., 50 *Maine-avenue, Ocean-grove, New Jersey, U.S.A.*

1907 [1] Wright, Alexander William, *Dereholm, Oakleigh-road, New Southgate.*

1886 Yeatman, Alexander Alfred, 2 *Coleman-street,* E.C.

1895 [1] Yeldham, William James, *Prudential Assurance Company, Holborn-bars,* E.C.

1906 [1] Yeomans, Ernest Charles, *Australian Widows' Fund Life Assur. Soc, Melbourne, Australia.*

1903 [1] Young, Henry J., *Prudential Assurance Company, Holborn-bars,* E.C.

1897 [1] Younger, R. H., *Liverpool & London & Globe Insur. Co., 1 Dale-st., Liverpool.*

1904 [1] Zumstein, Herbert Christian, *Australian Mutual Provident Society, Melbourne, Australia.*

CORRESPONDING MEMBERS.

Belgium.

BRUSSELS.

M. George H. Adan, *Directeur-Général de la Royale Belge Compagnie Anonyme d'Assurances à Forfait sur la vie et contre les Accidents; Rue Royale (Coin Impasse du Parc).*

M. Am. Bégault, F.A.S., *Vice-Président de l'Association des Actuaires Belges. Membre Correspondant de l'Institut des Actuaires Français; 72 Rue du Lac.*

M. Léon Hamoir, *Directeur-Général de la Cie. des Propriétaires Réunis; 16 Rue de Loxum.*

M. Fl. Hankar, *Directeur - Général de la Caisse Générale d'Epargne et de Retraite; 50 Rue du Fossé-aux-Loups.*

M. Omer Lepreux, F.A.S., *Directeur-Général Honoraire de la Caisse Générale d'Epargne et de Retraite. Directeur de la Banque Nationale de Belgique. Président du Comité Permanent des Congrès Internationaux d'Actuaires. Président de l'Association des Actuaires Belges. Membre Correspondant de l'Institut des Actuaires Français. Membre de la Commission Permanente des Sociétés Mutualistes et de la Commission Centrale de Statistique.*

Belgium—*contd.*

BRUSSELS—*contd.*

Membre de la Commission des Accidents du Travail. Ancien Capitaine du Génie Chargé de Cours à l'Ecole Militaire; 39 Rue de Turin.

Denmark.

COPENHAGEN.

Prof. Thorvald Nicolai Thiele, Dr. Phil., *Stockholmsgade 25.*

France.

PARIS.

M. Paul Guieysse, F.A.S., *Président de l'Institut des Actuaires Français. Député du Morbihan; 2 Rue Dante.*

M. Léon Marie, F.A.S., *Sous-Directeur, Le Phénix Compagnie d'Assurances sur la vie. Membre Correspondant de l'Association des Actuaires Belges. Membre Agrégé de l'Institut des Actuaires Français; 28 Rue de Châteaudun (IXe).*

M. Albert Quiquet, F.A.S., *Actuaire, La Nationale Compagnie d'Assurances sur la vie. Membre Correspondant de l'Association des Actuaires Belges. Secrétaire de l'Institut des Actuaires Français; 17 Rue Laffitte (IXe).*

M. Alfred Thomereau, *8 Rue le Peletier.*

Germany.

GOTHA.

Dr. Johannes Karup, *Actuary of the Gotha Life Assurance Company.*

Dr. Karl Samwer, *Manager of the Gotha Life Assurance Company, Ohrdruferstrasse, 4.*

Hungary.

BUDAPEST.

M. Julius Altenburger, *Consulting Actuary. Corresponding Member of the "Institut des Actuaires Français," and of the "Association des Actuaires Belges." Member of the "Versicherungs-Wissenschaftliche Vereinigung of Vienna"; vii. Hajtsár ut, 20.*

Italy.

FLORENCE.

M. Guido Toja, *Ingénieur. Directeur Général des Compagnies Italiennes d'Assurances sur la Vie et Contre L'Incendie "La Fondiaria." Membre du Comité permanent des Congrès Internationaux d'Actuaires. Membre Correspondant de l'Association des Actuaires Belges. Membre de la "Commission Reale per la Valutazione dei disavanzi delle Casse Pensioni perroviarie Italiane." Membre du "Circolo Matematico di Palermo." Ancien Professeur chargé du Cours de "Matematica Attuariale à l'Università Bocconi di Milano." Piazza Vittorio Emanuele.*

Russia.

ST. PETERSBURG.

M. Serge de Savitch, *Professor of Mathematics in the Electro-Technical Institute and in the University of St. Petersburg. Manager of the Tariffs' Service of the United Russian Insurance Companies; Nikolaewskaia, 35.*

Spain.

MADRID.

Dr. José Maluquer y Salvador, *Insurance Counsellor (Ex-Actuary at the Home Office). Member of the Institute of Social Reforms; calle de Campomanes, 10.*

Sweden.

STOCKHOLM.

M. Anders Lindstedt, Dr. Phil., *Professor of Mathematics and Director of the Polytechnicum. Counsellor of the Regency for Insurance Questions. Member of the Royal Swedish Academy of Science; etc.*

Switzerland.

ZURICH.

Dr. Gottfried Schaertlin, *Manager of the "Schweizerischen Lebensversicherungs-und Rentenanstalt."*

United States.

NEW YORK.

Mr. David Parks Fackler, M.A., F.A.S., *Past-President of the Actuarial Soc. of America* (1891-93). *Consulting Actuary; 35 Nassau-street.*

*** *It is requested that any inaccuracy in the foregoing list may be pointed out to the* ASSISTANT SECRETARY.

JOURNAL

OF THE

INSTITUTE OF ACTUARIES

Opening Address by the President, GEORGE FRANCIS HARDY, Esq.

[Delivered 30 November 1908.]

SINCE you did me the great honour of electing me as President, for the time being, of this Institute, I have many times looked forward to my task this evening with considerable diffidence and some trepidation, and I hope you will not suspect me of a merely conventional sentiment when I express the wish that the duty of addressing you had fallen into abler hands.

It is impossible to stand here and address you this evening without thinking of those who have preceded me in this Chair, and have so ably upheld the dignity of their office and of the Institute. I feel it an honour to succeed them, and a matter for sincere congratulation that so many of our past Presidents are still with us, giving us the benefit of their matured experience and taking an active share in the work of the Institute, not excepting, if I may be allowed to say so, the oldest of them all in years, whose signature to a certain parchment—now, I fear, a somewhat ancient document—gave me the right to consider myself an actuary.

Next to those personal feelings which, perhaps, naturally arise first in one's mind, one's strongest sentiment on an occasion such as this is that of pride in the Institute of which he is a member, in its long and honourable record of enterprise and achievement, and in its present abundant vitality and promise for the future.

It is just over sixty years since the historic meeting at the *Guardian* office practically launched the Institute upon its career.

The story of its founding and of the negotiations leading up to that far-reaching event has been told more than once in the pages of our *Journal.* Its advent was received with anything but cordiality by a section of the actuarial world of that day, and union was not achieved until a generation later when we obtained our charter, but as we read the account of the negotiations for the establishment of the Institute we recognize an eagerness and enthusiasm in its founders, a determination not to be deterred by indifference or opposition, which were a happy augury for its future success.

The objects set before the Institute by those gentlemen are as much an evidence of their wisdom and foresight as is the existence of the Institute itself of their enterprise and energy, and although they are not unfamiliar to the readers of the *Journal,* I make no apology for referring to them now. I will only select four which practically cover the ground. These are:—

(1) The development and improvement of the mathematical theories on which the practice of life assurance is based.

(2) The collection and arrangement of data connected with the subjects of the duration of life, health, and finance.

(3) The improvement and diffusion of knowledge and the establishment of correct principles relating to subjects involving money considerations and the doctrine of probability.

(4) The elevation of the attainments and status of the members of the profession.

It may be claimed on behalf of the Institute that throughout its long career it has steadily and with marked success pursued the aims thus set before it by its founders. It is not necessary, in this room, to examine this claim in any detail, the less so that this has been done by more than one of our past Presidents with more knowledge and skill than I could hope to bring to the task, and I shall only make a brief and very general reference to some of the work that has been done.

The development of actuarial science on its mathematical and practical side has been accomplished by the original work of our individual members, and by that constant interchange of ideas and criticism rendered possible by the papers and discussions at our meetings. These mainly fill the forty and odd volumes of the *Journal,* no one of which can, even now, be taken up without finding something of interest to the student and to the profession. In the aggregate they

deal with almost every question that is of importance to the actuary, and have from the first embraced questions of financial and practical importance, as well as those subjects which may be termed theoretical.

The collection, analysis and publication of statistical data has been the work not only of the Institute officially—although in this respect it has done invaluable service, which would have been far beyond the powers of any individual efforts, by its two great mortality investigations, the later in co-operation with our sister Society in Edinburgh—but also of individual members who have made innumerable contributions of data, often of the highest value, to the *Journal.* One has only to consult the Index to the *Journal,* under the heading Original Tables, to find how extensive and valuable these contributions have been. The readiness with which so many of our members have freely given to the profession the data resulting from investigations undertaken in the course of their work is only in harmony with that public spirit to which the Institute owes its existence, which has always animated its members in the past, and we are confident will always do so in the future.

The improvement and diffusion of actuarial knowledge, and the raising of the attainments and status of our members have been in view in all the operations of the Institute, but particularly so in its special education and training for the responsible work of their profession of successive generations of actuaries, many of whom have in the past lent additional lustre to the Institute, but none of whom will fail to acknowledge the debt they owe to it. In many respects this work is perhaps the most important which the Institute has done, and we must include under this head the publication of the official Text-Books, whose value has been recognized far beyond the limits of our membership.

Finally must be mentioned the work which the Institute has effected, not perhaps so immediately obvious but actually of the greatest importance, in the influence it has exercised upon legislation as regards various matters affecting Insurance, Friendly Societies, Taxation and Finance.

One result to which the work of the Institute has mainly contributed is of such public importance that it deserves more than a passing reference. In carrying out the programme so briefly summarized, the Institute has been the most powerful influence in securing the stability and, speaking generally, the sound management of the Life Assurance Societies of this

country, if indeed a wider claim may not be made, and in this respect has rendered an immense service to the public, the extent of which may be gauged, but is not fully measured, by the large financial interests involved, corresponding to the magnitude of the operations and resources of these societies. The change that has been effected in the status of the Life Offices between 1848 and 1908 is sufficiently striking. About the time of the formation of the Institute, a crowd of companies was coming into existence, and a little later the existing companies numbered nearly 200. Some light is thrown on the character of some of these companies by the fact that in the ten years, 1852-61, about 130 ceased to exist. What business the offices collectively did, or what was the extent of their combined income and funds, it is not possible to say with any accuracy, as in those days they did not all court publicity as to the extent of their operations, nor as to their financial position, but we may probably put down the total funds at about £40,000,000, and the total premium income at something over £4,000,000. These figures seem small indeed when compared to those of the present day, when the aggregate life and annuity funds amount to about £350,000,000; the annual premiums to about £40,000,000; and each year some £22,000,000 are distributed in respect of death claims and matured policies. Put shortly, during the 60 years that have elapsed since the Institute was founded, the aggregate income and funds of the life offices have been increasing very nearly in geometrical progression at a rate which has, speaking roughly, doubled the figures every twenty years.

It hardly needs saying that it is of the highest national importance that the societies to which are entrusted interests of such magnitude should be carried on not only with integrity but with the best knowledge and skill, and it is satisfactory to know that in the majority of cases, including some of the soundest and most successful (from the point of view of the beneficiaries), this is being achieved under the responsible guidance of men who have been trained for their work by the Institute.

Such is a very brief statement of the record of the Institute in the past. While we look back upon that record with pride, it is also our business to look forward, and to ask ourselves in what direction our work lies for the future? There will doubtless be new developments, but in many respects this work will follow

the same lines as hitherto. We cannot regard actuarial science as complete, either upon its financial or mathematical side. There occur in the history of most branches of science periods and phases where after a prolonged advance there comes a distinct pause, if not a tendency to stagnation; as, for example, in the science of astronomy, just prior to the introduction of the methods of spectrum analysis; in biology, just before Darwin; and, we may perhaps add, in the theory of evolution prior to the introduction and application of the statistical principles and methods which we owe in the the first instance to Mr. Francis Galton, and, not to mention other workers, the complete development of which is due to Professor Karl Pearson. Such a period of inertia is generally overcome by the advent of fresh general conceptions or of some fresh mass of material demanding analysis and new methods of work. Both these factors are operative in actuarial science to-day.

The new statistical theories and methods to which I have just referred have already done much for statisticians. They have shown the importance of considering not merely average results, but also those laws which appear to govern deviations from such average in individual cases or groups; and they have thrown a flood of light upon the nature of those laws. They go far to show that in any homogeneous data the distribution of such deviations of any measurable quality round its mean value may be represented by a single mathematical law, of which the old so-called law of error is a particular case, the exact form of the distribution depending merely upon the values of the three constants of a certain differential equation. They have further, in the mathematical theory of correlation, not only given us the means of determining the probability that certain characters in a group are correlated, but have also provided us with a numerical measure of the degree of such correlation.

It may be too early to say whether the particular family of curves which Prof. Pearson has investigated practically exhausts the observed laws of variation. Probably in the case of actuarial statistics where the age is the independent variable their application may be found to be limited by certain disturbing influences peculiar to this class of statistics. But it is clear that these new methods and the conceptions underlying them have a direct bearing upon much of our statistical work, and will be in many cases of great service to us.

It is satisfactory that the Institute has not passed by

unnoticed these new methods, which have so important a bearing upon the classification and interpretation of vital statistics. I need only refer to the reprint in the *Journal* of the papers by Miss Beeton and Mr. Yule jointly with Prof. Pearson on "The Inheritance of Longevity, &c.", and on the "Correlation between the Duration of Life and the number of Offspring",* and to the lucid introduction to the whole subject contained in Mr. Elderton's book, "Frequency Curves and Correlation", published under the auspices of the Council.

Even those older and more discussed questions of actuarial theory, upon which there may, at one time, have been a tendency to regard the last word as said, will have to be regarded from new points of view. Notwithstanding the numerous and valuable papers which have from time to time been written on the subject of Life Office Valuations, and Surplus Distribution, there still remain many points, not only of detail, but of principle, open to further discussion.

While, however, I do not think that the theory of our subject can be regarded as complete, I believe, nevertheless, that some of the most important work we have to do in the immediate future lies in the proper application of actuarial principles, statistical and financial, to the many practical questions which arise from time to time not only in the ordinary course of our work but also outside the sphere of life assurance.

Some such questions directly interesting to actuaries and of great moment to the public have been brought to the front by the events of the past year. Perhaps the most important are those raised by the recent passing of the Old Age Pensions Act. The question of the desirability of some action by the State in this matter has been, more or less officially, under consideration for the past twenty-five years, various committees and commissions having reported on it to Parliament. Both political parties may be said to have been committed to a measure in one form or another, and it can hardly be a matter for surprise that at last a Government has taken the plunge. This question, or the larger problem of which this is a part, has on several occasions engaged the attention of the Institute, and has been repeatedly referred to in the *Journal*. So long ago as 1891 we were indebted to Mr. Young for a valuable exposition of the German law of Insurance against Invalidity and Old Age from the financial and administrative points of view.† Mr. R. P. Hardy in his paper upon "The Formulæ for Benefits upon the principle of

* *J.I.A.*, vol. xxxv, pp. 112, 458.

† *J.I.A.*, vol. xxix, p. 269.

Collective Assurance"* has attacked successfully an important branch of the theory of the subject, while the question has been referred to in one or two Presidential Addresses, notably in that of Mr. Manly in 1896.

The recent Act has not yet come fully into operation, and it is therefore too early to say how far the preliminary estimates of cost will prove trustworthy. It already seems pretty clear that they will be exceeded in the sister island, and the enthusiasm with which the Act has been there welcomed perhaps excuses the recent question in the House as to whether the number of applications indicated the longevity of the inhabitants or their powers of imagination.

The Bill when first introduced, as was perhaps to be expected when the difficulties of the question, financial and other, are considered, presented many anomalies, not all of which have been removed during its passage through the House, and some of these in any case will have to be dealt with in the near future.

Apart from these, however, the Act as finally passed admittedly deals with only one part of a great problem. Old age is not the worst enemy which the wage-earner has to fear. Invalidity and unemployment are, in fact, more near and more dangerous to him because (in each individual case) more incalculable.

If it is of moment to the worker that these enemies should be guarded against, it is also of moment to the State, which loses not only by the temporary stoppage in production, but by the consequent disorganization of industry and the deterioration of the worker, even more than by the costly temporary expedients periodically demanded to meet crises of severe depression such as at present.

In these circumstances it is natural that there should be a somewhat widespread expectation that the Old Age Pensions Act will be succeeded by a supplementary and contributory scheme dealing with invalidity and earlier pensions and, conceivably, unemployment. Whether this will prove to be so or not will presumably depend upon the views arrived at by the Royal Commission which it is the intention of the Government to propose.

It is satisfactory to know that an enquiry of this nature is to precede any further legislation, and it is to be hoped that it will be exhaustive and final; that it will not be occupied by the examination of numberless rival schemes as in the case of a former enquiry, but will elicit and classify the facts which are the

* *J.I.A.*, vol. xxx, p. 79.

first requirement if any proposed scheme is to rest upon a sound financial basis.

Leaving out of the question for the moment the problem of unemployment, as regards provision against sickness, invalidity or death, we are not without ample data in the combined experience of our friendly societies; and this part of the ground is already largely covered by the operations of existing institutions. We cannot say, however, to what extent the ground is still unoccupied, nor do we know, and this is said without any disrespect to some of the most useful institutions in the country, how much remains to be done to bring the existing agencies to a state of efficiency. We know, however, that the position of the friendly societies has made great progress during the past generation, and I am strongly inclined to the view that any scheme that may eventually be produced, if it is to be successful, must take account of and in some way utilize these existing institutions.

According to the Report of the Deputy-Registrar of Friendly Societies for the year ending 1906, the friendly societies and affiliated orders making returns in that year represented nearly 6,000,000 members, and possessed funds amounting to £42,000,000, paying annually to their members for sickness and invalidity about £4,000,000 per annum, to which should be added about £500,000 paid by the trade unions. These figures compare with 14,000,000 estimated as the numbers in 1906 insured under the German State scheme, with funds of some £76,000,000, and annual payments for sickness and invalidity of about £6,000,000 exclusive of medical attendance.

In point of funds, when the difference of population is allowed for, the comparison is not unfavourable; but the numbers would suggest that a considerable section of our wage-earners are not members of registered friendly societies, and probably do not therefore make any provision by insurance against sickness and invalidity.

A further question upon which information is needed is as to how far these societies are efficiently managed and solvent. In the case of some of the larger affiliated orders, their valuations are made by well-known and competent actuaries, and we know that their condition is, on the whole, satisfactory. Leaving these out of account, it must be confessed that we are very much in the dark as to the remainder, and although many of them are known to be sound and well managed, it is certain that

there are a large number of which this cannot be said. Out of about 1,000 valuations received by the Registrar in 1906, fifty-nine per-cent showed an aggregate deficiency of over £1,250,000, while forty-one per-cent showed surpluses aggregating over £1,000,000. These figures, so far as they go, are not very satisfactory, and there are some disturbing symptoms. It is to be noted that only fourteen per-cent of the valuations were made by public valuers, and it is further to be noted that of the 1,000 societies above-mentioned, over sixty per-cent (no doubt, however, the smaller ones) possessed scales of contributions uniform at all ages at entry.

On the other hand, where comparison was possible with an earlier valuation, over sixty per-cent of the societies showed an improved financial position. When the large affiliated orders are taken into account, there can be little doubt that the financial position of Friendly Societies as a whole has been improving for many years past, and that this process is still going on, and this, I believe, is the opinion of the Chief Registrar. Unfortunately, this improvement has been largely due to the non-survival of the unfit, which is a painful process to those most nearly concerned.

When these preliminary questions have been dealt with, some of the gravest difficulties will still remain, as the Government will then have to determine whether a case is made out for State interference upon the basis of a contributory scheme, whether such scheme is to be optional in principle, taking the form of encouragement by subsidy of private societies, or whether there is to be a compulsory scheme, embracing the whole of the wage-earning class.

Present indications seem to point in the direction of the abandonment by the State of the policy of *laissez-faire,* and no doubt the force of example in other countries will tell in this direction, and the somewhat widespread feeling that it is illogical and unfair to provide a free pension for those over 70, and to refuse to recognize the claims of those who are helplessly invalided before reaching that age. I need hardly say, however, that social, political, and financial, as well as actuarial, considerations must be well weighed, before a conclusion can be reached as to the desirability of State interference.

In the event of a scheme for optional, subsidized insurance, important questions will arise as to how existing and approved societies can be utilized. Obviously a selection would have to be made, as the State cannot recognize and certainly not subsidize

societies which cannot satisfy it that they are solvent and are likely to remain so; and for this purpose I need hardly say that something more would be required than the kind of valuation now sometimes thought sufficient by the societies themselves.

If selected societies are to be subsidized, the question will arise as to how far benefits should be standardized, and as to what form such subsidy is to take. The suggestion has sometimes been made that the State should guarantee a minimum rate of interest upon the funds of approved societies, but it seems probable that under this plan the cost to the State would be disproportionate to the advantage to the societies, who can and already do, in many instances, earn distinctly higher rates upon their funds than that at which the State borrows. This would appear to point to an addition to the contributions as the necessary method of subsidy.

If a general scheme of compulsory insurance is to be adopted, it will be even more unjust to ignore existing societies of proved value and status, while, on the other hand, it will become more than ever necessary to have ample security for their continued solvency and sound management. The claims of the smaller and yet sound societies would also have to be considered, and possibly some method of grouping them adopted, to secure a sufficient basis of operations necessary to stability.

Many other questions will arise, such, for example, as the treatment of lives already invalided, or so far advanced in age that premiums for insurance against invalidity would be too onerous. This last question, indeed, would be one of great practical difficulty in any scheme.

The problem of unemployment stands upon an entirely different footing to that of invalidity or old age, and prolonged and careful enquiry will be needed before anyone can say, not whether it is desirable, but whether it is at all possible to deal with this problem on the principle of insurance. We are at present almost without data; and in a recent debate in Parliament the most divergent figures were given as representing the numbers involved. We require a census of unemployed which shall enable the facts to be so classified as to show the intensity of unemployment in various districts and occupations; how far and in what class of cases it is sporadic, periodic, or chronic; and to what extent it can be assigned to various causes. The trade unions are practically the only institutions which deal with this question from what is virtually an insurance point of view, but they

represent less than 2,000,000 members, or about one-fifth of the persons affected, and they deal with the problem in its most manageable form. Whether unemployment as a whole can be dealt with upon the principle of insurance is open to very grave doubt.

I need hardly say that these brief remarks are not offered as a detailed discussion of questions which in some aspects are essentially political and controversial, but enough perhaps has been said to indicate the nature of some important questions that must be dealt with before any Commission can hope to produce a satisfactory scheme. Whatever scheme, if any, be finally adopted, it will be the general feeling here that it should be of such a nature as to put a premium upon thrift rather than to discourage it.

Another public question in which we as actuaries are greatly interested is that of the vital statistics of the country, a subject suggested by the recent publication of the two volumes of the supplement to the Registrar-General's 65th Report, covering the 10 years 1891 to 1901. Those who have perused these volumes will have seen at once that in several respects they are a great advance upon their predecessors. The new English Life Table No. 6 contained in the first of these volumes was very recently discussed in this room, following upon the reading of Mr. King's census paper,* and it will generally be admitted to represent the mortality of the general population much more closely than any of the earlier English Life Tables. Apart from this table and the new Healthy English Table the main features of the volumes are the statistics of mortality in different localities and occupations. As regards the first group of tables, whereas in former reports we had to be content with what have been termed the crude death rates, uncorrected, that is, for the difference in age distribution of the population in the various registration districts, the proper correction is now brought into account, and it is for the first time during the whole history of these tables possible to make comparisons of death rates in various parts of the country with any degree of confidence in our conclusions, although even now some care must be exercised, for while the mortality of various localities has been corrected for deaths in institutions as well as for differences in age distribution, these two corrections have not been combined. Perhaps the most striking conclusion to be drawn from such comparisons as may be made by the aid of these corrected death rates is the

* *J.I.A.*, vol. xlii, p. 225.

difference between the best districts and the general average, or, to put the matter in another form, the difference between the rural and town populations. The average corrected mortality rates for the districts included in the new Healthy English Table was, for the 10 years, 13 per 1,000 on a population of 4,500,000; the highest rate included being 14 per 1,000. If we assume the standard of life under healthy conditions to be represented by the higher of these two rates, we find the excess mortality above this standard in the remainder of the country represents about 120,000 deaths annually.

In the statistics of the mortality of occupations, which are of value to us as being at present the only data of this kind of any extent available in this country, a distinct improvement has been effected, always assuming that the figures can be relied upon, by taking account of the retired members of each occupation or profession, and in this way a much better agreement has been secured between the registered deaths and the enumerated populations, and an important criticism of the former returns has been met, namely, that in many cases the mortality in a given occupation may under the old form of table be unestimated, because persons retiring from ill-health would cease to be included in the group. It is to be noticed, however, that these improvements amount in themselves to a strong condemnation of previous returns, since they go far to show that any deductions that may have been made from these earlier returns were necessarily untrustworthy. It is perhaps worth suggesting that something approaching uniformity should be arrived at between the returns for different parts of the Kingdom. The improved methods to which I have referred in the English returns, do not appear in those for Scotland or Ireland, and although it is true that, so far as the corrections for age distribution is concerned, these can be made by investigators for themselves, this can only be done by an enormous amount of individual labour. What strikes one chiefly in these returns is that, notwithstanding the improvements introduced, there has been too much adherence to traditional forms. It seems very doubtful whether the immense mass of detailed tables printed, showing the deaths at various ages and from various causes in every individual registration district, can be of any great public utility, the items in most of these tables consisting of small numbers from which no trustworthy conclusions can be drawn, although the labour and expense of

their tabulation must be considerable. What is needed is some simple means of showing at once which districts are healthy and which unhealthy, and in what degree, and this might easily be done by a much shorter series of tables or by suitable diagrams, neighbouring and similar registration districts being suitably grouped, and the deaths being corrected both for age distribution and for deaths in institutions.

In the closely allied branch of public statistics connected with the Census, the Institute has in the past endeavoured, by official representation, to bring about certain much needed reforms, but so far without any very definite results. The nature of these reforms is pretty well agreed upon by statisticians generally, and is fully indicated in Mr. Ryan's important paper on Census Reform,* and in the subsequent discussion. All that can be done is to continue, when the opportunity arises, to bring pressure to bear in the proper quarter. Actuaries and others interested in the Census have always had a natural desire to obtain the bare facts ascertainable from the enumeration, before these have been subjected to processes of grouping or smoothing. It is still more desirable that a serious effort should be made to eliminate from the census figures and the Registrar-General's returns the systematic errors which at present vitiate the age statements and possibly other information. We need, in fact, such an analysis of, for example, the age statements as will suffice to determine the extent and direction of the personal errors by which the returns are effected, and this can only be done by introducing the card system, and treating the census as a continuous and not a spasmodic operation.

Now that far-reaching problems such as those relating to schemes for state aided insurance are coming to the front, it is more than ever important that the census figures should be freed from systematic errors. The next Census will be one of the utmost importance, both as to the facts to be ascertained and their treatment, and it is much to be desired that due weight should be given to the views of those statisticians who have made a special study of this question.

As regards the work of the Institute in the future in respect to the collection and publication of data, it might seem that, after the extensive experience which has so recently been published, little remains to be done for the present. There are still, however, some important gaps in our knowledge as regards the mortality of insured lives, more particularly with respect to the

* *J.I.A.*, vol. xxxvi, p. 329.

proper classification of extra risk cases, whether due to personal defects, to family history, special occupations, or climates. In the archives of the offices there must exist a mass of material bearing on all these points, and though we cannot hope that this will be made available for general use in the near future, it is not too much to expect that it should not be permanently lost sight of. Although some attention was paid to this question at the time the Institute 20 Offices' Experience was collected, nothing very satisfactory was accomplished. The mere separation of risks into two classes of healthy lives and diseased lives was quite inadequate as regards the latter for any practical purpose, as no classification of them was attempted. The data which has so recently been published as respects normal risks has given to us a set of tables showing the rates of mortality prevailing among the holders of various classes of policies, which may be expected to fulfil all requirements on this head for some years to come, but that investigation left abnormal risks of all kinds on one side.

With respect to special rates of mortality prevailing in various occupations, fair conclusions may perhaps be drawn from the Registrar-General's returns, already referred to, in the supplement to the Sixty-fifth Report, if we except a few special occupations, and as regards the extra rates of mortality arising from unhealthy climates, there is a fair amount of miscellaneous information available, some of it accurate in character, although not perhaps very extensive in amount. Our information on this subject, however, still remains very fragmentary, and, except for certain special localities, not very trustworthy.

In the rating up of lives on account of personal defect or family history, there is a great want of uniformity in the practice of various offices, due to the fact that there are hardly any materials available upon which to base sound conclusions. These materials, nevertheless, exist, and I believe it would amply repay the offices if a sufficient body of them could be collected and analyzed. One difficulty in the way, perhaps, is that one can hardly expect that offices which have made a special study of this class of business, and by their own enterprise and at their own risk have amassed a quantity of valuable information on the subject, will be prepared to make the results of their experience available for the use of competitors. At the same time I do not think that this difficulty, although it may be a real one, should stand in the way of something being done in this direction by

the offices generally. Meanwhile, I am not, I think, betraying any confidence in stating, what I am sure the profession as a whole will be very glad to know, that one of our members, who has unusual opportunities for the collection of the necessary facts, is making an investigation upon this very subject. When he publishes his results, I am quite sure that much valuable information will for the first time be made available for the profession, and light thrown upon many points hitherto obscure.

In addition to this class of information, there exists in the offices materials for throwing light upon many important aspects of the problem of heredity. I have already referred to the paper by Miss Beeton and Professor Pearson, transcribed from the Proceedings of the Royal Society into the *Journal*, on the Inheritance of Longevity, a subject which has obviously a most direct bearing upon our work. In this research the authors found great difficulty in obtaining sufficient data suitable for their purpose. But the records of any single life office would probably avail to produce a much more extensive and trustworthy set of observations than those upon which the authors eventually had to rely. And this is true of other similar enquiries. In many cases it would not be necessary that an unwieldy amount of material should be brought together. It would be quite possible, from a comparatively moderate collection of data from the contributing offices, to arrive at some definite and valuable information on many points which are now somewhat obscure, such as the effect on the expectation of life of various features in family or personal history.

Of the events of the past year specially of interest to the actuarial profession, in addition to those already referred to, there are two which call for some notice; the first by its public, the second by its domestic importance.

The first is the American monetary crisis of last autumn, the immediate effect of which was a further fall in the values of Stock Exchange securities, including what may be termed the investment securities, largely held by life offices, which had already so considerably declined from the high-water mark of 1897-8. Since the beginning of the year there has been some recovery, but the question whether this recovery is to be permanent, if we can speak of permanency in these matters, depends upon events which the actuary cannot be expected to forecast.

I do not propose to say anything as to the causes of the crisis of last autumn, which are now pretty well understood, but as that crisis profoundly affected the whole financial, and through it the whole commercial world, it will be of some interest to consider the effect of the consequent fall in securities upon the finances of the life offices.

In the ten years prior to the close of 1898, the gross profit on investments brought into the Revenue accounts of the various life offices, so far as this can be ascertained from the Board of Trade returns, represented about 2 per-cent of the average aggregate values of the Stock Exchange Securities held by the offices, or considerably less than a quarter per-cent per annum. Even this estimated sum probably includes some profit on reversions. When we consider the actual extent of the appreciation that must have occurred during those years, over an annual average holding in all about £80,000,000, it is manifest that the offices generally exercised a wise restraint in the revaluation of securities, of which they have reaped the benefit in subsequent years.

From the year 1899 the movement in prices has been reversed, and taking the offices as a whole the assets were written down between that date and the end of 1906 by about £1,500,000, while investment reserve funds and the like were increased by a nearly similar sum, indicating a total loss on these investments, brought into account, of about £3,000,000.

How far this last figure will be augmented, when the results for 1907 are fully known, is uncertain, but probably by something over £2,000,000, making a total depreciation of over £5,000,000 in the last eight years. This aggregate no doubt includes losses on other classes of securities, but probably not to any considerable extent. If this sum appears large, it must be looked at in relation to the total of the funds affected, and it will then be seen to represent about a half per-cent per annum on these investments during the eight years. As contrasted with the extent of the fall in investment securities during the period, we shall be struck by the smallness of the amount that the companies have, in the aggregate, found it necessary to write off, a state of things obviously due to their caution in the preceding period of advancing prices.

It may be asked how far the financial position of life companies has been affected by this fall in securities, or may be affected by any further extension of the movement. The answer

must be, I think, that, paradoxical as it may sound, the fall in prices has strengthened the position of the offices.

While the reasons for this conclusion will be apparent to the actuary, they may not be so readily grasped by the public. The rise in the rate of interest which has necessarily accompanied the fall in prices will, even if the latter prove permanent, more than compensate the offices for the reduction in capital values. This rise is distinctly shown in the revenue accounts of the companies, and is much larger than can be accounted for by the automatic effect of the writing down of securities that has taken place. In other words, the rate of interest has risen on other classes of investment, such, for example, as mortgages, which we may term non-fluctuating investments; and if the capital depreciation of the former class of securities should prove permanent, there can be little doubt that the higher rates of interest on this class will also be permanent. I am, of course, speaking throughout only of the depreciation of investments with secured income, not such investments as Ordinary Railway Stocks, which have fallen partly from actual and anticipated reduction in dividends.

The gradual rise in securities up to 1898, and the consequent continuous lowering of the rate of interest, very properly led to a general reduction in the rates employed for valuations; and the logical result of a reversal of the former movement would be a corresponding rise in valuation rates, although, owing to the conservative management of the offices generally, this step has not, so far as I am aware, been taken in any instance.

The exact financial effect upon a life office, as regards existing contracts, of a fall in values and corresponding rise in rate of interest has not been fully worked out, but it is easy to arrive at approximate general conclusions. In an office of average age and constitution, if we suppose a rise in the rate of interest from $3\frac{3}{4}$ per-cent to 4 per-cent, corresponding (so far as these figures have reference to Stock Exchange securities) to an average fall of about 6 per-cent in prices, a corresponding change in the valuation rate, say from $2\frac{3}{4}$ per-cent to 3 per-cent, leaving the same interest margin, would result in a reduction of, roughly, $2\frac{1}{2}$ per-cent in the estimated net liability. It is unlikely, however, that the funds will be reduced in this proportion, as the shrinkage in values will affect Stock Exchange securities only, and these to a less extent than might appear from the above estimate, as many of them may consist of stocks redeemable

at the end of comparatively short periods, and therefore not subject to much fluctuation. In the meantime, by the valuation of lower net premiums, a larger provision has automatically been made for future expenses and profits, while the new business transacted will clearly be done with a larger margin for profit than under the lower rate of interest. Hence such an office would be, in all probability, in a stronger position than when its securities stood higher and its rate of interest lower; and *à fortiori* will this be the case when the valuation rate has been maintained at its old level.

There may well be in practice very good reasons against taking what I have ventured to call the logical course of raising the valuation rate to correspond to the writing down of investment values. Such a change, however logical, would probably be regarded as a sign of weakness, and would certainly arouse the criticism of the amateur expert, which in these days is not to be lightly disregarded. But apart from this consideration, there can be no doubt that actuaries are in the main impressed with the necessity of maintaining their office reserves at as high a level as is practicable, and as a consequence the fall in prices during the past few years has in a few cases affected bonus results.

It would seem probable from the considerations adduced that the offices have, taken as a whole, arrived, perhaps more by instinct than calculation, at that balance between Stock Exchange and non-fluctating securities, which in the long run will best insure them against any adverse effects of fluctations in interest rates and capital values.

The question of the proper method of valuing Stock Exchange securities in the balance sheet has been but little discussed, probably from the feeling that the circumstances of individual offices differ so greatly that no method can be laid down as universally applicable.

There is a general agreement that it is indispensable that assets should not be brought into the balance sheet at a higher figure than the market prices of the day, the only exceptions that can be admitted to this rule being those investments in which the income is absolutely secure, which are redeemable at fixed dates and prices, in which cases the rate of interest may fairly be equalized over the whole term of the investment.

It has sometimes been said that the first consideration in investing the funds of a life office was the security of the principal. If we are speaking of permanent investments, I would

prefer to say the first consideration is the security of the income. The prospect of an averagely constituted life office having to realize any appreciable portion of its securities to meet claims is extremely remote. The income of the offices as a whole, at the present moment, excluding the industrial companies, is about £40,000,000 per annum, and the outgo some £30,000,000; and even were they to cease taking new business, it would be some years before the funds would have to be drawn upon for the payment of claims.

I perhaps owe you some apology for dwelling upon considerations which may appear somewhat obvious. In view, however, of the magnitude and importance of the monetary crisis of last autumn, and of the attention it has attracted in consequence of its severe effects in the financial and commercial worlds, it is not, I think, out of place to point out that, so far from its having adversely affected the financial position of the life offices, it has probably left them stronger, and I believe there was never a time when the position of our leading offices was firmer than it is to-day.

The event of domestic import to which I alluded is the change in the syllabus of the examinations, and the accompanying variation in the preliminary qualification required from students. As you are well aware, the Council have had the question of the examinations before them for a long time, and I can personally testify to the amount of attention and study which they have given to it.

The changes that they have made will, I believe, commend themselves on consideration to the members of the Institute as a whole, whether Students, Associates or Fellows. The nature of these changes is fully described in a memorandum in the July number of the *Journal*,* preceding the revised Rules and Syllabus of Examinations. I need, therefore, say little about them; but it may be desirable to say a few words as to the object of the Council in making these alterations. Their object has been twofold. In the first place, to secure that candidates for the later examinations shall commence their preparation for those examinations with some previous knowledge of the elements and principles of the subjects to be studied. By this means it is hoped that they will enter the examination room better prepared, and that there will be a smaller percentage of failures than has been the case in recent years. Their object in the second place is to secure that those

* *J.I.A.*, vol. xlii, p. 428.

who may stop short of the complete qualification, and who may content themselves with passing the first two examinations and securing the Associateship, shall not have been trained purely in theoretical subjects, but shall have made such acquaintance with financial questions, if only of an elementary character, as shall the better qualify them for practical work in subordinate posts.

With a view to assisting students reading for Part II in their study of the subjects forming §§ 4 and 5 of the new syllabus, they have arranged for a series of lectures, which, as you are aware, have already been announced, upon those special subjects during the current session. They consider themselves very fortunate in having obtained the services in this connection of Mr. Burn, and while the lectures are intended primarily to meet the requirements of second-year students, they will not be limited by this object, but it is intended that they should, when complete, form a text-book upon the whole of these subjects, which will not only be useful for students, but also serve as a work of reference for all of us.

Concurrently with the changes in the syllabus above referred to, the Council have determined that in future gentlemen who may desire to join the Institute as Probationers or Students shall give more definite evidence of general education and culture than has hitherto been demanded of them. They will be expected to have passed either the London Matriculation, or some other similar examination, according to a list prepared by the Council, before being eligible for election. The object here, again, is to secure that our members down to the youngest student be not merely mathematicians, but shall have some acquaintance with the humanities, and shall possess that general culture, without which merely technical knowledge is insufficient to equip them properly for their future work.

The question of examinations has always been one that has closely occupied the attention of the Council, even in the earliest days of the Institute. The first examination, I believe, took place in 1850, and so soon as the following year it was determined to enlarge the single examination to a series of three, at which number they stood until a comparatively recent period. It is perhaps worth noting that in 1851 five candidates presented themselves for examination for the degree of associate, and the whole of them satisfied the Examiners that they were "fully entitled to receive it." Whether this satisfactory result was due to the exceptional quality of the Students of the time (which is

quite possible), or to the exceptional humanity of the Examiners, I cannot undertake to say, but the proportion of successful candidates has sadly fallen off since those times. It is hoped that the changes now made in the syllabus and in the regulations will secure that, even if there should be some diminution in the number of gentlemen presenting themselves for examination, the proportion of successful candidates will be larger than in recent years.

I cannot do better, in closing my remarks on this subject, than quote the advice to Students given by the Council in 1851: "If "the labour and anxiety of aspirants are somewhat increased and "protracted by these regulations, there is every reason to believe "that their reward will be the greater. They will ultimately "enjoy a higher degree of public consideration, and they will "have the satisfaction of knowing that they have well earned it."

Another important modification in the arrangements for examinations is the appointment of a Board of Examiners. Complaints have sometimes been made as to a want of uniformity in the standard of examination. I am not prepared to say that these complaints have been entirely groundless, because, as a matter of fact, it must be somewhat difficult to secure such uniformity of standard with the system of appointing Examiners annually, with its resultant changes in the *personnel*. This difficulty, however, will be obviated for the future.

I need not read to you the names of the gentlemen who kindly consented, at the invitation of the Council, to sit upon this Board, as they will be regularly published in the *Journal*; but I may go so far as to say that the Board is a thoroughly representative one, and that I think there can be no doubt that the names will give absolute satisfaction, and will command entire confidence.

The task of examining the number of candidates that have in recent years presented themselves is a very onerous one; and no one is better aware of this than the gentlemen who have consented to act on the Examining Board. It the more becomes us to express to them the thanks, not only of the Council, but of the Institute as a whole, for undertaking this very important work.

Before closing these remarks, I must refer briefly to the prospective arrangements for the now current Session.

With regard to the papers to be read, it is a pleasure to report that we shall lead off with a paper by Mr. George King,

on "A New Method of Constructing and Graduating Mortality and Other Tables", to which we have been looking forward with interest since the date of his paper last session on Census Tables, to which it may be regarded as supplementary. In the extent of his contributions to our proceedings, Mr. King must be almost unrivalled, and these have been as valuable as they have been varied. We are also anticipating papers by Mr. E. H. Brown, on "A New Method of Approximate Valuation of Whole-Life Assurances with Allowance for Selection"; from Mr. Eldridge, on "Statistics of Accidents and Diseases of Employment and Compensation, &c., under the Workmen's Compensation Act of 1906"; from Mr. Vyvyan Marr, on "Contributory Pension Schemes"; and from Mr. A. T. Winter, on "Tropical Mortality."

Many of us are looking forward with pleasure to the Congress to be held in Vienna in June next. It is to be hoped that any clouds which at the present moment may obscure the political horizon in that quarter will by then have been completely dispersed. But whether this be so or not, I am sure that nothing will interfere with the friendly relations that unite actuaries the world over; and all our members who attend the Congress, and I hope they will be many, may count upon a very cordial welcome from our friends in Vienna. Those of us who have not yet made the acquaintance of that beautiful city will doubtless be eager to do so, and those of us who have already done so will be only too pleased to renew that acquaintance. The Congress will take place at an admirable time of the year, from the 7th to the 13th June, in the height of the Vienna Season; and our hosts there have already made extensive arrangements for the comfort, entertainment and enjoyment of any of us who may visit them.

I will not detain you with any detailed statement of papers to be read, but you will be glad to know that the Institute will be well represented in this respect, and our cordial thanks are due to those gentlemen who have so readily come forward in reply to the invitation to read papers. Their names are a sufficient guarantee that the reputation of the Institute in this matter will be in very good keeping.

MESSENGER PRIZE ESSAY (1905).

Abstract of Essay "On the Methods of ascertaining the Rates of "Mortality amongst the General Population of a Country, "District, or Town, or amongst different classes of such "population, by means of Returns of Population, Births, "Deaths, and Migration." By CHAS. H. WICKENS, A.I.A., *of the Commonwealth Bureau of Census and Statistics, Melbourne, Victoria, Australia.*

[To this Essay was awarded the Messenger Prize (1905), the conditions of which were printed in the *Journal* for January 1905 (vol. xxxix, pp. 127, 128). The following Abstract has been prepared by the Author at the suggestion of the Council.]

SECTION I.—INTRODUCTORY.

1. IN the following pages it is proposed to consider various methods of investigating the mortality experience of a general population; that is, of obtaining a measure of the death toll which is being levied on a whole community, or some specified portion of it.

2. The most obvious method of measuring an experience of this nature is that of ascertaining the ratio of the number of deaths to the total population from which such deaths have been derived, thus obtaining what is known as a "crude death-rate." It will be seen hereafter that rates of this nature, obtained from aggregates of deaths and population, are frequently unreliable in their indications, and at all times require to be used with the greatest caution. It may be pointed out, however, that even the most refined rates are neither more nor less than ratios of deaths to population, the difference being that in their case the data have been extensively analyzed, and the deaths occurring in any special section of the community are compared only with the portion of the population to be found in that section, as, for instance, the deaths of persons of a particular race with the number of persons of that race in the community under observation, or the deaths in a given year of age with the number exposed to risk of death at that age.

3. The main points for consideration in an enquiry of this nature are—

(*a*) The objects of a general mortality investigation.

(*b*) The data required.

(*c*) The sources of information and methods of collection.

(*d*) The preparation of the data.

(*e*) The computation of rates.

Section II.—The Objects of a General Mortality Investigation.

4. The objects of such an investigation are numerous, but the most important may be said to be—

(*a*) Those connected with financial transactions dependent on the contingencies of human life.

(*b*) Those pertaining to matters of Public Health.

(*c*) Those relating to Biological Researches.

5. In financial transactions the results of a general investigation are frequently of great value at times when the appropriate special experience is either non-existent, incomplete, or insufficient in extent. With the accumulation of the requisite special data, the specialized results usually supersede the general, although in such cases as those of industrial assurances and old age pension schemes, where the field of application is very extensive, general results will probably always be of considerable value. Notable instances of the practical use of the results of general investigations for financial purposes are to be found in the cases of the Northampton, the Carlisle, and the English Life Tables. What is aimed at in an investigation of this nature for financial purposes, is the construction of an instrument which shall be at once an index to past experience, and a guide to the future, the accuracy of the former aspect furnishing to a great extent the basis for estimating the reliability of the latter.

6. From the standpoint of public health the object of a general mortality investigation is that of obtaining reliable evidence of excessive mortality in the community or any section of it, and of procuring such details as will enable the causes of the excess to be ascertained, and will thus prepare the way for remedial measures.

7. For biological purposes such investigations furnish valuable information concerning the laws governing the duration of human life, and the variations in such laws due to race, climate, marriage, &c., and thus contribute towards the solution of the problem of determining the direction and force of the evolution of the human species.

Section III.—The Data Required.

8. The essential data for a general mortality investigation are the number of years of life spent by the members of the community during a given period, and the number of deaths occurring in the community during the same period. In order, however, to meet all the requirements specified in the foregoing section, it becomes necessary to submit these data to somewhat extensive analysis, since the grouping together and treating on a common basis of masses of heterogeneous materials must necessarily furnish results which can only be considered as giving a rough indication of the mortality experienced, and should be used with the greatest caution for comparative purposes.

9. While it is probable that all the varied circumstances and conditions of a person's life have a bearing more or less remote on the prospects of longevity, there are some which, either on account of their intrinsic significance, or because of their lending themselves readily to statistical investigation and measurement, are, for the purposes of an enquiry of this nature, of greater importance than others. Chief amongst these are: Race, Sex, Age, Occupation, Conjugal Condition and Locality. To these may be added "Physical Incapacity", arising from disease, violence, or inherent physical defects.

10. *Race.*—The distinction of race is one which immediately suggests the likelihood of distinct mortality experience, but it is only in special cases that the race distinction of any community is so clearly defined as to render possible an elaborate investigation on this basis. To attempt, for instance, to differentiate between Celtic and Teutonic mortality in the United Kingdom would be to undertake an impossibility. In other cases, however, such a differentiation is not only possible, but is, from many points of view, eminently desirable.

11. In countries like India and Ceylon, for instance, where there are two main divisions of Europeans and Natives, of which the latter largely predominates, it is evident that a distinction

should be made in all mortality investigations, since in any combined result the largeness of the native figures would have the effect of swamping the European experience, thus destroying valuable evidence of the effect of transplanting the European race in tropical climates.

12. Other instances of a similar nature which may be mentioned are—

(*a*) The United States of America, where the principal races represented may be said to be, European, African Negro, Asiatic, and Native Indian.

(*b*) The Commonwealth of Australia, where the races represented are European, Asiatic, Polynesian and Aboriginal Native.

(*c*) New Zealand, where the Maori population forms a distinct feature.

(*d*) South Africa, where the native races exist in large numbers side by side with imported Europeans and Asiatics.

13. In these and all similar cases details should as far as possible be obtained separately for each race. It will be readily understood from what has been stated that the distinctions here suggested are only those wide ones which can be ascertained with reasonable certainty. To attempt excessive refinement in such matters, would, by the multiplicity of detail involved, tend rather to confuse than to simplify the question, while the liability to error would frequently be increased by the paucity of of the data and the uncertainty of the classification.

14. *Sex.*—When due consideration is given to the constitutional differences between the members of the two sexes, to the varying conditions under which they live, to the dissimilarity of their ordinary occupations, and to the special risks incurred by either sex that are not incurred by the other, it is evident that marked differences in the rates of mortality experience are inevitable, and that in all investigations concerning mortality, details for the two sexes should be kept separate and distinct. In many of the newly settled parts of the globe the pioneers consist largely of males, while in the older countries, and even in some of the more populous centres of the newer, there is a preponderance of females. To institute comparisons in such cases, without making allowance for sex distribution, will evidently lead to unreliable conclusions.

15. *Age.*—One of the most important matters in connection with a mortality investigation is that of classifying the data according to age, and hence no returns of either population, or deaths in which full age particulars are wanting, can be considered complete. For example, in making use of death rates for comparative purposes, it is absolutely necessary that due allowance be made for age distribution, as otherwise the conclusions drawn from the results might be quite untenable, whilst for financial purposes it may be said that without such classification the mortality experience would be of little or no practical value.

16. *Occupation.*—Whether viewed from a financial, public health, or biological standpoint, the influence of occupation on mortality is a matter which calls for careful investigation.

For financial purposes the results of such investigations are of considerable value in connection with matters concerning friendly societies and similar bodies, while in enquiries respecting compensations to workmen, details of death from various causes in each occupation are of great assistance. Again, in considering the suitability or otherwise of the results of a general investigation for any particular financial purpose, due weight must be allowed for the manner in which the data were distributed as regards occupation.

17. From a public health point of view it is necessary to ascertain whether the death toll which any particular occupation is levying on those dependent on it is excessive, while in considering the relative salubrity of two districts as indicated by their respective death rates, due account should be taken of the prevailing occupations.

18. From a biological standpoint, statistics of occupational mortality extending over a series of years, furnish interesting indications of the improvement or deterioration of the race which is being brought about in consequence of the occupations in which the members of the community are engaged.

19. *Conjugal Condition.*—Where practicable the mortality experience should be investigated according to conjugal condition in conjunction with age, as the results so obtained are of considerable value.

20. From a financial point of view the matter is of importance in connection with the basis on which widows' and orphans' funds are established, and if particulars as to issue are included in the death records, material of special value

is rendered available for determining complex questions concerning allowances to orphans.

21. From a public health standpoint details according to conjugal condition have an important bearing on questions relative to deaths in childbirth, deaths from diseases peculiar to women, and especially from cancer of the female organs of generation.

22. Such particulars are also of value for biological purposes in considering the effects of marriage, childbearing, &c., on longevity.

23. The classes into which, wherever practicable, the data should be grouped are those of "married", "widowed", "divorced", "never married (of marriageable age)", "never married (of unmarriageable age)."

24. *Locality.*—For many of the purposes for which the results of a general investigation are required, it becomes necessary to obtain separate particulars for distinct localities or groups of localities. One of the principal of such distinctions is that which separates the urban from the rural experience, the former being usually more unfavourable than the latter, owing largely to the insanitary conditions consequent on aggregations of population. In countries where, owing to differences in altitude or in latitude there are marked variations in the climatic conditions, or where, owing to the proximity of the ocean, of forest lands, or of deserts, different districts are subject to special influences, it becomes necessary to ascertain the effect produced on the mortality experience of the community by such variations and influenees.

25. *Physical Incapacity.*—Statistics of physical incapacity are unfortunately very incomplete, owing to the practical impossibility of obtaining reliable particulars as regards the living. Where registration of deaths is practised the records furnish an account of the final effects of such incapacity, and if, in these cases, duration of disease could, in addition, be stated, valuable material would be available for ascertaining the effect of diseases at different ages. In most of the cases, however, in which such information would be of the greatest value, it would probably be impossible for either the patient or the medical attendant to say exactly when the disease started.

26. Apart from such considerations, details concerning causes of death are of very great value in connection with any general mortality investigation. Thus, in using a given

experience for financial purposes, it is of importance to know whether such agencies as war, pestilence, and famine have been in evidence in bringing about the mortality experienced, and, if so, to consider what allowance should be made for the possibility of such occurrences in the future.

27. From the public health point of view statistics of causes of death are invaluable, both as indicating directions in which the amelioration of existing conditions is required, and also as furnishing evidence of results produced by work already carried out.

SECTION IV.—SOURCES OF INFORMATION AND METHODS OF COLLECTION.

28. At the present day the principal sources of information concerning the number of years of life experienced by a given community are, primarily, the census returns of population procured periodically by actual enumeration, and, secondarily, the returns of births, deaths, and migration, while the most satisfactory source of information respecting deaths is that furnished by continuous registration.

CENSUS RETURNS OF POPULATION.

(A) *Ancient and Medieval.*

29. Although the numbering of the people was an occasional occurrence amongst the ancient Jews and Egyptians, and regular enumerations were, at a later date, carried out by the Greeks and subsequently by the Romans, it may be said that the practice of census taking, as that term is now understood, is a comparatively modern one.

30. The enumerations above referred to were undertaken mainly for purposes of war and taxation, and in most cases the numbers returned related only to those who were available for the former or were liable for the latter. Thus for most of the purposes for which the modern census is required, these enumerations would be of little value.

31. Census taking appears to have been practically unknown during the Middle Ages, although the compilation of the Breviary of the Charlemagne, and the preparation of the Domesday Book of William the Conqueror may be considered as partaking of the nature of the census.

(b) *European.*

32. *England and Wales.*—The first census of England and Wales was that taken on 10 March 1801. This has been followed by enumerations made decennially in the first year of each decade, each such enumeration being provided for by a fresh legislative enactment. Under the provisions of the "Equalisation of Rates Act, 1894", a special census of the Administrative County of London is now taken by the County Council at a date intermediate between the dates of the General Census of Great Britain, but the details obtained thereat are very meagre.

33. *Scotland.*—Censuses of Scotland have been taken on the same dates as those of England and Wales. From 1801 to 1851 inclusive, provisions for the Scotch Census were contained in the same Act as those for the Census of England and Wales, while from 1861 to 1891 inclusive, separate legislation was passed for the two divisions. In 1901, however, a reversion occurred to the original method of including all the provisions for the Census of Great Britain in the same Act. The actual work of collecting and compiling the Census of Scotland has always been quite distinct from that of England and Wales.

34. *Ireland.*—The first attempt to take an official census of Ireland was made in 1813, but owing to defective organization it proved abortive. The first authoritative and complete census of Ireland was taken in 1821, but the next census taken in 1831 was for various reasons considered unsatisfactory. From the Census of 1841, at which the Family or Occupier's Schedule was introduced, a very complete census has been taken decennially, the collection being effected by means of the Royal Irish Constabulary. As in the case of Great Britain, census taking is provided for afresh by the Legislature every ten years.

35. *Sweden.*—The earliest of the modern European Censuses appears to have been that taken in Sweden in 1751. This was followed by a census every three years (with three omissions) up to 1775, after that by a quinquennial census until 1860, and a decennial census from the last mentioned year up to the present time.

36. *Norway.*—The Norwegian Census, which is now taken in the tenth year of each decade, was instituted in 1815.

37. *Belgium.*—A special statistical service was created in Belgium in 1831, and a law for regulating the mode of census, enumeration, and the keeping of the population register was

passed in 1856. The Belgian Census is now taken in the tenth year of each decade.

38. *France.*—The first regular Census of France was taken in 1801, the second in 1806, the third in 1821, and the fourth in 1831. From the last-mentioned year onwards a census has been taken quinquennially, with the single exception that, owing to the Franco-Prussian war of 1870, the census which, in the ordinary course, would have been taken in 1871, was deferred until 1872.

39. *Spain.*—Census taking in Spain has proceeded very irregularly. The first three complete censuses were those of 1787, 1797 and 1857 respectively. A gap of 13 years also occurred between the census of 1887 and the latest enumeration made in 1900.

40. *Italy.*—A Bureau of Statistics was created in 1860, and the first general Census of Italy was taken on 31 December 1861, succeeded by further censuses in 1871, 1881 and 1901.

41. *Switzerland.*—The original constitution of the Swiss Federation required a census once in twenty years, but in 1860 a federal law was passed which prescribed a decennial enumeration. This, however, does not appear to have been strictly complied with, as the census of 1880 was followed by one in 1888, and this latter by the census of 1900.

42. *Germany.*—A central bureau was established in Prussia in 1805, through which population reports were obtained, and an enumeration of most of the German States appears to have been made in 1816. The German Census is now taken quinquennially in the fifth and tenth years of each decade.

43. *Austria.*—Prior to 1851 no complete Census of Austria had been taken, the only population returns obtained being those connected with military conscription. Censuses are now taken decennially, the last enumeration being that of 1900.

44. *Greece.*—The first census of modern Greece is stated to have been taken in 1836, and to have been succeeded by annual enumerations until 1845. From that year onwards censuses appear to have been taken at irregular intervals.

45. *Russia.*—Incomplete enumerations of the Russian population were made during the eighteenth and the early part of the nineteenth centuries, but the first and only complete census of the Russian Empire was that taken in 1897.

(c) *Asiatic.*

46. *India.*—Although provincial censuses attaining a considerable degree of accuracy had been undertaken in various

parts of the Empire in the earlier portion of the nineteenth century, the first synchronous census of all India was that taken in 1881. This has since been followed by similar enumerations in 1891 and 1901. A feature of the Indian Census method is the preliminary enumeration which is made some few weeks prior to the actual census day. At this enumeration all the required particulars are duly entered for every person then resident in each enumerator's district, so that all that is required on census day is the bringing up to date of these returns. This is effected by striking out the particulars concerning those who have died or departed in the meantime and adding others for those who have since arrived or been born.

47. *Ceylon.*—The first complete census of the island was taken in 1824 followed by further enumerations in 1871, 1881, 1891, and 1901. Both in scope and machinery the Census of Ceylon bears a strong resemblance to that of India, and on the last occasion the Indian method of preliminary enumeration and subsequent revision was adopted.

(D) *African.*

48. *Cape Colony.*—The first ordinary census of the Colony was taken in 1865, the second in 1875, the third in 1891, and the fourth in 1904. The intention to take a census in 1901 in harmony with the rest of the British Empire was frustrated by the abnormal and unsettled condition of the country, due to the war then being carried on in the Transvaal and Orange River Colonies.

49. *Natal.*—The first Census of Natal was taken in 1891, and the second in 1904, the South African war having, as in the case of Cape Colony, rendered a census in 1901 impracticable.

50. *Mauritius.*—The first census of the island was taken in 1846 and the second in 1851, the latter being succeeded by decennial enumerations at or about the date of the Census of the United Kingdom.

(E) *American.*

51. *Canada.*—The earliest Canadian Census was that taken in 1665 in the province of Quebec, then known as La Nouvelle France. Provincial censuses were of frequent but somewhat irregular occurrence down to 1871, when the first enumeration of practically the whole Dominion was made. From that date onwards a decennial Dominion Census has been taken.

52. *United States.*—Provision for a decennial census of the United States was made in the Constitution of 1787, and the first census was taken in 1790. This has been succeeded by others at decennial intervals, the twelfth census being taken for 1 June 1900. The method of enumeration in the United States has invariably been that of entry of particulars by the canvassing enumerator after enquiries made either *vivâ voce* or by post.

(F) *Australasian.*

53. *Commonwealth of Australia.*—The federation of the Australian States was accomplished on 1 January 1901, and all future censuses will be carried out by the Commonwealth Government. In an Act passed in 1905 the Commonwealth Parliament has made provision for a census to be taken in 1911 and thereafter in every tenth year. Prior to this each of the component States was responsible for its own census taking, which was effected at somewhat irregular intervals down to 1881, when the date coincided with that of the Census of the United Kingdom. A similar course was followed in connection with the censuses of 1891 and 1901. The earliest Australian Census was that taken in the State of New South Wales in 1828.

54. *New Zealand.*—The first census of New Zealand was taken in 1858, and was succeeded by others taken at irregular intervals down to 1881, from which year onwards a quinquennial enumeration has been made.

(G) *Methods of Enumeration.*

55. The various methods of enumeration may be classified on two distinct bases—

(i) According to the manner in which the information obtained at first hand is recorded;

(ii) According to the manner of determining the locality to which each person enumerated is to be credited.

56. As regards responsibility for the original record, it may be broadly stated that there are two distinct methods of enumeration, which may be respectively designated—

(*a*) The householder method; and

(*b*) The canvasser method;

while in respect of the locality to which the population enumerated is to be credited there are—

(*a*) *De facto* enumerations in which each person is credited to the locality in which he is living at the date of enumeration; and

(*b*) *De jure* enumerations in which he is credited to the locality in which he usually resides.

57. In an enumeration on the householder method, the occupier of each dwelling is held responsible for furnishing a written record of the desired particulars relative to the inmates of the dwelling occupied by him; while in one carried out on the canvasser method, the whole of the original entries are made by the enumerator as the result of direct enquiries instituted by him either *vivâ voce* or, in exceptional cases, by post.

58. Two important points in favour of the householder method are—

(i) That it effects a division of the labour of writing out the desired particulars;

(ii) That such division renders possible the furnishing of particulars for all parts of the country for the same point of time, and thus minimizes the likelihood of omissions, duplications or errors due to defective memory.

On the other hand, the principal drawbacks to the method lie in the difficulty experienced—

(i) In making clear to the householder exactly what is required of him, and

(ii) In obtaining his ready assistance and co-operation.

59. In support of the canvasser method, it may be urged that, with reasonable care in the selection of enumerators, the collection would be carried out by persons who would not only have a clear knowledge of what was required and how it should be recorded, but who would also have the ability to readily obtain the desired information by means of questions well chosen and carefully asked. The great drawback to the canvasser method lies in the fact that the enumeration is not synchronous for the whole country, and that it thus becomes difficult to avoid omissions and duplications due to changes of address during the

period the enumeration is in progress, and errors due to defective recollection of the circumstances as they existed on census day. There are, of course, many countries in which, owing to the general illiteracy of the population, the only method possible is the canvasser method.

60. A careful consideration of the various points in favour of and against these two methods leads, in countries where both are equally practicable, to the conclusion that for the purpose of obtaining a complete enumeration a census involving only a few simple inquiries should be taken by the householder method, and that for more complicated and difficult questions a special canvass unconnected with the general census should be made.

61. In considering the relative merits of *de facto* and *de jure* enumerations, it may be said that on the score of simplicity in collection the former method is the better, as it only requires from the collector for any given district a statement concerning the persons who spent the census night in that district. On the other hand, the *de jure* enumeration requires details for those who usually reside in the district, although temporarily absent from it on census day, and would require the exclusion of those only temporarily present at that date. This necessitates somewhat cumbrous provisions for records of vacant houses and subsequent communication with the absentees. Further difficulties arise in connection with the determination of what should constitute temporary residence in a locality, and what may be considered as a person's "usual place of abode."

62. The chief objections to the *de facto* method of enumeration are that under it visitors are included in the population while temporary absentees are excluded. To overcome these difficulties it is desirable to select as date of enumeration a point of time at which the displacement of population is at a minimum, and to make such enumerations as frequently as practicable.

63. A careful examination of the leading features of the *de facto* and *de jure* enumerations serves to indicate that, useful though the information aimed at by the latter method undoubtedly is, by far the simplest and most accurate enumeration is that carried out on the *de facto* principle.

64. Under certain circumstances it has at times been found necessary in carrying out a census to adopt special means of enumeration. Thus in the census of the natives of the Gold Coast in 1891, grains of Indian corn and cowrie shells were made use of for enumeration purposes, while in India bamboos

have on some occasions been similarly employed. Another method which has at times been put in practice in enumerating some of the remote districts of India has been that of taking a complete census of a few typical villages and obtaining an estimate for the whole district by taking into account the number of such villages. This method has also been suggested as applicable to some parts of Africa.

65. Another special method of obtaining census figures which has occasionally been employed, but which, from its liability to abuse, and the extreme uncertainty of any data on which to base it, should invariably be discountenanced, is that of making an addition, by way of percentage or otherwise, to the enumerated population to allow for those who were unrecorded. From the nature of the case it is evident that any such estimate must be extremely unreliable, and practically amounts to an attempt to make the census results agree with a pre-conceived idea of what the population should be.

Death Returns.

(a) *Family Records.*

66. These records, although of considerable value in connection with certain class statistics of mortality such as those relating to the Peerage, &c., are of little importance as regards the general population.

(b) *Burial Records.*

67. It is to the records of burials that we owe the earliest attempts at a comprehensive determination of mortality experience. In London the publication of such records appears to have been commenced as early as 1532, and from 1603 to 1849 their weekly publication continued uninterruptedly.

68. These records, however, cannot be regarded as furnishing any reliable estimate of the total number of deaths actually occurring in the area to which they relate. The principal causes of discrepancy are stated by Dr. Ogle to be (*a*) burial in the country of persons who have died in London; (*b*) non-inclusion of burials of persons who were not members of the Established Church; (*c*) neglect of parish clerks to supply returns. For these reasons he estimates that, to obtain complete figures, an addition of about 44 per-cent would require to be made to the

records for the 17th and 18th centuries, and that the bills from 1832 onwards would need an addition of at least 100 per-cent.

69. In certain German towns similar but fuller records appear to have been made known from a very remote period, and it was on the basis of the burial records of Breslau that Halley constructed his celebrated Mortality Table.

(c) *Civil Registration.*

70. The most reliable records of deaths are those obtained by means of compulsory civil registration, effected as early as possible after death has taken place. In the absence of complete registration for the whole of a country, it has been suggested, in the case of the United States of America, that a continuous sample registration might be resorted to, and that if the districts chosen for the purpose be fairly representative ones, the death rates ascertained therein might with reasonable accuracy be taken as denoting those of the larger areas from which the sample districts had been selected.

(d) *Census returns of Deaths.*

71. A further means of death enumeration is that of obtaining at a census, particulars of the deaths which have occurred during a period prior to the date of the census. Thus, in the Acts relating to the first five censuses of Great Britain, provision was made for abstracts of burials and baptisms to be furnished by the clergy for somewhat extensive periods, usually ten years, prior to the respective dates of enumeration. In these cases, however, the real work of collecting the particulars had been performed by the keepers of the various records. In the United States of America, on the other hand, it has been the practice since 1850 to attempt the collection of death statistics at first hand in connection with the ordinary census, but although the period covered by such inquiries has not exceeded the twelve months immediately preceding the date of the census, the results obtained have invariably been extremely defective, and furnish strong argument in favour of prompt registration, if reliable statistics of deaths are required.

Birth Returns.

72. In many matters connected with the determination of rates of mortality in a general population, reliable records of

births furnish valuable assistance, especially in connection with (*a*) estimate of intercensal population, (*b*) computation of rates of infantile mortality, (*c*) construction of life tables, (*d*) interpolation for individual ages, and (*e*) correction of inaccuracies in census returns for early ages. The main sources of such information are similar to those available in the case of death returns, being family records, baptismal records, civil registration, and census returns.

(A) *Family Records.*

73. As in the case of deaths, family records of births are of little practical value for the collection of statistics of the general population.

(B) *Baptismal Records.*

74. Birth statistics derived from baptismal records can only, even in the most favourable cases, be considered as furnishing very rough approximations. An interesting case of a celebrated investigator having been misled by such records is that of Dr. Price, who, in the construction of his Northampton Table, accepted the number of baptisms as representing the number of births, overlooking the fact that, owing largely to the presence in the district of a considerable number of Baptists, many of the children born did not appear in the Church baptismal registers.

(C) *Civil Registration.*

75. By far the most satisfactory and reliable method of collecting birth statistics is that of compulsory registration within a short period of the date of birth. The civil registration of births came into force in England in 1837, but it was not until 1875 that such registration was made compulsory. It has been estimated by Dr. Farr on the basis of census returns of population, that, during the period of voluntary registration, about 5 per cent. of the total number of births escaped record.

(D) *Census Returns.*

76. The collection by the census authorities of abstracts of baptismal records has already been mentioned in speaking of deaths, but, in addition to this, the census details collected in reference to population and deaths have at times been used to

determine indirectly the number of births, although a direct inquiry relative to births does not appear to have ever been made in connection with a census.

77. In connection with the census of the United States, where, as previously stated, death statistics are collected, it is usual to furnish an estimate of the number of births which have occurred during the year preceding the date of the census, by adding to the number of children returned as being under one year of age at that date, the number reported as having died whose births had occurred during the period under review. Such an estimate is recognized by the census authorities themselves as being liable to considerable errors arising from defective returns of both population and deaths under one year.

Civil Registration in various Countries.

78. The motives which have led to the establishment of the civil registration of births and deaths are different in different countries, but are in the main either legal, with the object of providing reliable records concerning the lawful ownership of property; or statistical, with the object of ascertaining the natural increase or decrease of the population. There is little doubt that in the early days of registration the legal motive preponderated, and the statistical results were merely a by-product. At the present time, however, the statistical aspect has assumed increasing importance and, where the results obtained are used for purposes of public health, is in many cases of equal if not greater value than the purely legal.

79. In England and Wales civil registration was instituted in 1837 under the Registration Act of 1836. In Scotland it was brought into operation in 1855, in Ireland in 1864, in the Isle of Man in 1848, in Jersey in 1842, and in Guernsey in 1840.

80. An excellent summary of the provisions for the "Registration of Births, Deaths and Marriages in the British Dominions beyond the seas" has been prepared by the Registrar-General in England on the basis of replies received by him to inquiries sent to the registration authorities of the various portions of the Empire. This summary was published as an appendix to the Registrar-General's 65th Annual Report (1902), and was also issued separately as an abstract. From it the following particulars have, in the main, been derived—

Civil Registration in British Dominions beyond the Seas.

Country	When introduced	Extent of Application	Whether Compulsory or Voluntary	Period within which notification is required to be given: Births	Period within which notification is required to be given: Deaths
EUROPEAN:					
Gibraltar	Births, 1848; Deaths, 1869	General	Compulsory	21 days	8 days
Malta and Gozo	1862	General	Compulsory	5 days	"without delay"
Cyprus	1895	General	Compulsory	31 days	24 hours
ASIATIC:					
Ceylon	1847	General	Compulsory	42 days	5 days
Straits Settlements	Singapore, Penang, and Wellesley, 1869, Malacca, 1870, Dindings, 1885	General	Compulsory	14 days	12 hours
Federated Malay States	1901	General	Compulsory	14 days	12 hours
Labuan	...	European Births. Deaths of Natives.	Voluntary	...	...
North Borneo	1884	General	Compulsory	...	...
Hong Kong	1873	General	Compulsory	...	...
Wei-Hai-Wei	1900	Europeans	Compulsory	...	...
AFRICAN:					
Cape Colony	1894 (Partial registration at earlier dates)	General	Compulsory	42 days	36 hours
Natal (General)	1868	Natives, with a few exceptions, are exempt	Compulsory	...	...
Natal (Natives)	1902	Natives	Compulsory	3 days	3 days
Orange River	1902	General	Compulsory	28 days	36 hours
Transvaal	1900	General	Compulsory	...	...
Basutoland	...	Europeans	Compulsory	...	...
Bechuanaland Protectorate	...	Deaths of Europeans	Compulsory	...	...
Mashonaland & Matabeleland	1902	Aboriginal Natives, with a few exceptions, are exempt	Compulsory	Urban, 42 days; Extra-Urban, 3 months	Urban, 48 hours; Extra-Urban 3 months
Gambia	1886	General	Compulsory	...	...
Sierra Leone	1857	General	Compulsory	42 days	5 days
Lagos	1867	General	Compulsory	21 days	2 days
Southern Nigeria	...	Natives, with certain exceptions, are exempt	Compulsory	...	...
St. Helena	1853	General	Compulsory	42 days	8 days
Mauritius	1793	General	Compulsory	45 days	24 hours
Seychelles	1794	General	Compulsory	30 days	24 hours

Civil Registration in British Dominions beyond the Seas—continued.

Country	When introduced	Extent of Application	Whether Compulsory or Voluntary	Period within which notification is required to be given	
				Births	Deaths
American:					
Ontario	...	General	Compulsory	30 days	"before interment"
British Columbia	...	All races except Indians	Compulsory	60 days	"before interment"
Manitoba	...	General	Compulsory	30 days	"before interment"
N.W. Territories	...	General	Compulsory	1 month	...
New Brunswick	...	General	Compulsory	30 days	...
Newfoundland	1891	General	Compulsory	14 days	10 days
Bermudas	...	General	Compulsory	14 days	14 days
Bahamas	...	General	Voluntary	42 days	21 days
Jamaica	1878	General	Compulsory	42 days	5 days
Turks and Caicos Islands	...	General	Compulsory	...	...
Leeward Islands	1856 to 1869	General	Compulsory	14 days to 30 days	"forthwith" to 8 days
Windward Islands	1864 to 1869	General	Compulsory	7 days to 14 days	3 days to 7 days
Barbadoes	...	General	Compulsory	42 days	...
Trinidad	1847	General	Compulsory	42 days	5 days
British Honduras	1885	General	Compulsory	42 days	4 days, or "as soon thereafter as can reasonably be done"
British Guiana	1868	General	Compulsory	...	...
Falkland Islands	1853	General	Compulsory	...	...
Australasia:					
New South Wales	1856	General	Compulsory	60 days	30 days
Victoria	1853	General	Compulsory	60 days	7 days
Queensland	1856	General	Compulsory	60 days	30 days
South Australia	1842	General	Compulsory	42 days	10 days
West Australia	1841	General	Compulsory	60 days	14 days
Tasmania	1839	General	Compulsory	60 days	8 days
New Zealand	1848 (Imperfect prior to 1875)	Maoris exempt	Compulsory	62 days	31 days
Fiji	1874	Europeans	Compulsory	2 months	1 month

81. In the annual Reports of the Registrar-General of England and Wales statistics of the registration of births and deaths for a series of years in the following countries are now published regularly—

British Empire.

England and Wales	South Australia
Scotland	Western Australia
Ireland	Tasmania
New South Wales	New Zealand
Victoria	Ceylon
Queensland	Jamaica

Foreign Countries.

Denmark	Servia
Norway	Netherlands
Sweden	Belgium
Russia	France
German Empire	Switzerland
Prussia	Spain
Austria	Italy
Hungary	Japan
Roumania	Chili

82. In dealing with the registration returns of births and deaths it is the almost universal practice to treat the number of registrations for any period as representing the number of births or deaths which have occurred during that period. This, it is clear, introduces a slight error, since the births or deaths registered in any period will contain some which occurred in a preceding period, while, on the other hand, some of those which occurred in the period will not be registered until a subsequent period. The number so brought forward or carried over will evidently be largely dependent on the time allowed for registration, and, consequently, since the allowance is usually greater for births than for deaths, it is probable that the registrations and occurrences for any period will be more nearly coincident in the case of deaths than in the case of births.

Migration Returns.

83. In any investigation concerning the mortality experience of a general population, the part played by migration is a matter

of considerable importance, both directly as affecting the numbers on which the computation of rates is based, and indirectly, as affecting the average healthiness of the population. In the case of a new country receiving settlers in large numbers, it is probable not only that the majority of the immigrants will be at the healthiest period of life, but also, that they will represent the healthiest of those at that period. There is thus evidently a selection in favour of the country to which such migration is being carried on, and a corresponding selection against the country from which the migrants are being derived. On the other hand if the country, district, or town towards which the migration is setting is a health resort, it is probable that the introduction of persons in various stages of ill-health will have the effect of increasing the death rates there, while diminishing them in those places from which the migrants have come, a selection similar to that mentioned above, but in the opposite direction.

84. Other cases which require consideration are those of the migration to towns in the prime of life and the return migration to the country at advanced ages, as well the cases of migration to a particular locality owing to the existence therein of some large public institution for the treatment of persons either bodily or mentally afflicted. Another case in which mortality rates are affected by migration is that in which persons who, when in good health, are engaged in some casual employment in a particular district, on becoming incapacitated return to their native districts for attention and treatment.

85. Although in the cases mentioned the general effect of the migration is readily seen, an exact numerical determination of the extent of such local migrations is in many instances by no means easily effected, and frequently all that can be done is to call attention to their bearing on the computed rates without making any attempt to accurately measure them.

86. Statistics of migration may be obtained either directly by means of records of arrivals and departures, or indirectly from an examination of the census returns of population. In Sweden and some other European countries, permanent changes of residence are required to be notified to the authorities by whom the facts are registered, but tranfers of a temporary nature do not appear to be recorded. The best known and most extensively applied form of the direct method is that of using passenger lists to record the arrivals and departures of oversea migrants. It is

to be feared, however, that much of the detail supplied on passenger lists is quite unreliable, and consists in many cases of guesses made by the purser, or any officer responsible for the task of filling up the lists, and it appears doubtful whether anything further than number, sex, port of embarkation and destination can be generally relied upon. Records of migration may also be obtained by procuring from the railway authorities returns of passengers passing through border stations, but here again it appears impracticable to attempt the collection of extensive particulars. Similar returns may be obtained in certain cases of migration by river or other internal waterway, but the collection of particulars of migration by these means would in most cases be quite impracticable.

87. Census returns of population classified according to birth-places also furnish valuable indications of the nature and effect of migration, and a comparison of such returns for two consecutive censuses will serve to show the net result of migration and mortality during the intervening period. Such returns, however, furnish no evidence of the influx and outflow of population, but only of the resulting residuum. In cases where length of residence in the country of enumeration is made a subject of census inquiry, further valuable information concerning this residuum is rendered available.

Section V.—Preparation of Data.

88. In connection with the preparation of the raw material for use in the computation of rates of mortality, the following matters present themselves for consideration:

(*a*) Tabulation of data.

(*b*) Detection, correction and prevention of errors.

(*c*) Interpolation of particulars for individual ages.

(*d*) Intercensal population estimates.

(*e*) Methods of calculating mean population.

(A) *Tabulation of data.*

89. The raw material of a general mortality investigation consists primarily of a detailed description of the individual members of the community, together with a corresponding description of those members who have died during the period to

which the investigation relates. The death records, if well kept, contain full particulars for all who die, but the population records are not in any case nearly as complete. All that any population census can attempt is, to give a sample list of such descriptions as at the date of enumeration. It then devolves upon the investigator to make the best estimate he can, concerning the complete experience of lives for the period.

90. For reducing the raw material to such form as will admit of quantitative results being obtained, there are in vogue two principal methods of tabulation, the schedule system and the card system, the latter being further divided according to whether the work is done by hand or mechanically.

91. The schedule system, which is the older, proceeds on the principle of placing on specially prepared schedules, a mark for each occurrence in the original lists of a given characteristic or group of characteristics, the totals of such marks giving the material requisite for the preparation of the tables required for publication.

92. Although very simple in principle, the schedule system is by no means well suited for the purpose to which it is applied, some of the principal drawbacks being the cumbrous character of the forms required, the liability to omission or duplication in recording, and the difficulty of checking.

93. The card system involves the transfer of full details from the original lists to cards, and may at first sight appear to involve more labour than the schedule system. The advantages attaching to the use of cards are, however, so great that the work of copying is paid for many times over in the facility and accuracy with which the subsequent tabulation can be effected. The adoption of the card system for the tabulation of population returns and vital statistics, although not yet brought about in the census and registration departments of the United Kingdom, is gradually extending, and in 1901 the Indian Census was, for the first time, tabulated by means of cards. The innovation appears to have been very satisfactory for it was estimated that, in addition to other advantages, a saving in cost of £45,000 was thereby effected.

94. The earliest instance of the use of cards for statistical purposes appears to have been that in which Mr. A. G. Finlaison, of the National Debt Office, London, in the year 1852, carried out by means of cards an investigation into the mortality experience of Friendly Societies.

95. As far as can be ascertained, cards were first used for census purposes at the Austrian Census of 1871, taken under the direction of Professor Von Mayr. Since then the card system has been extended to all quarters of the globe for the tabulation of statistics of various kinds, and particularly in connection with population and vital statistics. It is worthy of note, too, that there is no instance on record of a change from the card to the schedule system.

96. In Australia the card system is very largely used for statistical purposes, and in addition to census and vital statistics is extensively employed in connection with statistics of Friendly Societies, crime and hospitals. Cards were first used in Australia for census purposes at the Victorian Census of 1881, but at the succeeding censuses of 1891 and 1901 they were employed in all the Australian States and the Colony of New Zealand. They were also used at the Censuses of Cape Colony taken in 1891 and 1904, and, as previously noted, were employed for the first time at the Indian Census of 1901. In all these cases the cards used were marked, sorted, and counted by hand. In Canada and the United States a card system involving mechanical marking and electrical counting has been in operation at the last two censuses, namely, those of 1891 and 1901 in Canada, and 1890 and 1900 in the United States. Canada appears to be the only portion of the British Possessions that has adopted the mechanical method of card tabulation for census purposes.

97. An important feature of the card system is the fact that it enables minute subdivisions of the data to be effected with a minimum of labour and a maximum of accuracy. By means of an initial sorting into well-defined groups common to many inquiries, any special investigation need concern itself only with those cards relating to the particular matter in hand.

98. In cases where hand marking is practised, the cards employed for census purposes in different countries vary considerably in form, the main distinction being that while some provide one blank space for each item of information to be written in, others provide separate specially marked compartments for the more important classes under each item, and a blank for writing in the less important, the former being recorded by means of a mark across the proper compartment.

99. Where very large numbers of cards are involved, mechancial methods of dealing with them have in recent years

been adopted in certain cases, thus effecting a very considerable saving in labour. One of the best known of the mechanical devices used for this purpose is the Hollerith electrical tabulating machine, a description of which is given by Mr. W. F. Willcox, the Chief Statistician of the American Census Office, in a paper read by him on the 29th December 1899, before the American Economic Association, and subsequently published in pamphlet form.

100. His description is as follows:—

" A card some three by six inches is used to record the " facts for each person in the United States. A section of the " card is reserved for the information in reply to each question " on the schedule, the answer to which is to be used in the " tables, and a part of each section is reserved for each possible " answer to the question. Thus a narrow strip near the left of " the card is reserved for the answer regarding race or colour. " In the census volumes five race divisions, namely, White, " Black, Chinese, Japanese, and Indian, are recognized, and the " answer to this question for every person must be entered under " one or other of these five classes."

101. " Entry is made on the card by punching a little hole " in the proper place. One hole and only one is to be made in " each section, that is, the person must be of some race, some " sex, some age, &c."

102. " The card is then placed in a machine and a system " of blunt pins brought down upon it, one pin at every place " where a hole might occur. Wherever a hole has been made, " the corresponding pin passes through into a cup containing " mercury, elsewhere the pins are pushed back by the resistance " of the card. Each cup is connected with a counter having a " dial capable of registering to 10,000. The electric current " passes through one section of the card at the place a hole " has been made, and into the mercury, then through the counter " connected with the mercury cup, then to the next section and " through the hole there made and the counter connected with " it, and so on.

103. " If in any section no hole has been punched, the " circuit is broken at that point, and nothing on the card is " recorded, the bell introduced at another point in the circuit " fails to ring and the card is thrown aside for correction. The " machine is thus a device for simultaneous mechanical tallying, " and can be modified and combined with various accessory " devices, according to the needs of the particular case."

104. In a paper on "The Electrical Tabulating Machine", read before the Royal Statistical Society, London, Dr. Hollerith, the inventor of the machine, mentions, amongst other things, how, by a simple use of the electrical relay, the counting of various details in combination can be effected, the number of items which can be combined being only limited by the number of counters and relays. He also describes an electrical device used for a preliminary sorting with a view to reducing the number of counters required in the actual process of tabulation, and points out that punching errors involving certain inconsistencies would be detected by the machine, and the cards concerned would be rejected.

105. It will be seen that the Hollerith device, although usually spoken of as a tabulating machine, is really a counting machine, and the work done by it is that of counting the number of holes in the various portions of the cards. In this operation the work of sorting is involved in only a subordinate way, and the process bears a certain resemblance to the schedule method of tabulation, the records of the electrically manipulated counter taking the place of ticks or dots made by hand.

106. Another interesting account of a compilation by means of cards dealt with mechanically is given in *J.I.A.*, vol. xxxvii, in a contribution by Mr. D. P. Fackler, on the special mortality investigation instituted by the Actuarial Society of America. According to this method the cards are punched and then sorted, the machines for both purposes being shown and described in the contribution referred to.

107. In addition to the census offices of the United States and Canada, those of Austria and Russia have adopted the Hollerith machine, while the advisability of adopting it has been seriously considered by several other countries. In England it has been contended that the simplicity of the schedule renders mechanical tabulation less valuable than where more complicated or more extensive details are concerned. In this contention, however, sight appears to be lost of the fact that complication in tabulating arises more from the combination of particulars than from the number of enquiries; and any device which promises to facilitate such combinations, and thus enable the information to be thoroughly analyzed with a minimum of labour and a maximum of accuracy, is at least deserving of a trial. Again, in some British Colonies it has been urged that the numbers involved are too small to warrant the use of the machine. Here

also it appears doubtful whether, unless the numbers are very small indeed, mechanical tabulation might not with advantage be adopted. In India a further objection on the score of expense has been raised; and it is pointed out in the 1901 Census Report that, whereas the initial cost of the requisite machinery would be considerable, native clerical labour is so abundant and cheap that the work could be much more economically performed by hand. In this case the gain in reduced supervision and in reliability of results appears to have been overlooked, for it is stated in another part of the report that there is a great tendency for the average tabulator to "fudge" his results when, by so doing, he could avoid the investigation of some error which a comparison of results has rendered apparent.

(B) *Detection, Correction and Prevention of Errors.*

108. It is here proposed to briefly consider some of the principal circumstances which lead to inaccuracies in furnishing the required information and in interpreting the results, and to investigate methods for securing the detection and correction, or better still, the prevention of such inaccuracies.

109. *Race.*—In most cases no special enquiry as to race is made. In some instances birthplace statistics alone are relied upon to furnish the desired information, while in others name and religion are also taken into account. Even with this latter precaution, results so obtained must be considered as approximations only. Occasionally a special instruction is given on the census schedule that if a person belongs to some specified race a note to that effect is to be placed in the column provided for the birthplace. Such a course is unsatisfactory, since any enquiry which necessitates for a section of the community an addition to be made in a given column to the information of a similar character obtained for the rest of the community, will usually result in defective information. If an enquiry concerning race is to be made, there should, on the census schedule and death certificate, be a special column for race, not a provision for inclusion in certain cases with some other item.

110. *Sex.*—As regards sex it might be imagined that so simple an enquiry could not possibly result in the furnishing of erroneous returns. There are, however, several instances on record of more or less serious errors of classification according to sex.

111. An interesting case is noted in the Report on the Census of England and Wales for 1901, where it is pointed out that owing to a preponderating tendency on the part of tabulators to enter females in the columns intended for males, the number of females had, at each census since 1851, been overstated. The estimated amount of the over-statement for 1851 has not been placed on record, but for the succeeding censuses the estimated errors are, 12,908 in 1861; 10,373 in 1871; 8,995 in 1881; 7,500 in 1891; and 5,117 in 1901.

112. Similar errors will occasionally occur where the card system of tabulation is in use, through particulars relative to a male being entered on a card intended for a female or *vice versâ*. The only reliable preventive of errors of this nature is a thorough and systematic independent check of all copying or tabulating. To trust solely to the accuracy of the original copyist or tabulator is to take a risk which practical experience in such matters invariably shows to be much too great.

113. In connection with the Indian Census it has at times been suggested that a disproportion found to exist between the sexes was due to incomplete enumeration of unmarried females of nubile age, the presence of such girls having been concealed owing to the degradation which in some parts of India is considered to attach to the household containing them. This view is not supported, however, by the census authorities of 1901, who attribute the discrepancy in part to misstatement of age and in part to heavy female mortality resulting from early pregnancy.

114. In the Report on the Western Australian Census of 1901 a sex error in the population estimate is noted, and is stated to have been probably due to insufficiency of details on passenger lists. The remedy in such a case as this is evidently that adopted, namely, the employment of more complete lists, combined with a careful scrutiny of the information furnished.

115. *Age.*—There is probably no subject of enquiry so liable to error as that of age, although there is little doubt that the majority of such errors are simply due to ignorance on the part of the informants. A person who knows that he is about 40, but does not know his age exactly, will put himself down as 40, while one who only knows that he is between 40 and 50 will usually put himself down as 45. Such errors evidently tend to swell the numbers recorded at ages ending with 0 or 5, and to

correspondingly reduce the numbers recorded at certain of the adjacent ages.

116. Certain of these errors are probably due to laziness or indifference, while others again are due to nothing but vanity, this latter feature exhibiting itself in the desire of certain persons of middle age to appear younger than they really are, and the desire of others of advanced age to appear older than they are. It is probable that in many cases neither the understatement nor the overstatement just referred to is due to actual dishonesty, but that the desire that a thing should be has led to the belief that it is. The record of ages is a matter which is more liable to errors of this nature than almost any other that could be mentioned, since the advance of age is continuous, but is unaccompanied by any definite marks of progress.

117. Other inaccuracies arise from misapprehension of the terms of the enquiry, and current year of age is occasionally stated in reply to a question as to age last birthday. Such errors are most frequent in the case of young children and lead to distorted returns for such ages, particularly at 0, 1, and 2, the figures shown for age 0 being frequently less than that for age 1, notwithstanding the heavy death rate invariably incidental to the first year of life.

118. Finally, there are cases where the misstatement—sometimes in one direction, sometimes in the other—is wilful, and is made to conceal previous misstatements or to secure certain advantages.

119. A notable instance of age error, and one to which considerable attention has been devoted, occurs in the Censuses of England and Wales, in the case of women recorded as being aged 20 to 25. For several successive censuses there has, on each occasion, been recorded a greater number of females aged 20 to 25 than could possibly have survived out of the number aged 10 to 15 at the preceding census, and, consequently, in the absence of a large net immigration of females of that age, it is clear that extensive misstatement has taken place.

120. Errors due to accumulation at quinquennial ages are rendered very conspicuous by the graphic method of plotting to scale the number at each age, the result being an extremely serrated line whose high points from about age 20 onwards are those corresponding to ages ending in 0 and 5, and whose low points occur at ages immediately preceding and succeeding these.

121. Since errors of this nature are not likely seriously to

affect the results for remote ages, it is evident that by means of suitable grouping much better results may be obtained for age groups than for single ages. The determination of a suitable grouping must, in each case, be dependent upon a general consideration of the circumstances likely to lead to such errors in the community under review, taken in conjunction with the evidence which the records for single ages furnish of the effects produced by such circumstances. It is thus evident that in all cases the initial tabulation should be one at single ages, and that an original tabulation in age groups proceeds on the principle of determining the composition of the groups without obtaining the particulars which are essential for correctly ascertaining the nature of the grouping most suitable to the particular investigation.

122. A careful examination of the returns for single ages will usually serve to show that the accumulation at any quinquennial age has, in very large measure, been drawn from the age immediately preceding and that immediately following it, and consequently that these two ages should be associated with the quinquennial age in any grouping that may be adopted. It will thus appear that the best grouping for general purposes is that in which the quinquennial age occupies the central position. Under such a method, particulars for single ages might in all cases be given for ages 0 to 7 inclusive, and in quinquennial groups 8–12, 13–17, 18–22, &c., from that point onwards. As previously pointed out, however, it is not desirable to lay down a hard-and-fast rule in this matter.

123. The grouping adopted by the Registrar-General of England and Wales has at times been the subject of unfavourable comment on the grounds of want of uniformity at different stages, commencing, as it does, with single ages, and then proceeding through quinquennial and decennial groups to a final group of indefinite extent. While it is admittedly undesirable to have any grouping that is not preceded by details for individual ages, there appears to be no valid reason for objecting to variations in the size of the several groups. On the contrary, the size of the group at any stage should be determined, not by any preconceived uniform standard, but by general considerations concerning the special circumstances of the case, and an examination of the details for individual ages.

124. It may be noted, in passing, that in all instances the original tabulation should furnish the number of cases in which

age is unspecified, and should not, as is sometimes done, give only the results obtained after an adjustment has been made in the data to allow for unspecified ages.

125. The most satisfactory correction for quinquennial accumulation is that of suitable grouping and subsequent interpolation for single ages, while for errors arising at infantile ages the best method of correction is probably that of the Registrar-General of England and Wales, who, by means of birth and death records for a series of five years, redistributes the census particulars for ages under 5. There does not appear to be any suitable correction for errors due to vanity and wilful misstatement, except in so far as such errors are affected by the grouping and redistribution referred to above.

126. To diminish the number of age errors it has been proposed to ask for "date of birth" instead of "age last birthday." This suggestion appears worthy of careful consideration, and has the distinct advantage that for any given person the "date of birth" is constant throughout life, while "age last birthday" varies annually.

127. *Occupation.*—An attempt to ascertain the effect of occupation upon mortality can usually only be considered satisfactory up to the age of 65, owing to the vagueness with which occupations are frequently stated, both in the census returns and in the death records for persons above that age. In many countries, too, such an enquiry would have to be restricted to males, since the majority of adult females are married, and their occupation would usually be stated as "Married Woman", "Domestic Duties", &c., notwithstanding the fact that a considerable portion of their early life may have been spent in some specific occupation. It may in addition be pointed out that the vagueness of designation referred to above is a defect which does not attach solely to advanced age, but it is to some extent in evidence at all ages in all census and registration returns.

128. The importance of obtaining accurate census details of occupations, particularly in these days of highly specialized callings, affords very strong grounds for the contention in favour of the system of enumeration by trained canvasser as against that by the more or less casual householder.

129. In considering the effect of occupation on mortality the relative heaviness or lightness of the occupation requires to be taken into account, since it is probable that the magnitude of the death rates in some occupations is due less to the inherent

unhealthiness of the occupations themselves, than to the fact that their lightness enables persons of weak constitution to find employment in them. The nature of the calling thus brings about, amongst those entering on occupations, a selection in favour of the heavier, as against the lighter, occupations. The death rate amongst the heavier occupations is further reduced by damaged lives drifting from them into and unduly swelling the mortality of the lighter occupations.

130. To obtain absolutely reliable information concerning occupational mortality, it would be necessary to take into account age at entry, physical condition at entry, and duration of employment, for the various occupations, details which the census returns and death records fail to supply. While it is thus impracticable in most cases to make due numerical allowance for these factors, their existence should always be borne in mind in making comparisons of death rates for different occupations. In all investigations concerning occupational mortality, extensive analyses with respect to age should be made, as otherwise, utterly misleading conclusions will be drawn owing to the marked difference in age distribution exhibited by the data for different occupations.

131. *Conjugal Condition.*—It has at times been contended that, because the recorded death rates are higher for single than for married males, therefore marriage has the effect of lengthening life. Such a conclusion is erroneous, since the married constitute a select class, while the single comprise some who are voluntarily celibate, and others who from physical incapacity could not undertake the responsibilities of matrimony.

132. *Locality.*—The chief obstacle to the correct determination of locality death rates is the difficulty of correctly allocating the number of years of life and the number of deaths experienced. Estimates of the population of specified localities for intercensal periods are frequently very unreliable, whilst, owing to migration, deaths are often recorded in one locality while the corresponding number of years of life is credited to another. For the former defect, more frequent enumerations would appear to be the only remedy, while for the latter a complete correction could only be obtained by means of elaborate statistics of length of residence.

133. *Physical incapacity.*—In determining and comparing death rates connected with various forms of physical incapacity, errors are apt to occur through deficiency and lack of uniformity

in the certification and classification of the causes of death. As regards certification, the verdicts of coroners' juries are frequently extremely indefinite, whilst vagueness of description is also not infrequent in the case of medical certificates. With reference to classification, it is clear that if the death statistics of two communities, tabulated according to cause, are to be strictly comparable, the systems of classification should be identical, and scope for differences of opinion on the part of different classifiers should as far as practicable be eliminated, particularly in connection with multiple causes of death.

134. For the purpose of minimizing this class of error it may be suggested—

(i) That in all cases causes of death be explicitly stated.

(ii) That in cases of deaths from multiple causes, the more probable cause be indicated, and also that a compound tabulation be effected, showing the number of such cases and the associated causes.

(iii) That a universal classification of causes of death be adopted.

(c) *Interpolation of particulars for individual ages.*

135. For obtaining particulars for individual ages from data for grouped ages, methods of interpolation have at various times been employed—

(1) The statistical method.

(2) The analytical method.

(3) The graphic method.

The English census authorities usually employ the first two of these in combination, while the third is that which was used by Milne in constructing the Carlisle Table.

136. The distinctive features of these three, as applied to population returns, are, that the statistical method bases the distribution of the groups upon the normal proportion of survivors at individual ages, as ascertained by means of birth and death statistics; the analytical method proceeds on the assumption, express or implied, that the population at or above any age may be stated as a function of that age; while the

graphic method is based on the assumption that, if the true population at each age were plotted to scale with the years of age as abscissæ, the line joining the extremities of the ordinates would be a smooth curve. It will thus be seen that, as regards the processes of computation involved, the three methods referred to may be described respectively as the arithmetical, the algebraical, and the geometrical.

The Statistical Method.

137. The principle involved in this method is that of computing the number, who, on the basis of births and deaths registered, would at the date of the census be living at each age in a given group, and then distributing the number actually recorded in that group over the individual ages in proportion to to the number of survivors so ascertained. With complete registration of births and deaths, and in the absence of migration affecting the group, a distribution of this nature would be absolutely correct, but in practice such ideal conditions would usually not hold.

138. It is of interest to note that, where migration at the ages under review is experienced, this method introduces a double error, although probably not of serious extent, except where the migration has been considerable. In the first place such migration will affect the death statistics and render them unsuitable for the determination of a normal distribution, and in the second, the effect of the migration, if at all extensive, will be such as to render unsuitable the application of a normal distribution even if correctly determined.

139. At the Censuses of England and Wales prior to that of 1901, the statistical method of interpolation was generally employed for ages 0-1 to 4-5 inclusive, but on the occasion of that census the method was extended so as to apply to ages 0-1 to 24-25, two groupings, of 0-1 to 14-15, and 15-16 to 24-25, being made use of.

The Analytical Method.

140. Under this head may be grouped all those plans for distribution which are based on the assumption, express or implied, that the numbers at or above successive ages may be represented by a definite mathematical law.

141. It has been usual in connection with the Censuses of England and Wales to prepare the greater portion of the estimated distribution by the analytical method, and, prior to the census of 1901, such particulars for all ages from 5 upwards were computed by means of formulas of interpolation applied to the logarithms of the number of persons above given ages. At the 1901 Census, the statistical method was extended as far as age 24-25, and the analytical method was used to obtain a distribution from age 25 onwards. The assumption involved in this distribution was that, if N_x denote the number enumerated above age x, $\log N_x$ will, for values of x between certain limits, represent a series having a constant fourth difference, and consequently, that N_x may be denoted by $10^{k+ax+bx^2+cx^3+dx^4}$ within the limits referred to.

142. In his memorandum on the Age Tables of the Indian Census of 1901, Mr. G. F. Hardy effects a distribution based on all three of the methods mentioned; from age 0 to age 9 on the statistical method; from 10 to 54 on the graphic method; and from 55 onwards on the analytical method.

143. In his analytical treatment it is assumed that $\log N_x = k + ax + bx^2 + mc^x$, giving $N_x = 10^{k+ax+bx^2+mc^x}$. The value of c was taken as $10^{\cdot 039}$ and the values of k, a, b, and m, were obtained from the numerical values of N_x when $x=15$, 45, 55 and 65. The character of the function, Mr. Hardy says, "accords very closely with the nature of the normal population "curve, and having four unknowns (when the value of c is "assumed) it is sufficiently flexible."

The Graphic Method.

144. In the analytical method dealt with above it is assumed that between certain age limits, the population is a function of the age, and that consequently, when such populations are plotted to scale, the result will be a smooth curve. The graphic method of distribution aims at the production of a smooth curve without any intermediate analytical process, and endeavours to represent as nearly as practicable the original data, amended only by the removal of certain errors due to inaccuracies or deficiencies in the returns supplied.

145. One of the best known instances of the use of the graphic method is that in which it was adopted by Milne in the construction of the Carlisle Table. The application of this

method consists in erecting, on abscissæ proportional to the number of years in the several age groups, rectangles, the areas of which are proportional to the population in the respective groups. The upper boundary of the figure is made up of alternating horizontal and vertical lines, and the object of the method here outlined is the substitution of a smooth curve for the serrated boundary in such a manner that the area of the figure remains unaltered, and also that the areas corresponding to the various groups are changed as little as is consistent with the production of a reasonably smooth curve. If a year of age be taken as the unit of abscissa, the ordinate to the curve measured from the centre of the appropriate unit will give the population for the corresponding year of age.

146. A practical difficulty in connection with this method is that of reading the ordinates true to a number of significant figures. In a paper on the investigation by Messrs. Moors and Day into the mortality experience of New South Wales and Victoria (*J.I.A.*, vol. xxxvi, p. 151) it is stated that, in applying Milne's method, a pine table 5 ft. by 3 ft. was obtained and cross-ruled into half inch squares. This is said to have worked admirably, the curves being traced out by means of a length of cotton pinned into the required position.

(D) *Intercensal Population Estimates.*

147. For the purposes both of providing data for a current intercensal period, and of supplying particulars for intermediate dates in a completed period, it becomes necessary to adopt some means of estimating the population for dates other than those on which censuses are taken. The following are some of the methods which either have been or may be adopted—

(i) The geometrical progression method.

(ii) The arithmetical progression method.

(iii) The statistical method.

(iv) The combined progression method.

(v) The habitation method.

148. In the following account of these methods it has, for the sake of simplicity, been assumed that a census is taken once in 10 years, but the reasoning will apply equally well to an intercensal period of any other length.

The Geometrical Progression Method.

149. This method is based on the assumption that, during a given intercensal period, the population increases or decreases throughout at a uniform rate, or, in other words, that if the population at the commencement of such a period, and that at equidistant points of time throughout the period were set down, the resulting series would be a geometrical progression.

150. In the practical application of this method to obtain particulars for any specified point of time in a completed census period, the procedure is as follows:—

Let $$\phi_{\overline{0}}\ ,\ \phi_{\overline{1}}\ ,\ \phi_{\overline{2}}\ ,\ \ldots\ \phi_{\overline{10}}$$

denote the population at the end of years 0, 1, 2, . . . 10, of a decennial census period, $\phi_{\overline{0}}$ and $\phi_{\overline{10}}$, thus representing the population enumerated at the initial and terminal censuses. Further, let i denote the annual effective rate of increase per unit of population experienced throughout the period, so that—

$$\phi_{\overline{10}} = \phi_{\overline{0}}\ (1+i)^{10}$$

then $$\log(1+i) = \frac{\log\phi_{\overline{10}} - \log\phi_{\overline{0}}}{10}$$

from which i may be readily determined.

151. By means of the value of i so ascertained, the estimated population, $\phi_{\overline{n}}$, at any point of time n, where n is any number whole or fractional less that 10, is obtained by the formula $\phi_{\overline{n}} = \phi_{\overline{0}}\ (1+i)^n$.

152. To obtain an estimate for a current intercensal period this formula will also be used, i being the rate of increase ascertained as above for the preceding completed period.

153. This method has been used by the Registrar-General of England and Wales for many years, both for furnishing estimates for the current period, and also for effecting retrospective corrections in estimates already made.

The Arithmetical Progression Method.

154. This method differs from the geometrical progression method in assuming for equal intervals of time a uniform numerical increase instead of a uniform *rate* of increase. Thus, if as before, $\phi_{\overline{0}}$, $\phi_{\overline{1}}$, $\phi_{\overline{2}}$, . . . $\phi_{\overline{10}}$ denote the population at the

end of years 0, 1, 2, . . . 10, and if I denote the uniform annual increment during the period,

then $$\phi_{10} = \phi_{0} + 10 \times \mathrm{I}$$

and $$\mathrm{I} = \frac{\phi_{10} - \phi_{0}}{10};$$

while $\phi_{n} = \phi_{0} + n\mathrm{I}$, where n is any number whole or fractional less than 10. For a current intercensal period, I will denote the numerical increase ascertained as above for the preceding completed period.

THE STATISTICAL METHOD.

155. Since the population at any date consists of the number at the date of the preceding census, increased by the births and arrivals and diminished by the deaths and departures which have since taken place, it is clear that if reliable statistics of these events are available accurate estimates can readily be obtained.

156. If, as before, ϕ_{n} denote the population at the point of time n in an intercensal period, and if β_{n}, θ_{n}, a_{n}, and δ_{n} denote the number of births, deaths, arrivals, and departures between the points of time 0 and n in that period, we shall have—

$$\begin{aligned} \phi_{n} &= \phi_{0} + (\beta_{n} + a_{n}) - (\theta_{n} + \delta_{n}) \\ &= \phi_{0} + (\beta_{n} - \theta_{n}) + (a_{n} - \delta_{n}) \end{aligned}$$

The quantity $(\beta_{n} - \theta_{n})$ is termed the "natural increase of the population", while $(a_{n} - \delta_{n})$ is generally known as the net immigration.

157. If for any decennial census period the initial and terminal censuses and the migration and vital statistics were absolutely accurate, the terminal census would furnish the same result as the population estimate for that date; that is to say—

$$\phi_{10} = \phi_{0} + (\beta_{10} - \theta_{10}) + (a_{10} - \delta_{10}).$$

In almost all cases, however, it will be found that the estimated exceeds the enumerated population, thus indicating an error in some one or more of the contributing items. From the nature of the case it is evident that there are no reliable means of directly ascertaining the nature and extent of the error in any particular item, and as none of the records, whether of

population, births, deaths, arrivals, or departures can lay claim to absolute accuracy, the discrepancy may be said to have been made up of inaccuracies of greater or less magnitude in all the contributing items. The chief source of inaccuracy is probably the record of departures, owing to certain emigrants by sea leaving without previously booking their passages.

158. The convenient assumption, therefore, is usually made that all the items are correct except the emigration figures, and the necessary adjustment is consequently made by suitably amending the value of δ_{10} in the above formula.

159. Thus, if the terminal census result fell short of the estimate for that date by u, it would be necessary to substitute $\overline{\delta_{10}}+u$ for δ_{10} in the formulas shown above; that is to say, to increase the departures for the period in the ratio of $\overline{\delta_{10}}+u$ to $\overline{\delta_{10}}$. This difference u is generally spoken of as being due to "unrecorded departures", and $\frac{100u}{\delta_{10}}$ is the "percentage allowance for unrecorded departures" made use of both for correcting the figures for the completed period, and also, with or without modification, for furnishing current estimates.

160. The statistical method has been used for intercensal population estimates in Australia and New Zealand from the earliest times, and has, on the whole, given very satisfactory results. It is evident that in a new country where a large proportion of the fluctuation of population is due to migration, the geometrical progression method is quite inapplicable, while variations in migration from one census period to the next are such as to render the arithmetical progression method unsuitable.

The Combined Progression Method.

161. Although a method of estimating population based on a combination of the geometrical and the arithmetical progression methods does not appear to have ever been used in practice, such a method would seem to be worthy of consideration, and would, probably, in many cases be more in accordance with the actual state of things than either the geometrical or the arithmetical progression method.

162. If increase of population depended solely on excess of births over deaths, and if birth and death rates varied but slightly from year to year, the population for a series of years

might be represented approximately by a geometrical progression. If, on the other hand, the increase were due solely to excess of arrivals over departures, and such arrivals and departures varied from year to year within fairly narrow limits, an arithmetical progression might be applied. Since, however, these forces are usually in operation in conjunction with each other, it would appear that a combined progression would represent the position of affairs more nearly than either taken alone.

163. Thus, if $\phi_{\overline{n}|}$ be given the value previously assigned to it, and if it be assumed that each year the population increases by a constant proportion plus a constant number, we shall have $\phi_{\overline{1}|}=\phi_{\overline{0}|}(1+i')+\mathrm{I}'$, where i' denotes the constant annual proportionate rate of increase per unit of population, while I' denotes the further annual numerical increment.

$$\text{Similarly}\quad \phi_{\overline{2}|}=\phi_{\overline{1}|}(1+i')+\mathrm{I}'$$

$$=\phi_{\overline{0}|}(1+i')^2+\mathrm{I}'(1+i')+\mathrm{I}'$$

$$\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot$$

$$\text{and}\quad \phi_{\overline{10}|}=\phi_{\overline{0}|}(1+i')^{10}+\mathrm{I}'(1+i')^9+\mathrm{I}'(1+i')^8+\ldots+\mathrm{I}'$$

$$=\phi_{\overline{0}|}(1+i')^{10}+\mathrm{I}'\frac{(1+i')^{10}-1}{i'}$$

$$\therefore\quad \mathrm{I}'=i'\frac{\phi_{\overline{10}|}-\phi_{\overline{0}|}(1+i')^{10}}{(1+i')^{10}-1}$$

In this equation the values of $\phi_{\overline{0}|}$ and $\phi_{\overline{10}|}$ are known, being the populations enumerated at the initial and terminal censuses, while the values of I' and i' have to be determined.

164. A first value for i' may be obtained by dividing the average annual recorded excess of births over deaths for the decennium by the average initial population for the several years, computed on the basis of an increase in arithmetical progression. So computed, the average initial population would evidently be

$$\phi_{\overline{0}|}+\frac{4\frac{1}{2}}{10}\{\phi_{\overline{10}|}-\phi_{\overline{0}|}\}.$$

From the value of i' thus determined, a first value of I' may be obtained, and from these two values a closer approximation may be made to the average initial population, thus leading to a closer approximation to the value of i', and consequently also to the

value of I′. Having determined the values of i' and I′, the values of $\phi_{\overline{n}|}$ may be readily obtained from the formula—

$$\phi_{\overline{n}|} = \phi_{\overline{0}|}(1+i')^n + I'\frac{(1+i')^n-1}{i'} = \left(\phi_{\overline{0}|} + \frac{I'}{i'}\right)(1+i')^n - \frac{I'}{i'}$$

The average initial population required for the purposes of the above calculation is thus—

$$\frac{\phi_{\overline{0}|} + \phi_{\overline{1}|} + \phi_{\overline{2}|} + \ldots + \phi_{\overline{9}|}}{10}$$

$$= \frac{1}{10}\left\{\left(\phi_{\overline{0}|} + \frac{I'}{i'}\right)\frac{(1+i')^{10}-1}{i'} - 10\frac{I'}{i'}\right\}$$

$$= \frac{1}{10}\left(\phi_{\overline{0}|} + \frac{I'}{i'}\right)\frac{(1+i')^{10}-1}{i'} - \frac{I'}{i'}$$

The Habitation Method.

165. A method which has frequently been employed, but the use of which is generally restricted to estimates for specific localities, and even then has served usually merely as a check on estimates obtained by other means, is that of basing the population on the number of habitations. If, at the date of a census, the average number of persons to each occupied house were computed, and if at some date during the currency of an intercensal period the number of such occupied houses could be ascertained from the records of a local governing body, or by other means, the multiplication by the average number of persons to a house ascertained at the census would furnish an approximation to the population of the locality.

Subdivisional Estimates.

166. In the preparation of intercensal estimates for subdivisions of the total population, such, for instance, as the population of a given district or town, or the number at a given age, or following a specified occupation, the method adopted for the whole population may of course be used for the subdivision, but a difficulty presents itself in the fact that under certain circumstances the aggregate of the subdivisional estimates will differ from the estimated total. Thus, on the assumption of a

uniform rate of increase throughout an intercensal period for the whole population, and a separate uniform rate for each of a group of subdivisions making up the total, it will almost invariably be found that this is the case. On the other hand, on the assumption of a uniform numerical increase for the whole population, and a separate uniform numerical increase for each subdivision, the aggregate of the subdivisional estimates will always agree with the estimated total.

167. For example, if the total population at two consecutive censuses be $\phi_{\overline{0}}$ and $\phi_{\overline{10}}$, the rate of increase per unit for the intercensal period will be $\frac{\phi_{\overline{10}}}{\phi_{\overline{0}}} - 1$, and the total population $\phi_{\overline{n}}$ at any given point of time n between the censuses, will, on the assumption of a uniform rate of increase, be

$$\phi_{\overline{0}} \left\{ \frac{\phi_{\overline{10}}}{\phi_{\overline{0}}} \right\}^{\frac{n}{10}} \text{ or } \frac{\{\phi_{\overline{10}}\}^{\frac{n}{10}}}{\{\phi_{\overline{0}}\}^{\frac{n}{10}-1}} .$$

Similarly, if $\qquad {}_a\phi_{\overline{0}}\,,\ {}_b\phi_{\overline{0}}\,,\ {}_c\phi_{\overline{0}}\,,\ \ldots\ {}_m\phi_{\overline{0}}$

and $\qquad {}_a\phi_{\overline{10}}\,,\ {}_b\phi_{\overline{10}}\,,\ {}_c\phi_{\overline{10}}\,,\ \ldots\ {}_m\phi_{\overline{10}}$

are the corresponding subdivisional populations at the two censuses, so that

$${}_a\phi_{\overline{0}} + {}_b\phi_{\overline{0}} + {}_c\phi_{\overline{0}} + \ldots + {}_m\phi_{\overline{0}} = \phi_{\overline{0}}$$

and

$${}_a\phi_{\overline{10}} + {}_b\phi_{\overline{10}} + {}_c\phi_{\overline{10}} + \ldots + {}_m\phi_{\overline{10}} = \phi_{\overline{10}}\,,$$

and if a separate subdivisional rate be assumed for each subdivision, the aggregate of the subdivisional estimates at the point of time n years from the initial census will be

$$\frac{\{{}_a\phi_{\overline{10}}\}^{\frac{n}{10}}}{\{{}_a\phi_{\overline{0}}\}^{\frac{n}{10}-1}} + \frac{\{{}_b\phi_{\overline{10}}\}^{\frac{n}{10}}}{\{{}_b\phi_{\overline{0}}\}^{\frac{n}{10}-1}} + \frac{\{{}_c\phi_{\overline{10}}\}^{\frac{n}{10}}}{\{{}_c\phi_{\overline{0}}\}^{\frac{n}{10}-1}} + \ldots + \frac{\{{}_m\phi_{\overline{10}}\}^{\frac{n}{10}}}{\{{}_m\phi_{\overline{0}}\}^{\frac{n}{10}-1}}\,,$$

while the estimated total will be

$$\frac{\{\phi_{\overline{10}}\}^{\frac{n}{10}}}{\{\phi_{\overline{0}}\}^{\frac{n}{10}-1}}\,, \text{ or } \frac{\{{}_a\phi_{\overline{10}} + {}_b\phi_{\overline{10}} + {}_c\phi_{\overline{10}} + \ldots + {}_m\phi_{\overline{10}}\}^{\frac{n}{10}}}{\{{}_a\phi_{\overline{0}} + {}_b\phi_{\overline{0}} + {}_c\phi_{\overline{0}} + \ldots + {}_m\phi_{\overline{0}}\}^{\frac{n}{10}-1}}$$

and these expressions are only necessarily equal if

$$\frac{{}_a\phi_{\overline{10}}}{{}_a\phi_{\overline{0}}} = \frac{{}_b\phi_{\overline{10}}}{{}_b\phi_{\overline{0}}} = \ldots = \frac{{}_m\phi_{\overline{10}}}{{}_m\phi_{\overline{0}}}\,,$$

that is, if the rates of increase for the several subdivisions are identical with each other, and consequently identical with that for the whole population.

168. On the other hand, if the same symbols be used to denote the various subdivisional and total populations, and the assumption of a uniform numerical increase be made, the total population at the given point of time n will be

$$\phi_0 + \frac{n}{10}\{\phi_{10} - \phi_0\}.$$

Similarly, the populations of the various subdivisions will be

$$ {}_a\phi_0 + \frac{n}{10}\{{}_a\phi_{10} - {}_a\phi_0\},\quad {}_b\phi_0 + \frac{n}{10}\{{}_b\phi_{10} - {}_b\phi_0\},$$

$$\ldots\ {}_m\phi_0 + \frac{n}{10}\{{}_m\phi_{10} - {}_m\phi_0\},$$

the aggregate of which is evidently

$$\phi_0 + \frac{n}{10}\{\phi_{10} - \phi_0\},$$

which, as shown above, is the estimated total.

169. In order to avoid the discrepancies referred to, it has been suggested that intercensal subdivisional estimates should be based upon the ratios of the subdivisional to the total population as ascertained at the two censuses, and that the assumption should be made that the variation in such ratio during the intercensal period has taken place in arithmetical progression. Thus, if, as before, the estimated total population at the given point of time n be denoted by ϕ_n, the various subdivisional estimates for this point of time would be in the form

$$\left[\frac{{}_a\phi_0}{\phi_0} + \frac{n}{10}\left\{\frac{{}_a\phi_{10}}{\phi_{10}} - \frac{{}_a\phi_0}{\phi_0}\right\}\right]\phi_n,$$

and their aggregate would be

$$\left[\frac{\phi_0}{\phi_0} + \frac{n}{10}\left\{\frac{\phi_{10}}{\phi_{10}} - \frac{\phi_0}{\phi_0}\right\}\right]\phi_n = \phi_n,$$

that is, the aggregate of the subdivisional estimates agrees with the estimated total, and this result is quite independent of the method by which the estimated total has been determined.

170. An ingenious method of facilitating the computation of such estimates in cases where a uniform rate of increase in the

total population is assumed, has been devised by Mr. A. C. Waters, and may be described as follows :

As shown above, the estimated population of a subdivision at a given point of time n may be denoted by

$$\left[\frac{{}_a\phi_0}{\phi_0} + \frac{n}{10}\left\{\frac{{}_a\phi_{10}}{\phi_{10}} - \frac{{}_a\phi_0}{\phi_0}\right\}\right]\phi_n,$$

while in addition, on the assumption of a uniform rate of increase in the total population we have

$$\phi_n = \phi_0 \left\{\frac{\phi_{10}}{\phi_0}\right\}^{\frac{n}{10}}$$

171. The subdivisional estimate thus becomes

$$\left[\frac{{}_a\phi_0}{\phi_0} + \frac{n}{10}\left\{\frac{{}_a\phi_{10}}{\phi_{10}} - \frac{{}_a\phi_0}{\phi_0}\right\}\right]\phi_0 \left\{\frac{\phi_{10}}{\phi_0}\right\}^{\frac{n}{10}}$$

$$= \frac{{}_a\phi_0}{\phi_0}\left(1 - \frac{n}{10}\right)\phi_0 \left\{\frac{\phi_{10}}{\phi_0}\right\}^{\frac{n}{10}} + \frac{n}{10}\cdot\frac{{}_a\phi_{10}}{\phi_{10}}\cdot\phi_0\cdot\left\{\frac{\phi_{10}}{\phi_0}\right\}^{\frac{n}{10}}$$

$$= {}_a\phi_0\left(1 - \frac{n}{10}\right)\left\{\frac{\phi_{10}}{\phi_0}\right\}^{\frac{n}{10}} + {}_a\phi_{10}\cdot\frac{n}{10}\cdot\left\{\frac{\phi_{10}}{\phi_0}\right\}^{\frac{n}{10}-1}$$

$$= {}_a\phi_0 \cdot f(n) + {}_a\phi_{10} \cdot \mathrm{F}(n),$$

where $f(n)$ and $\mathrm{F}(n)$ are functions of n which may be determined and tabulated once for all as soon as the quantities ϕ_0 and ϕ_{10}, the total populations at two consecutive censuses, are known, and where ${}_a\phi_0$ and ${}_a\phi_{10}$ denote the corresponding census figures for the subdivision. If, for instance, in the case of a decennial census the values of these functions were tabulated for all values of n from $\frac{1}{12}$ to 10, proceeding by intervals of $\frac{1}{12}$, the estimated population of a given subdivision at the end of any month of a completed intercensal period could readily be obtained by two multiplications and an addition.

(E.) *Methods of Calculating Mean Population.*

172. The most satisfactory interpretation of the term "mean population" appears to be that which treats of it as representing periods of human life actually experienced. In accordance with this view, the mean population of any community for a given period (month, quarter, year, decennium, &c.) is the number of such

periods of human life as have been spent by the members of the community during the month, quarter, year, &c., under review. Thus the statement that a mean population of a given town for a certain month was 1,000,000 should mean that during the month 1,000,000 months of human life had been experienced in the town. This interpretation of the term renders clear the meaning of many calculations which are based on the mean population of the community for a specified period and which, as ordinarily presented, convey to the reader a somewhat confused idea of the principle underlying the various processes.

173. The principal methods which have at various times been used for computing the mean population of a community for a given year are as follows—

(*a*) Take the arithmetical average of the population at the beginning and that at the end of the year.

(*b*) Take the population of the middle of the year as representing the mean for the year.

(*c*) Take the arithmetical average of the population at the beginning and that at the end of each month, and divide the sum of these monthly means by 12.

(*d*) Take the arithmetical average of the population at the beginning and that at the end of each quarter, and divide the sum of these quarterly means by 4.

174. If the population increased or decreased throughout the year in arithmetical progression, these four methods would evidently give identical results, but would not necessarily do so if any other rate of increase or decrease were in operation. Under ordinary circumstances it is probable that any one of these methods would give results of sufficient accuracy for most practical purposes.

175. In obtaining the mean for the year by method (*c*), the twelve monthly means, each of which represents so many months of human life actually experienced, are added together to obtain the total experience for the year, and are divided by 12 to reduce that experience from months of life to years. In a similar manner the quarterly means are treated under method (*d*). Further, having obtained the mean population for each of a series of years, say for a decennium, the mean for the decennium is found by adding the means for the several years, and dividing by 10, thus ascertaining the number of decennia of human life experienced in the ten years under review.

176. Where the population at any given date is assumed to be a function of the time elapsed since the date of a given census, it is evident that the number of years of life experienced, and consequently the mean population for any period, can readily be determined by means of integral calculus. In the following instances formulas for the decennial mean have been derived.

177. On the assumption of an increase or decrease in arithmetical progression, the mean population for the decennium is

$$\frac{1}{10}\int_0^{10}\{\phi_{\overline{0}|}+n\mathrm{I}\}dn$$
$$=\frac{1}{10}\left\{10\phi_{\overline{0}|}+\frac{100\times\mathrm{I}}{2}\right\}$$
$$=\phi_{\overline{0}|}+5\times\mathrm{I}$$
$$=\frac{\phi_{\overline{0}|}+\phi_{\overline{10}|}}{2}$$

that is, the mean population for the decennium will be obtained in accordance with method (*a*) above, applied to the results of the initial and terminal censuses.

178. On the assumption of an increase or decrease in geometrical progression, the mean for the decennium will be

$$\frac{1}{10}\int_0^{10}\{\phi_{\overline{0}|}(1+i)^n\}dn$$
$$=\frac{1}{10}\phi_{\overline{0}|}\frac{(1+i)^{10}-1}{\log_e(1+i)}$$
$$=\frac{\phi_{\overline{10}|}-\phi_{\overline{0}|}}{\log_e(1+i)^{10}}=\frac{\phi_{\overline{10}|}-\phi_{\overline{0}|}}{\log_e\dfrac{\phi_{\overline{10}|}}{\phi_{\overline{0}|}}}=\frac{\mathrm{M}(\phi_{\overline{10}|}-\phi_{\overline{0}|})}{\log_{10}\phi_{\overline{10}|}-\log_{10}\phi_{\overline{0}|}}$$

where $\mathrm{M}=\cdot 43429\ldots$, the modulus of the system of common logarithms.

179. If the intercensal population were determined by the combined progression method outlined in articles 161 to 164, the mean population for the period would be

$$\frac{1}{10}\int_0^{10}\left\{\left(\phi_{\overline{0}|}+\frac{\mathrm{I}'}{i'}\right)(1+i')^n-\frac{\mathrm{I}'}{i'}\right\}dn$$
$$=\frac{1}{10}\left\{\left(\phi_{\overline{0}|}+\frac{\mathrm{I}'}{i'}\right)\frac{(1+i')^{10}-1}{\log_e(1+i')}-10\frac{\mathrm{I}'}{i'}\right\}$$
$$=\frac{1}{10}\left(\phi_{\overline{\ }|}+\frac{\mathrm{I}'}{i'}\right)\frac{(1+i')^{10}-1}{\log_e(1+i')}-\frac{\mathrm{I}'}{i'}$$

180. In dealing with the preparation of intercensal subdivisional estimates, it has been pointed out that on the assumption of a geometrical rate of increase for the total population, and a subdivisional ratio to total, increasing or decreasing for the decennium in arithmetical progression, the population of the subdivision at any given intercensal date may be represented by

$$ {}_a\phi_{\bar{0}}\left(1-\frac{n}{10}\right)\left\{\frac{\phi_{\overline{10}}}{\phi_{\bar{0}}}\right\}^{\frac{n}{10}} + {}_a\phi_{\bar{0}}\cdot\frac{n}{10}\left\{\frac{\phi_{\overline{10}}}{\phi_{\bar{0}}}\right\}^{\frac{n}{10}-1} $$

181. Denoting $\frac{\phi_{\overline{10}}}{\phi_{\bar{0}}}$ by r, and $\frac{n}{10}$ by m, this expression becomes ${}_a\phi_{\bar{0}}(1-m)r^m + {}_a\phi_{\overline{10}}\, mr^{m-1}$. The mean population for the decennium will thus be

$$ \int_0^1 \{{}_a\phi_{\bar{0}}(1-m)r^m + {}_a\phi_{\overline{10}}\, mr^{m-1}\}\,dm $$

$$ = {}_a\phi_0\left\{\frac{r-1}{\log_e r} - \frac{r}{\log_e r} + \frac{r-1}{(\log_e r)^2}\right\} + {}_a\phi_{\overline{10}}\cdot\frac{1}{r}\left\{\frac{r}{\log_e r} - \frac{r-1}{(\log_e r)^2}\right\} $$

$$ = {}_a\phi_{\bar{0}}\cdot\frac{1}{\log_e r}\left(\frac{r-1}{\log_e r}-1\right) + {}_a\phi_{\overline{10}}\cdot\frac{1}{r\log_e r}\left(r-\frac{r-1}{\log_e r}\right) $$

$$ = {}_a\phi_0\cdot f_1(r) + {}_a\phi_{\overline{10}}\,\mathrm{F}_1(r) $$

where $f_1(r)$ and $\mathrm{F}_1(r)$ are functions of r whose values can be readily computed as soon as the value of r has been ascertained. These values will be applicable to any subdivisional population for the decennium under review.

Section VI. Computation of Rates.

182. The final stage in an investigation, such as that here dealt with, is the computation of the required rates. It has been previously pointed out that for the correct determination of these, it is necessary to have returns of both population and deaths, but in the absence of complete data computations have at times been based on death returns only, and at others, though less frequently, on population returns only:

(A.) *Computations based on Death Returns only.*

183. In connection with the earliest publication of the Bills of Mortality in England, the actual number of deaths appears to have

been considered a sufficient measure of the mortality experienced. With any population, however, which exhibits variations in number and modifications in sex and age constitution, such a measure is evidently of little value.

184. Another method adopted at times is that of obtaining the average age at death, the assumption being that a high average indicates a favourable, and a low average an unfavourable, condition as regards mortality. A little reflection, however, will show that as the average age at death is largely dependent on the average age of the persons exposed to the risk of death, a high average age at death might exist side by side with unfavourable sanitary provisions, and a low one with ideal conditions. For instance, in a home for the aged the average age at death must necessarily be high, while in an orphanage or industrial school the average age at death is, from the nature of the case, invariably low, but neither circumstance proves anything concerning the sanitary condition of the various institutions.

185. Another instance of the misleading use of death statistics alone for comparative purposes occurs in connection with statements exhibiting ratios of deaths at given ages to total deaths at all ages. Such statements are evidently open to the objection that an increase in the number for one age will tend to reduce the ratio for other ages, and thus give the impression of a decline in deaths at such ages.

186. Dr. Halley's Life Table, deduced from the death returns of Breslau, and Dr. Price's Tables from those of Northampton, are further instances of attempts to measure mortality by death returns alone.

187. It may be mentioned in connection with the use of average ages at death that a legitimate use of such averages occurs in ascertaining the age incidence of certain diseases. By such means an indication is furnished that diphtheria is a disease of childhood, phthisis of early adult life, and cancer of advanced age.

(B) *Computation based on Population Returns only.*

188. Instances of such computations are rare, but an excellent example of the determination of a measure of mortality, based on population returns only, is furnished by the Life Tables for India, compiled by Mr. G. F. Hardy in connection with his investigations concerning the ages and rates of mortality disclosed by the Indian Censuses. In the preparation of the

tables involved, due allowance, based on the experience of a number of years, is made for increases in the populations under review.

(c) *Computations based on Death and Population Returns.*

189. The most obvious measure of mortality based on death and population returns is the ratio of the total number of deaths for a given period to the total mean population for the same period. This measure is known as the "Crude Death Rate", and although in many respects defective, has yet certain advantages. It is, for instance, easily computed, and being expressed in one number can be quoted readily and compared with facility. Separate rates for each sex and age, on the other hand, are difficult to compute, clumsy to quote, and by no means easy to compare.

190. Certain methods of facilitating comparison, while yet taking due account of the various factors have, at different times, been employed, one of which is what has been termed the "index of mortality." In reality, the so-called "index of mortality" of any community is the crude death rate which that community would have experienced if its population had had the same age distribution as some standard population. The standard population chosen by the International Statistical Institute is that of Sweden for the census of 1890, the groups selected being those for ages 0 to 1, 1 to 19, 20 to 39, 40 to 59, and 60 and upwards.

191. For International Statistical purposes it would appear desirable to adopt Dr. Ogle's suggestion for taking as standard the population of a group of countries, whilst in view of the well-defined difference between male and female rates of mortality and the varying proportions of the sexes at different ages in certain populations, sex as well as age should be taken into account.

192. For the decennial supplement to the report of the Registrar-General of England and Wales, it is usual to compute for each country and district for the decennium under review, a measure of mortality known as "the rate in standard population", but which is really an "index of mortality", the standard population in this case being the mean population of England and Wales for the decennium.

193. Another method of taking account of the various factors affecting the death rate of a population is that which has been

adopted since 1883 by the Registrar-General of England and Wales in his annual summaries, and applied to the computation of "corrected death rates" for the principal English towns. For each town a standard death rate is computed, being the crude rate which would have been experienced in the town if the average death rate for each sex and age group for the whole of England and Wales during the last intercensal period had been in operation in the town at the date of the last census. The ratio of the "crude death rate" of England and Wales for the last intercensal period to this "standard rate" is the town's "factor for correction" for age and sex distribution, and the "crude death rate" of the town for any year of a current intercensal period is corrected by multiplying it by this factor.

194. The "index of mortality" furnishes a better measure of mortality than the "corrected death rate", but the latter provides a convenient means of approximation for an intercensal period, since the "factor for correction" may be computed for all required cases as soon as the census results and the registration figures for the decennium are known, and all that is subsequently required is the crude death rate of the town for each year. To obtain the corresponding "index of mortality" it would be necessary to have deaths for each year classified according to sex and age, as well as a similar distribution of the population of the town, thus involving in each case a considerable amount of calculation.

195. Numerous life tables, amongst which may be mentioned the Carlisle Table and the five English Life Tables, have been compiled on the basis of Death and Population Returns.

PERIODS OF OBSERVATION.

196. As regards the duration of the period of observation much will depend on the nature and extent of the data available, as well as on the object in view in making the investigation. It is clear that a period which might be suitable for a given purpose, might, for that very reason, be quite unsuitable for some other purpose. Thus, while the experience of a war, a famine, or an epidemic might be of great value for special purposes, it would usually be desirable to exclude the periods in which such exceptional events occurred.

197. For purposes connected with public health it is frequently desirable to know the season, or even the months or

weeks of the year, in which extra mortality has been experienced; whilst, on the other hand, when the results of the investigation are required for financial purposes, it is essential that the experience should be neither so limited that purely temporary fluctuations would be given undue weight, nor so extensive that permanent changes would be obscured.

198. From general considerations it would appear that the period which commences and closes with a census is more likely to furnish a satisfactory basis than one in which a census occupies a central point. In cases, however, where the table is required for the experience of a short period only, say for three or five years, or where the necessary details for a full intercensal period are wanting, the latter will often be the only basis available.

Infantile Mortality.

199. The method which has usually been adopted by statisticians for ascertaining the rate of infantile mortality for any year in any given community, is that of comparing the number of deaths of children under one year of age registered during the year with the corresponding number of births registered. On the supposition that the births are uniformly distributed throughout the several calendar years, and that they do not vary from year to year, such a comparison would give perfectly accurate results concerning the proportion of children born who fail to attain the age of one year. In actual experience, however, it will be found that neither supposition is correct; births are more numerous in some portions of the year than in others, and vary from year to year. Consequently, in most cases, a comparison of the number of deaths under one year of age for the year under review, with the mean number of births for that and the preceding year, would probably give a closer approximation to the true infantile rate than is obtained by the method usually adopted.

200. It is possible, also, that the restriction of the period of observation to the first year of age may lead to somewhat serious errors in the comparisons of different localities or different periods. Deaths under one year do not comprise the whole loss of infant life, and it is quite possible that a comparatively high rate of mortality for infants under twelve months may be accompanied by a normal or even a comparatively low rate for infants between one and five years of age. It thus

appears desirable that in all cases in which comparative rates of infantile mortality are being prepared, attention should be paid not only to the rates for age 0, but also to those for ages 1 to 4 inclusive.

201. Probably the best means of taking due account of the whole of this period would be the adoption of what is known as Professor Pell's method. For the application of this method, the requisite data are the number of deaths at each of the ages 0 to 4 recorded during the year under review, and the number of births for that and the five preceding years—sexes being distinguished in each case.

202. Thus, if the number of births in the year under review be denoted by $\beta_{(1)}$, and those in the five preceding years be denoted by $\beta_{(-1)}$, $\beta_{(-2)}$, $\beta_{(-3)}$, $\beta_{(-4)}$, and $\beta_{(-5)}$, respectively, and if, further, the number of deaths recorded in the given year at ages 0, 1, 2, 3, and 4, be denoted by θ_0, θ_1, θ_2, θ_3, and θ_4, and the rates of mortality at corresponding ages by q_0, q_1, q_2, q_3, and q_4, Professor Pell's method gives $q_0=\frac{\theta_0}{\frac{1}{2}\beta_{(-1)}+\frac{1}{2}\beta_{(1)}}$. According to the usual statistical practice, q_0, would be taken as $\frac{\theta_0}{\beta_{(1)}}$, but, as previously stated, $\frac{\theta_0}{\frac{1}{2}\beta_{(-1)}+\frac{1}{2}\beta_{(1)}}$ appears likely to give a closer approximation to the true rate.

Similarly,

$$q_1(1-q_0)=\frac{\theta_1}{\frac{1}{2}\beta_{(-2)}+\frac{1}{2}\beta_{(-1)}}$$

$$q_2(1-q_1)(1-q_0)=\frac{\theta_2}{\frac{1}{2}\beta_{(-3)}+\frac{1}{2}\beta_{(-2)}}$$

$$q_3(1-q_2)(1-q_1)(1-q_0)=\frac{\theta_3}{\frac{1}{2}\beta_{(-4)}+\frac{1}{2}\beta_{(-3)}}$$

$$q_4(1-q_3)(1-q_2)(1-q_1)(1-q_0)=\frac{\theta_4}{\frac{1}{2}\beta_{(-5)}+\frac{1}{2}\beta_{(-4)}}.$$

From these equations the values of q_0, q_1, q_2, q_3 and q_4 may readily be found. The most convenient quantity for comparative purposes will be the numerical value of

$$\{1-(1-q_4)(1-q_3)(1-q_2)(1-q_1)(1-q_0)\}$$

representing, on the basis of the given year's experience, the proportion of infants born who fail to reach the age of five years.

203. If β_1, β_{-1}, β_{-2}, β_{-3}, β_{-4}, and β_{-5} be identical, we have

$$q_0=\frac{\theta_0}{\beta_1},\quad q_1=\frac{\theta_1}{\beta_1-\theta_0},\quad q_2=\frac{\theta_2}{\beta_1-(\theta_0+\theta_1)},$$

$$q_3=\frac{\theta_3}{\beta_1-(\theta_0+\theta_1+\theta_2)},\quad q_4=\frac{\theta_4}{\beta_1-(\theta_0+\theta_1+\theta_2+\theta_3)},$$

and

$$\{1-(1-q_4)(1-q_3)(1-q_2)(1-q_1)(1-q_0)\}=\frac{\theta_0+\theta_1+\theta_2+\theta_3+\theta_4}{\beta_1}$$

Hence, if the number of births per annum is fairly constant, the proportion of infants who fail to reach the age of five years will be found approximately by ascertaining the ratio of the number of deaths under five years of age to the number of births for the year.

204. If it be required to obtain such rates for a longer period than a year, say for a quinquennium, the procedure will be similar to that outlined above, and we shall have

$$q_0=\frac{\theta_0}{\frac{1}{2}\beta_{-1}+\beta_1+\beta_2+\beta_3+\beta_4+\frac{1}{2}\beta_5}$$

$$q_1(1-q_0)=\frac{\theta_1}{\frac{1}{2}\beta_{-2}+\beta_{-1}+\beta_1+\beta_2+\beta_3+\frac{1}{2}\beta_4}$$

$$q_4(1-q_3)(1-q_2)(1-q_1)(1-q_0)=\frac{\theta_4}{\frac{1}{2}\beta_{-5}+\beta_{-4}+\beta_{-3}+\beta_{-2}+\beta_{-1}+\frac{1}{2}\beta_1,}$$

Where β_1, $\beta_{(2)}$, β_3, β_4, and β_5 denote the number of births for the successive years of the quinquennium, and θ_x and q_x denote the number of deaths and the rate of mortality for age x for the period. On the assumption of a comparatively uniform number of births from year to year the proportion failing to reach five years of age will be approximately represented by

$$\frac{\theta_0+\theta_1+\theta_2+\theta_3+\theta_4}{\beta_1+\beta_2+\beta_3+\beta_4+\beta_5};$$

that is, by the ratio of the number of deaths under five years of age to the total births for the quinquennium.

General Life Tables.

205. For the purpose of presenting a clear and comprehensive view of the mortality experienced by a community, the most accurate method is that of constructing a life table. The

complete data for the construction of such a table are the number of years of life which, during the period under review, have been experienced in the community in each year of age, and the number of deaths which, during the same period, have occurred in each year of age. The ratio of the number of deaths to the number of years of life is what is known as the central death rate for each year of age, and denoted by the symbol m_x. Where the requisite data have been carefully collected and compiled, this "central death rate" for each year of age is probably the most accurate measure directly obtainable of the mortality actually experienced in the period under review.

206. It is evident that, as regards the number of years of life experienced during a given period in any specified year of age, the contributions from the different members of the community will be somewhat varied. Thus, while some will have contributed a full year's experience, having entered upon and completed the year of age during the period under review, others will have furnished fractional experience only, owing to their having come under observation after the commencement of the year of age, or having disappeared by death, migration, or close of observations before the end of the year. In making use of the data for the construction of a general life table these fractional exposures may, for each year of age, be considered as equivalent to as many individual exposures for a complete year as are represented by the aggregate of the fractional periods. If the deaths in each year of age occurred always at the end of that year, the ascertained years of life might, for any given age x, be taken as representing the number of persons who entered upon it, and the ratio of deaths to years of life would give the value of q_x. It is, however, more reasonable to assume that the life experience contributed by each deceased person to the year of age in which he dies is, on the average, half a year. Thus if λ_x denote the number of years of life experienced between the ages of x and $x+1$, and θ_x denote the number of deaths recorded for the same year of age, the life experience contributed by those who did not die will be denoted by $\lambda_x - \frac{1}{2}\theta_x$, and this, on the consideration above referred to, would be taken as representing the number of persons who contributed a complete year of experience between the ages of x and $x+1$; that is to say, it would represent the number of survivors at age $x+1$. From this the number who would be considered as having commenced

the year of age can be readily ascertained by adding the number who died during the year, thus giving $\lambda_x - \frac{1}{2}\theta_x + \theta_x$ or $\lambda_x + \frac{1}{2}\theta_x$. The ratio of the number who survive the year of age from x to $x+1$ to the number who enter upon it, is thus

$$\frac{\lambda_x - \frac{1}{2}\theta_x}{\lambda_x + \frac{1}{2}\theta_x} \text{ or } \frac{1 - \frac{1}{2}\frac{\theta_x}{\lambda_x}}{1 + \frac{1}{2}\frac{\theta_x}{\lambda_x}} \text{ or } \frac{1 - \frac{1}{2}m_x}{1 + \frac{1}{2}m_x},$$

this last being the well-known expression for p_x in terms of m_x.

207. Since $\lambda_x + \frac{1}{2}\theta_x$ denotes the number under observation at the commencement of the year of age, and $\lambda_x - \frac{1}{2}\theta_x$ denotes the number at its close, the number at the centre of the year of age will, on the assumption of a uniform distribution of deaths throughout the year be denoted by λ_x, and the ratio of θ_x to λ_x will thus represent approximately the annual rate at which deaths occur at the centre of the year of age; hence the designation "central death rate." It is clear from the foregoing that, in determining the values of p_x it is unnecessary to compute m_x, as the required values can be obtained direct from the original data by means of the formula

$$p_x = \frac{\lambda_x - \frac{1}{2}\theta_x}{\lambda_x + \frac{1}{2}\theta_x}.$$

208. Having obtained p_x for all values of x from 0 upwards, the other life table functions may be readily tabulated by means of the well-known relations which they bear to p_x and each other.

209. For the purpose of minimizing the labour involved in the preparation of a life table, various "short methods" of computation have been devised, of which those of Dr. Farr and Dr. Hayward are probably the best known.

210. In the construction by Dr. Farr of life tables for the Metropolis, Surrey and Liverpool on the basis of the experience for the year 1841, the value of p_0 was calculated from birth and death returns, and those of m_1 to m_4 from death and census returns for single ages. For the age group 5-9 the ratio $(=m)$ of deaths to years of life was ascertained and the probability of

surviving five years at age five was deduced on the assumption that such probability is equal to $\left(\dfrac{1-\frac{1}{2}m}{1+\frac{1}{2}m}\right)^5$. For subsequent groups a similar procedure was followed, and from the values of ${}_5p_x$ so ascertained, the values of l_x for quinquennial intervals were determined. It is evident that this method may be employed with age groups of any size on the assumption that

$$(p_x \times p_{x+1} \times p_{x+2} \times \ldots \times p_{x+n-1})^{\frac{1}{n}} = \frac{1-\frac{1}{2}m}{1+\frac{1}{2}m}$$

where m denotes the ratio of deaths to years of life for the age group x to $x+n-1$ inclusive. If the values of p_x increased or decreased in geometrical progression, this assumption would be fully warranted since $\dfrac{1-\frac{1}{2}m}{1+\frac{1}{2}m}$ represents, approximately, the value of p for the central age of the group.

211. From the values of l_x so ascertained the values of ΣL_x are computed on the assumption that the number of years of life experienced between the ages of x and $x+n$ by these l_x persons is $\frac{n}{2}(l_x+l_{x+n})$, and from such values of ΣL_x the values of $\mathring{e}_x$ are derived by means of the formula $\mathring{e}_x = \dfrac{\Sigma L_x}{l_x}$.

212. For the earlier ages these values of $\mathring{e}_x$ are fairly close approximations, but at the higher ages they are usually far from correct. A modification, however, has been devised by Dr. Hayward, which, while simple in its application, is yet effective in producing results by the short method in close accord with those obtained by the extended process.

213. The modification is applied in determining the number of years of life experienced between the ages of x and $x+n$ by the l_x persons, which number is taken by Dr. Farr's method as $\frac{n}{2}(l_x+l_{x+n})$. By Dr. Hayward's method the interval of n years is subdivided into k parts, and the number of years of life is based on the values of l_x and l_{x+n} and also on the corresponding values

for each of the intermediate $k-1$ points. That is, denoting $\dfrac{1-\frac{1}{2}m}{1+\frac{1}{2}m}$ by p, the number of years of life experienced between x and $x+n$ will, by this method, be represented by

$$\frac{n}{k}\left\{\frac{l_x+l_xp^{\frac{n}{k}}}{2}+\frac{l_xp^{\frac{n}{k}}+l_xp^{\frac{2n}{k}}}{2}+\ldots\frac{l_xp^{\frac{(k-1)n}{k}}+l_{x+n}}{2}\right\}$$

$$=\frac{n}{k}\left\{\frac{l_x+l_{x+n}}{2}+l_xp^{\frac{n}{k}}\cdot\frac{1-p^{\frac{(k-1)n}{k}}}{1-p^{\frac{n}{k}}}\right\}$$

214. As the result of experiments, Dr. Hayward has ascertained that the following scheme furnishes a simple and accurate application of his method:

(*a*) For the two groups 5–9 and 10–15 $n=5$ and $k=1$.

(*b*) For each group 15–24 to 65–74 inclusive . . . . . $n=10$,, $k=2$.

(*c*) For the two groups 75–84 and 85–94 $n=10$,, $k=4$.

(*d*) For the final group of n ages, 95 and upwards, $k=n$, the value of p for this group having been obtained from those for the four preceding groups by means of finite differences.

Graduation.

215. In any general life table directly constructed from original data there will usually be found an unevenness in the rates of mortality, and a tendency to unaccountable fluctuation which does not appear to be in keeping with the uniformity of nature. The various uses, financial and other, to which a life table is applied, render it desirable that these inequalities should, as far as reasonable conformity to the original facts will admit, be so smoothed out as to prevent the occurrence of anomalous results in the use of the table.

216. It has, indeed, been contended by De Morgan and others that no method of graduation is warrantable, since any such process involves an attempt to make actual facts accord

with preconceived ideas. Such an objection does not appear valid, as the principal reason on which the adjustment is based is simply that derived from a contemplation of nature, namely, that nature does not proceed *per saltum*. The conclusion which is consequently drawn is, that where a leap appears to have occurred, it is due to some deficiency in data which should be remedied by the best means at our disposal.

217. Many different methods of adjustment have at various times been applied, but it would be beyond the scope of the present essay to do more than refer very briefly to any of them.

218. As the object in view in adjusting any life table is that of smoothing out what may be termed accidental irregularities, it is clear that the process of tabulating deaths and years of life in age groups, and subsequently distributing the results over single ages by means of interpolation by the analytical or the graphic methods already described, will have the effect of introducing a considerable degree of smoothness in the computed rates of mortality, and are thus entitled to rank as methods of graduation. In addition to these, however, there are three methods of graduation which have received general recognition, and which in practical operation are usually applied to one or other of the functions l_x, d_x, or q_x. These may be termed the summation, the analytical, and the graphic methods of graduation.

219. Probably the best known example of the summation method is that which was applied by Mr. Woolhouse to the graduation of the Institute of Actuaries' Tables in 1869. The summation method is one which lends itself well to the adjustment of general life tables, as it is simple in its application, and, while effective in reducing inequalities, is yet not so drastic as to conceal characteristic variations in mortality.

220. The analytical method of graduation is based on the assumption of a definite mathematical law of mortality, and the best known example of it is probably that with which the names of Gompertz and Makeham are associated. As this method involves the assumption that, between the age limits within which it is applied, the rate of mortality is a continuously increasing function, it is by no means suitable for the graduation of a general experience.

221. With reference to the graphic method, it may be said that as the criterion of the success of any graduation is the combination of smoothness of curve with closeness of fit with original data, and as the graphic method readily admits of due

effect being given to these two essential requirements, this method has claims for consideration which cannot be overlooked. The use of the graphic method for the graduation of life tables has been strongly advocated by Dr. Sprague. This method has also been largely used by numerous investigators for the adjustment of statistics of all kinds.

General Life Tables.

222. *England.*—The following is a brief summary of particulars concerning some of the principal general life tables published in England—

(*a*) Dr. Halley's Table, calculated from burial records of City of Breslau for the years 1687–91; published in London in 1693. Males and females combined.

(*b*) Dr. Price's First Northampton Table, based on records of 3,690 burials in parish of All Saints, Northampton, for 36 years, 1735–70.

(*c*) Dr. Price's Second Northampton Table, based on records of 4,689 burials in same parish for 46 years, 1735–80.

(*d*) Dr. Price's London Life Table, based on bills of mortality for the years 1759–68.

(*e*) Thomas Simpson's London Life Table, based on bills of mortality for the years 1728–37, with an adjustment to allow for adults settling in London.

(*f*) Swedish Life Table, prepared by Dr. Price in 1783 on basis of abstracts of living and dying in Sweden during a period of 21 years. Claimed to be the first national life table ever constructed—apparently the first table ever compiled by means of a comparison of the living and the dying.

(*g*) The Carlisle Table, constructed by Milne on the basis of tables of living and dying, prepared by Dr. Heysham for the parishes of St. Mary and St. Cuthbert, Carlisle. Experience of living appears to have been obtained by an enumeration in January 1780, and an estimate in December 1787. Numbers at respective dates were 7,677 and 8,677. Death experience comprised records for the nine years, 1779–87. In the construction of the table sexes were not distinguished. The proportion of females to males in the population is said to have been about 55 to 45.

(*h*) Dr. Farr's First English Life Table, published in 1843 in the Fifth Annual Report of the Registrar-General of England and Wales. This was the first National Life Table of England and Wales, and was computed on the basis of the Census Returns of 1841, and the birth and death records for that year, particulars for males and females being shown separately.

(*i*) Dr. Farr's Second English Life Table, published in 1853 in the Twelfth Annual Report of the Registrar-General. This table was based upon the Census Returns of 1841 and the deaths recorded in the seven years, 1838–44, males and females being dealt with separately.

(*j*) Dr. Farr's Third English Life Table, published as a separate volume in 1864, and included in 1867 in the Twenty-eighth Annual Report of the Registrar-General. This table was based upon the population censuses of 1841 and 1851, the death returns for the 17 years, 1838–54, and the birth returns for the 17 years, 1837½–53½.

(*k*) Dr. Ogle's Fourth English Life Table, published in 1885 in the Supplement to the Forty-fifth Annual Report of the Registrar-General. This table was based upon the results of the population censuses of 1871 and 1881, and the registrations for the ten years, 1871–80.

(*l*) Dr. Tatham's Fifth English Life Table, published in 1896 in the Supplement to the Fifty-fifth Annual Report of the Registrar-General. This table was constructed on the basis of the census returns of 1881 and 1891, and the registrations for the ten years, 1881–90.

(*m*) Dr. Farr's Healthy Districts Life Table, published first in a paper contributed to the Royal Society, London, in 1859, and subsequently printed in the Thirty-third Annual Report of Registrar-General. This table was compiled from the results of the census of 1851, and the deaths registered in the five years, 1849–53, in 63 districts of England and Wales, which showed during the period 1841–50 a mean annual "crude" death rate not exceeding 17 per 1000 of mean population.

(*n*) Dr. Tatham's Healthy Districts Life Table, published in 1897 in the second volume of the Supplement to the Fifty-fifth Annual Report of the Registrar-General. This table was compiled from the Census Returns for 1881 and 1891, and the deaths for the ten years 1881-90, in 263 districts in England and Wales, in which "corrected" death rates for the period did not exceed 15 per 1,000 of the mean population per annum.

(*o*) Numerous tables of a purely local character, such as the Surrey and Liverpool Tables of Dr. Farr, the Manchester Tables of Dr. Tatham, the Brighton Tables of Dr. Newsholme, and the Haydock Tables of Dr. Hayward.

(*p*) Dr. Hayward's tables of expectations of life for the whole of England and Wales for quinquennial ages for each of the decennial periods, 1841–50, 1851–60, 1861–70, 1871–80, 1881-90, 1891–1900, published in the *Journal of the Royal Statistical Society*, vol. lxiv, page 636, and vol. lxvi, page 366.

223. *Scotland.*—The principal general life tables compiled for Scotland are—

(*a*) Dr. Robertson's table, published in the Nineteenth Annual Report of the Registrar-General of Scotland. This was compiled from the Census and Death Returns for the year 1871.

(*b*) Dr. Adam's table, published in the *Journal of the Royal Statistical Society*, vol. lxvii, page 448. This was based on mean population and deaths in Scotland for the decennium, 1881–90.

(*c*) Dr. Dunlop's series of tables for the decennial periods 1861–70, 1871–80, 1881–90, 1891–1900, published in the Supplement to the Forty-eighth Annual Report of the Registrar-General of Scotland. These were constructed by Dr. Hayward's "shorter" method.

224. *India.*—Valuable life tables for the various provinces of India have been prepared by Mr. G. F. Hardy and published in the reports of the Indian Census authorities in connection with the censuses of 1881, 1891, and 1901.

225. *Ceylon.*—Two life tables of Ceylon have been published by the Registrar-General of that Colony, one in 1888, in his

Annual Report on the Vital Statistics of Ceylon for 1887, and the other in 1902 in his report on the 1901 census.

226. *United States of America.*—Prior to 1900 it was usual to include, in the reports on the decennial census, tables of expectation of life for various towns and districts, but on the occasion of the census of 1900 no attempt at the construction of such tables appears to have been made.

227. *Australia.*—Numerous tables have from time to time been compiled on the basis of the experience of the several States of the Commonwealth. Amongst these may be mentioned the following—

(*a*) Professor Pell's New South Wales Tables for 1856-66 and 1860–75.

(*b*) Burridge's Table for Victoria for 1871.

(*c*) Burridge's Table for New South Wales, Victoria, Queensland, and South Australia for 1870–81.

(*d*) New South Wales Census Office Table, based on census of 1891 and death records for four years.

(*e*) Moors & Day's table for New South Wales and Victoria, based on census of 1891 and deaths for 1889–93.

(*f*) Western Australian Census Office table, based on census of 1901 and death records for three years.

228. *New Zealand.*—An extensive Table, based on the census results of 1881, 1886 and 1891, and the death records for the 13 years, 1880–92, was prepared by Mr. Geo. Leslie of the New Zealand Government Life Insurance Department, and was published in the New Zealand *Journal of Insurance, Mining, and Finance,* in 1895.

LEGAL NOTES.

By ARTHUR RHYS BARRAND, F.I.A., *Barrister-at-Law.*

Does payment of commission cease on agent's death?

THE case of *Wilson* v. *Harper* [1908] 2 Ch. 370, dealing as it does with an important question in regard to agents' commission, is likely to be of interest to assurance officials, although the business to which the agency related was not that of assurance. The facts of the case

are as follows: The defendants agreed with one Wroe, a commercial traveller, to pay him a commission of 5 per-cent on all goods supplied to customers introduced by him, as long as they did business with such customers. This was done until Wroe's death in November 1906, but the defendants refused to continue the payment of this commission to his executors. The latter thereupon brought this action, asking for a declaration that the agreement was not determined by the death of Wroe, and that the defendants were liable to pay upon all orders received after his death from customers introduced by him.

Neville, J., gave judgment for the plaintiffs, and in doing so said: "It seems to me quite impossible to say that you are to "read in business documents of this kind an implied limitation "that no remuneration is to be paid to the executors of one of "the contracting parties. . . . You must read the agreement "to pay 'you' in the ordinary sense in which those words are "used, that is, 'to pay you, and, after your death, to pay your "executors.' I think what you are to pay is quite clear. You "are to pay 5 per-cent on the orders received from firms "introduced by Mr. Wroe. Therefore it seems to me it does "not matter whether he was dead or alive, although after he is "dead the agreement is terminated in this sense, that no "further customers can be introduced by anybody on the terms "of this document. . . . I think, therefore, that the plaintiffs "must succeed, but the declaration asked by the statement of "claim, that the agreement was not determined by the death, is "incorrect. I think it was determined by the death. I think "it is the remuneration that survived, full consideration having "been given for that in the work already done."

Survivorship in case of commorientes.

The question of the survivorship of one of two persons perishing in the same calamity came before the Court recently in the case of In the Goods of Good, 1908, 24 T.L.R. 493, and may be of interest to those who have to do with reversionary transactions. Here husband and wife left home together on 2 November 1907, and were not seen or heard of until 24 November, when their bodies were found, tied together with string, floating in the Thames; and a verdict of suicide was returned, without expressing any view as to which survived the other. The wife was entitled to certain reversionary interests, and had left a will in favour of her husband, if he should survive her, or, if he predeceased her, in favour of the

trustees of the National Gallery. The latter stated that they did not propose to make any claim to the estate. In these circumstances the Court was asked to grant leave to swear the death of the wife on or since 2 November 1907, and that there was no reason for believing that her husband had survived her; and Bucknill, J., gave leave to do so.

Incorrect statements by proposer in medical report.

The case of *Joel* v. *Law Union and Crown Insurance Company*, 1908, 24 T.L.R. 898, has attracted considerable attention among assurance officials, not only by reason of the important considerations involved in it, but also on account of the lengthy and weighty judgments delivered by the members of the Court of Appeal when it came before that Court. The following are the facts of the case. An assurance was effected with the defendants by one Robina Morrison, in November 1902, on the basis of a proposal dated 27 October 1902, and the correctness of the answers to the questions in this form was not disputed. At the end of the form she signed a declaration to the effect that, to the best of her knowledge and belief, the particulars given were true, and agreed "that this proposal and declaration shall be the basis of the contract." Certain questions contained in a printed form were also put to her by the medical officer of the defendants, and she was asked, *inter alia*, to give the names of any medical men consulted by her. In reply to this she gave the names of two doctors only, a Dr. Scott and a Dr. Hodson whom, she said, she had consulted for colds and measles respectively. She was also asked whether she had at any time suffered from "mental derangement", her answer to which was in the negative. As a matter of fact, the proposer had previously suffered from acute mania, and had consulted a Dr. Kinsey Morgan in December 1894, for nervous depression, and on a certificate signed by him had been in confinement in the house of a Dr. Leach from January to August 1895. Her answers were filled in by the medical officer upon the form referred to, at the end of which was the following declaration, which she signed: "I, the said "Robina Morrison do hereby declare with reference to the "proposal for assurance on my life and my declaration dated "30 October 1902 (presumably 27 October 1902), that the "answers to the foregoing questions are all true." The policy, which was executed on 4 November 1902, contained no

reference to the proposal or to either of the declarations. On 6 March 1906, the assured committed suicide while of unsound mind. The defendants contested the claim on the ground that the policy was void by reason of the assured's untrue statement that she had not suffered from mental derangement, and her omission to disclose the fact that she had consulted Dr. Kinsey Morgan. The jury, in response to questions left to them by the judge, found the following facts: (1) The assured did not know in October 1902 that she had suffered from mental derangement. (2) She did not, fraudulently, conceal from the defendants the fact that she had so suffered. (3) She foolishly, but not fraudulently, concealed from the defendants the fact that she had consulted Dr. Kinsey Morgan in 1894 for nervous depression. (4) The fact that she had so consulted Dr. Morgan was material for the assurance company to know. Upon these findings Lord Alverstone, C.J., after consideration, gave judgment for the defendants in the following terms: "The defendants "being willing, and offering to return the premiums received, "declare that the policy on the life of Robina Morrison was void "and of no effect, and that the policy should be delivered up to "the defendants to be cancelled."

The plaintiff appealed from this decision, and the Court of Appeal, consisting of Vaughan Williams, Fletcher Moulton and Buckley, LL.J., ordered a new trial. Each of the members of the Court delivered a lengthy judgment in which many remarks of interest to insurance companies were made. It is impossible here to give these judgments in full, or even to quote at any length from them, and the readers of these notes must therefore be referred to The Times Law Reports, vol. 24, page 898, or to the report in *The Times* of 4 August 1908, where they will find them set out *in extenso*. The following is, however, a brief summary of the grounds on which the appeal was allowed: (1) The answers to the questions contained in the medical report of 31st October, unlike those contained in the proposal of 27 October, did not form part of the basis of the contract; and it was therefore sufficient if they were answered to the best of the knowledge and belief of the proposer. (2) That as the medical officer was expressly authorized and instructed to put the questions with any necessary explanations, and as he was not called as a witness, although present in Court, there was no evidence as to what explanations or modifications were made by him when putting the questions; and therefore no evidence

that the proposer had failed to disclose to him all that was necessary.

The Court also expressed itself somewhat strongly on the subject of proposers for life assurance being required to warrant the truth of statements of which, from their very nature, they could have no absolute knowledge, and as to which they could only speak to the best of their knowledge and belief.

Damages under Lord Campbell's Act.

Attention has on previous occasions been drawn to the anomalous rule of law by which, in assessing damages under what is popularly known as Lord Campbell's Act, the amount of any assurances against accident has to be taken into account and deducted from the gross amount of the pecuniary damage suffered by reason of the death. The matter is referred to in *J.I.A.*, xli. 202 and 578, and the attempts to remedy the evil both by public and private legislation pointed out. As is well known, certain companies had, by means of private Acts of Parliament, succeeded in securing exemption from this rule for their own policies, but now by an Act of the present year the rule is abrogated in respect of all companies. By section 1 of the Fatal Accidents (Damages) 1908, 8 Edw. VII, ch. 7, it is enacted that "In assessing damages in any action, "whether commenced before or after the passing of this Act, "under the Fatal Accidents Act, 1846, as amended by any "subsequent enactment, there shall not be taken into account "any sum paid or payable on the death of the deceased under "any contract of assurance or insurance, whether made before or "after the passing of this Act."

Amendment of Friendly Societies Act, 1896.

By the Friendly Societies Act, 1908, 8 Edw. VII, ch. 32, several important amendments were made in the Friendly Societies Act, 1896, (in the new Act referred to as the principal Act). Those which are likely to be of most interest to our readers are as follows:

Assurances on children under one year of age.

Section 2. (1) A person of, or under one year of age, may be admitted as a member of a registered society or branch, and accordingly in section thirty-six of the principal Act (which relates to the membership of minors) the words "but above one year of age" shall be repealed.

(2) Where the rules of a registered friendly society or branch, in force at the commencement of this Act, provide for

the admission as members of persons from the minimum age authorized by the principal Act, the rules shall be construed as providing for the admission as members of persons from birth.

Amount of assurances and annuities. *Section* 3. In section forty-one of the principal Act, "three hundred pounds" shall be substituted for "two hundred pounds", and "fifty-two pounds" for "fifty pounds" as the maximum amount a member or person claiming through a member of a registered friendly society or branch is entitled to receive by way of gross sum and by way of annuity respectively.

Investments. *Section* 4. In sub-section (1) of section forty-four of the principal Act (which relates to the manner in which the funds of registered societies and branches may be invested) the following paragraph shall be added after paragraph (*e*): "or (*f*) in any investment in which trustees are for the time being by law authorized to invest trust funds."

Nominations. *Section* 5. The following sub-section shall be added to section fifty-six of the principal Act (which relates to the power of members to dispose by nomination of sums payable on death).

(6) A nomination, or a variation or revocation of a nomination by writing under the hand of a member of a registered branch and delivered at or sent to the registered office of that branch, or made in a book kept at that office, shall be effectual notwithstanding that the money to which the nomination relates or some part thereof is not payable by that branch, but is payable by the society or some other branch.

Old Age Pensions Act. Another Act already passed in the present session of Parliament, and which is of the greatest interest and importance to actuaries and other insurance officials, is the Old Age Pensions Act, 1908, 8 Edw. VII, ch. 40. The very fact of its importance and novelty renders it perhaps inadvisable to attempt a summary of it here, and it has been thought better that the Act itself should be printed *in extenso* in the *Journal*. The full text of the Act will be found on the following pages (90 to 98).

An Act to provide for Old Age Pensions.

[8 Edw. VII. ch. 40] [1st August 1908.]

BE it enacted by the King's most Excellent Majesty, by and with the advice and consent of the Lords Spiritual and Temporal, and Commons, in this present Parliament assembled, and by the authority of the same, as follows:—

Right to receive old age pension.

1.—(1) Every person in whose case the conditions laid down by this Act for the receipt of an old age pension (in this Act referred to as statutory conditions) are fulfilled, shall be entitled to receive such a pension under this Act so long as those conditions continue to be fulfilled, and so long as he is not disqualified under this Act for the receipt of the pension.

(2) An old age pension under this Act shall be at the rate set forth in the schedule to this Act.

(3) The sums required for the payment of old age pensions under this Act shall be paid out of moneys provided by Parliament.

(4) The receipt of an old age pension under this Act shall not deprive the pensioner of any franchise, right, or privilege or subject him to any disability.

Statutory conditions for receipt of old age pension.

2.—The statutory conditions for the receipt of an old age pension by any person are—

(1) The person must have attained the age of seventy:

(2) The person must satisfy the pension authorities that for at least twenty years up to the date of the receipt of any sum on account of a pension he has been a British subject, and has had his residence, as defined by regulations under this Act, in the United Kingdom:

(3) The person must satisfy the pension authorities that his yearly means as calculated under this Act do not exceed thirty-one pounds ten shillings.

Disqualification for old age pension.

3.—(1) A person shall be disqualified for receiving or continuing to receive an old age pension under this Act, notwithstanding the fulfilment of the statutory conditions—

(*a*) While he is in receipt of any poor relief (other than relief excepted under this

provision), and, until the thirty-first day of December nineteen hundred and ten unless Parliament otherwise determines, if he has at any time since the first day of January nineteen hundred and eight received, or hereafter receives, any such relief: Provided that for the purpose of this provision—

(i) any medical or surgical assistance (including food or comforts) supplied by or on the recommendation of a medical officer; or

(ii) any relief given to any person by means of the maintenance of any dependant of that person in any lunatic asylum, infirmary, or hospital, or the payment of any expenses of the burial of a dependant: or

(iii) any relief (other than medical or surgical assistance, or relief herein-before specifically exempted) which by law is expressly declared not to be a disqualification for registration as a parliamentary elector, or a reason for depriving any person of any franchise, right, or privilege;

shall not be considered as poor relief:

(*b*) If, before he becomes entitled to a pension, he has habitually failed to work according to his ability, opportunity, and need, for the maintenance or benefit of himself and those legally dependent upon him:

Provided that a person shall not be disqualified under this paragraph if he has continuously for ten years up to attaining the age of sixty, by means of payments to friendly, provident, or other societies, or trade unions, or other approved steps, made such provision against old age, sickness, infirmity, or want or loss of employment as may be recognised as proper provision for the purpose by regulations under this Act, and any such provision, when made by the husband in the case of a married couple living together, shall

as respects any right of the wife to a pension, be treated as provision made by the wife as well as by the husband:

53 & 54 Vict. c. 5.

(*c*) While he is detained in any asylum within the meaning of the Lunacy Act, 1890, or while he is being maintained in any place as a pauper or criminal lunatic:

(*d*) During the continuance of any period of disqualification arising or imposed in pursuance of this section in consequence of conviction for an offence.

(2) Where a person has been before the passing of this Act, or is after the passing of this Act, convicted of any offence, and ordered to be imprisoned without the option of a fine or to suffer any greater punishment, he shall be disqualified for receiving or continuing to receive an old age pension under this Act while he is detained in prison in consequence of the order, and for a further period of ten years after the date on which he is released from prison.

61 & 62 Vict. c. 60.

(3) Where a person of sixty years of age or upwards having been convicted before any court is liable to have a detention order made against him under the Inebriates Act, 1898, and is not necessarily, by virtue of the provisions of this Act, disqualified for receiving or continuing to receive an old age pension under this Act, the court may, if they think fit, order that the person convicted be so disqualified for such period, not exceeding ten years, as the court direct.

Calculation of means.

4.—(1) In calculating the means of a person for the purpose of this Act account shall be taken of—

(*a*) the income which that person may reasonably expect to receive during the succeeding year in cash, excluding any sums receivable on account of an old age pension under this Act, that income, in the absence of other means for ascertaining the income, being taken to be the income actually received during the preceding year;

(*b*) the yearly value of any advantage accruing to that person from the use or enjoyment of any

property belonging to him which is personally used or enjoyed by him;

(*c*) the yearly income which might be expected to be derived from any property belonging to that person which, though capable of investment or profitable use, is not so invested or profitably used by him; and

(*d*) the yearly value of any benefit or privilege enjoyed by that person.

(2) In calculating the means of a person being one of a married couple living together in the same house, the means shall not in any case be taken to be a less amount than half the total means of the couple.

(3) If it appears that any person has directly or indirectly deprived himself of any income or property in order to qualify himself for the receipt of an old age pension, or for the receipt of an old age pension at a higher rate than that to which he would otherwise be entitled under this Act, that income or the yearly value of that property shall, for the purposes of this section, be taken to be part of the means of that person.

Mode of paying pensions.

5.—(1) An old age pension under this Act, subject to any directions of the Treasury in special cases, shall be paid weekly in advance in such manner and subject to such conditions as to identification or otherwise as the Treasury direct.

(2) A pension shall commence to accrue on the first Friday after the claim for the pension has been allowed, or, in the case of a claim provisionally allowed, on the first Friday after the day on which the claimant becomes entitled to receive the pension.

Old age pension to be inalienable.

6. Every assignment of or charge on and every agreement to assign or charge an old age pension under this Act shall be void, and, on the bankruptcy of a person entitled to an old age pension, the pension shall not pass to any trustee or other person acting on behalf of the creditors.

Determination of claims and questions.

7.—(1) All claims for old age pensions under this Act, and all questions whether the statutory conditions are fulfilled in the case of any person claiming such a

pension, or whether those conditions continue to be fulfilled in the case of a person in receipt of such a pension, or whether a person is disqualified for receiving or continuing to receive a pension, shall be considered and determined as follows:—

(*a*) Any such claim or question shall stand referred to the local pension committee, and the committee shall (except in the case of a question which has been originated by the pension officer and on which the committee have already received his report), before considering the claim or question, refer it for report and inquiry to the pension officer:

(*b*) The pension officer shall inquire into and report upon any claim or question so referred to him, and the local pension committee shall, on the receipt of the report of the pension officer and after obtaining from him or from any other source if necessary any further information as to the claim or question, consider the case and give their decision upon the claim or question:

(*c*) The pension officer, and any person aggrieved, may appeal to the central pension authority against a decision of the local pension committee allowing or refusing a claim for pension or determining any question referred to them within the time and in the manner prescribed by regulations under this Act, and any claim or question in respect of which an appeal is so brought shall stand referred to the central pension authority, and shall be considered and determined by them:

(*d*) If any person is aggrieved by the refusal or neglect of a local pension committee to consider a claim for a pension, or to determine any question referred to them, that person may apply in the prescribed manner to the central pension authority, and that authority may, if they consider that the local pension committee have refused or neglected to

consider and determine the claim or question within a reasonable time, themselves consider and determine the claim or question in the same manner as on an appeal from the decision of the local pension committee.

(2) The decision of the local pension committee on any claim or question which is not referred to the central pension authority, and the decision of the central pension authority on any claim or question which is so referred to them, shall be final and conclusive.

8.—(1) The local pension committee shall be a committee appointed for every borough and urban district, having a population according to the last public census for the time being of twenty thousand or over, or for every county (excluding the area of any such borough or district), by the council of the borough, district, or county.

Local pension committee, central pension authority, and pension officers.

The persons appointed to be members of a local pension committee need not be members of the council by which they are appointed.

(2) A local pension committee may appoint such and so many sub-committees, consisting either wholly or partly of the members of the committee as the committee think fit, and a local pension committee may delegate, either absolutely or under such conditions as they think fit, to any such sub-committee any powers and duties of the local pension committee under this Act.

(3) The central pension authority shall be the Local Government Board, and the Board may act through such committee, persons, or person appointed by them as they think fit.

(4) Pension officers shall be appointed by the Treasury, and the Treasury may appoint such number of those officers as they think fit to act for such areas as they direct.

(5) Any reference in this Act to pension authorities shall be construed as a reference to the pension officer, the local pension committee, and the central pension authority, or to any one of them, as the case requires.

Penalty for false statements, &c., and repayment where pensioner is found not to have been entitled to pension.

9.—(1) If for the purpose of obtaining or continuing an old age pension under this Act, either for himself or for any other person, or for the purpose of obtaining or continuing an old age pension under this Act for himself or for any other person at a higher rate than that appropriate to the case, any person knowingly makes any false statement or false representation, he shall be liable on summary conviction to imprisonment for a term not exceeding six months, with hard labour.

(2) If it is found at any time that a person has been in receipt of an old age pension under this Act while the statutory conditions were not fulfilled in his case or while he was disqualified for receiving the pension, he or, in the case of his death, his personal representative, shall be liable to repay to the Treasury any sums paid to him in respect of the pension while the statutory conditions were not fulfilled or while he was disqualified for receiving the pension, and the amount of those sums may be recovered as a debt due to the Crown.

Regulations and expenses.

10.—(1) The Treasury in conjunction with the Local Government Board and with the Postmaster-General (so far as relates to the Post Office) may make regulations for carrying this Act into effect, and in particular—

(*a*) for prescribing the evidence to be required as to the fulfilment of statutory conditions and for defining the meaning of residence for the purposes of this Act; and

(*b*) for prescribing the manner in which claims to pensions may be made, and the procedure to be followed on the consideration and determination of claims and questions to be considered and determined by pension officers and local pension committees or by the central pension authority, and the mode in which any question may be raised as to the continuance, in the case of a pensioner, of the fulfilment of the statutory conditions, and as to the disqualification of a pensioner; and

(c) as to the number, quorum, term of office, and proceedings generally of the local pension committee and the use by the committee, with or without payment, of any offices of a local authority, and the provision to be made for the immediate payment of any expenses of the committee which are ultimately to be paid by the Treasury.

(2) The regulations shall provide for enabling claimants for pensions to make their claims and obtain information as respects old age pensions under this Act through the Post Office, and for provisionally allowing claims to pensions before the date on which the claimant will become actually entitled to the pension, and for notice being given by registrars of births and deaths to the pension officers or local pension committees of every death of a person over seventy registered by them, in such manner and subject to such conditions as may be laid down by the regulations, and for making the procedure for considering and determining on any claim for a pension or question with respect to an old age pension under this Act as simple as possible.

(3) Every regulation under this Act shall be laid before each House of Parliament forthwith, and, if an address is presented to His Majesty by either House of Parliament within the next subsequent twenty-one days on which that House has sat next after any such regulation is laid before it, praying that the regulation may be annulled, His Majesty in Council may annul the regulation, and it shall thenceforth be void, but without prejudice to the validity of anything previously done thereunder.

(4) Any expenses incurred by the Treasury in carrying this Act into effect, and the expenses of the Local Government Board and the local pension committees under this Act up to an amount approved by the Treasury, shall be defrayed out of moneys provided by Parliament.

Application to Scotland, Ireland, and the Scilly Isles.

11.—(1) In the application of this Act to Scotland, the expression "Local Government Board" means the Local Government Board for Scotland; the expression

"borough" means royal or parliamentary burgh; the expression "urban district" means police burgh; the population limit for boroughs and urban districts shall not apply; and the expression "Lunacy Act, 1890", means the Lunacy (Scotland) Acts, 1857 to 1900.

(2) In the application of this Act to Ireland, the expression "Local Government Board" means the Local Government Board for Ireland; ten thousand shall be substituted for twenty thousand as the population limit for boroughs and urban districts; and the expression "asylum within the meaning of the Lunacy Act, 1890", means a lunatic asylum within the meaning of the Local Government (Ireland) Act, 1898.

61 & 62 Vict. c. 37.

(3) In the application of this Act to the Isles of Scilly, those isles shall be deemed to be a county and the council of those isles the council of a county.

Commencement and short title.

12.—(1) A person shall not be entitled to the receipt of an old age pension under this Act until the first day of January nineteen hundred and nine and no such pension shall begin to accrue until that day.

(2) This Act may be cited as the Old Age Pensions Act, 1908.

SCHEDULE.

Means of Pensioner.	Rate of Pension per Week.
Where the yearly means of the pensioner as calculated under this Act--	*s.* *d.*
Do not exceed £21	5 0
Exceed £21, but do not exceed £23 12*s.* 6*d.* ...	4 0
Exceed £23 12*s.* 6*d.*, but do not exceed £26 5*s.* ...	3 0
Exceed £26 5*s.*, but do not exceed £28 17*s.* 6*d.* ...	2 0
Exceed £28 17*s.* 6*d.*, but do not exceed £31 10*s.*...	1 0
Exceed £31 10*s.*	No pension

ACTUARIAL NOTES.

I.

A Note on Complete Annuities. By W. PALIN ELDERTON, F.I.A., *Assistant Actuary of the Star Life Assurance Company.*

1. THE recent actuarial notes on the subject of complete annuities by Mr. Lidstone and Mr. R. D. Anderson remind one of the difficulty which the subject presents to students, and this is my excuse for sending a contribution to the discussion in case a general treatment may prove of some use. I may preface my remarks by saying that it seems so easy to apply Lubbock's or Woolhouse's formula directly to the evaluation of a complete annuity payable m times a year, that I am personally a little averse to troubling about other methods. Many of these alternative approximations are of considerable interest, but as it seems that the whole problem of approximate work in actuarial mathematics is to express either an area in terms of a series of ordinates, or a series of mn ordinates in terms of a series of n ordinates, I think there is a little danger of having a general method hidden behind less systematic methods, if we spend much time on these interesting alternatives.

2. I am inclined to believe that half the difficulty of the mathematical part of the work would be removed if we kept before our minds the forms of the Lubbock and Woolhouse formulæ we intended to use. For convenience of reference these are reproduced here.

Woolhouse's formula, No. 21 of *Text-Book*, Part II, Chapter xxiv, rearranged—

$$\int_a^b u_h dh = \frac{1}{m}\left(u_{a+\frac{1}{m}} + u_{a+\frac{2}{m}} + \ldots + u_b\right) - \frac{1}{2m}\left(u_h\right)_a^b - \frac{1}{12m^2}\left(\frac{du_h}{dh}\right)_a^b + \ldots \quad \text{(I)}$$

Lubbock's formula modified,

$$\int_a^b u_h dh = \frac{1}{m}\left(u_{a+\frac{1}{m}} + u_{a+\frac{2}{m}} + \ldots + u_b\right) - \frac{1}{2m}\left(u_h\right)_a^b - \frac{1}{12m^2}\left(\Delta u_h\right)_a^b$$
$$+ \frac{1}{24m^2}\left(\Delta^2 u_h\right)_a^b + \ldots \quad \text{(II)}$$

This last expression does not appear to be given in the *Text-Book*, but is useful and can be derived from the more usual expression

$$\frac{1}{m}\left(u_{a+\frac{1}{m}}+\ldots+u_b\right)=\left(u_{a+1}+\ldots+u_b\right)-\frac{m-1}{2m}\left(u_h\right)_a^b-\frac{m^2-1}{12m^2}\left(\Delta u_h\right)_a^b$$
$$+\frac{m^2-1}{24m^2}\left(\Delta^2 u_h\right)_a^b+\ldots \quad . \quad . \quad \text{(III)}$$

by making m infinite :

$$\int_a^b u_h dh=\left(u_{a+1}+\ldots+u_b\right)-\frac{1}{2}\left(u_h\right)_a^b-\frac{1}{12}\left(\Delta u_h\right)_a^b+\frac{1}{24}\left(\Delta^2 u_h\right)_a^b-\ldots$$
$$=\frac{1}{m}\left(u_{a+\frac{1}{m}}+\ldots+u_b\right)-\frac{1}{2m}\left(u_h\right)_a^b-\frac{1}{12m^2}\left(\Delta u_h\right)_a^b+\frac{1}{24m^2}\left(\Delta^2 u_h\right)_a^b-\ldots$$

either by changing the unit or by substitution from the ordinary Lubbock expression (III).

By means of (I), (II) and (III), with the addition of a form similar to (III) in differential coefficients, the applications throughout the *Text-Book* can be effected; but for continuous annuities (I) and (II) are all that are required.

3. We may now turn to the general proposition in complete annuities, but in case this note may be read by students having no practical experience, I may remark that the form used in practice is

$$\mathring{a}_x^{(m)}=a_x+\frac{m-1}{2m}+\frac{1}{2m}A_x,$$

which is quite accurate enough for all ordinary purposes, and can be seen at once if it is borne in mind that A_x is merely used as an approximation to $\overline{A}_x$.

4. Consider $\mathring{a}_x^{(m)}$, and see what exactly is wanted,

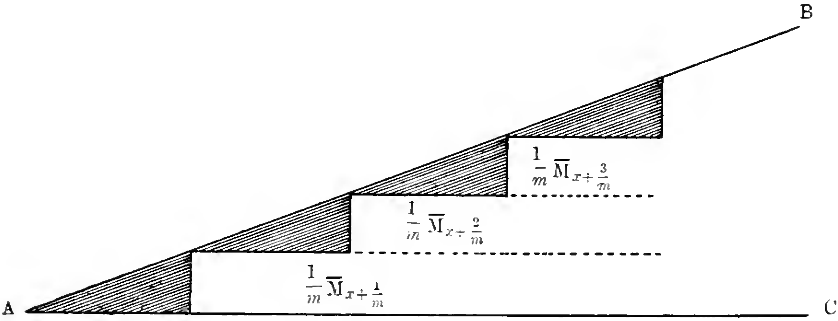
$$\mathring{a}_x^{(m)}=\frac{1}{mD_x}\left\{D_{x+\frac{1}{m}}+D_{x+\frac{2}{m}}+\ldots\right\}$$
$$+\frac{1}{D_x}\left\{\int_x^\infty \overline{M}_x dx-\frac{1}{m}\left(\overline{M}_{x+\frac{1}{m}}+\overline{M}_{x+\frac{2}{m}}+\ldots\right)\right\} \quad . \quad . \quad \text{(IV)}$$

From (IV) all the approximations required can be reached. In order to obtain (IV) it is merely necessary to remember that the adjustment for the complete portion would be a regularly increasing assurance during the whole of life (payable at the

moment of death) but at each $\frac{1}{m}$ th of a year a payment of annuity is received, therefore the assurance is to be decreased at each interval by $\frac{1}{m}$. The present value of the regularly increasing assurance is $\frac{1}{D_x}\int_x^{\infty}\overline{M}_x dx$, where $\overline{M}_x$ is defined by the relation $\overline{A}_x=\frac{\overline{M}_x}{D_x}$ and the deductions are

$$\frac{1}{m}\frac{\overline{M}_{x+\frac{1}{m}}}{D_x},\ \frac{1}{m}\frac{\overline{M}_{x+\frac{2}{m}}}{D_x},\ \&c.$$

A graphic illustration of (IV) may render this clearer.



AB shows the regularly increasing assurance, but what is wanted is a series increasing regularly for $\frac{1}{m}$, and then stopping and starting again, *i.e.*, the shaded part; the deductions, expressed in terms of $\overline{M}$, are shown in the figure. The $\frac{1}{m}$ has to be used as a coefficient because the unit of time dealt with is $\frac{1}{m}$ th of a year.

Now, returning to formula (IV), we notice that, since the Lubbock and Woolhouse formulæ (I) and (II) express $\int_a^b u_h dh$ in terms of $\frac{1}{m}\left(u_{a+\frac{1}{m}}+u_{a+\frac{2}{m}}+\ldots+u_b\right)$, we can eliminate either $\int\overline{M}dx$ or $(D-\overline{M})_{\frac{1}{m}}+(D-\overline{M})_{\frac{2}{m}}$, &c., where, in the latter case, we treat $(D-\overline{M})$ as a single function.

5. Eliminating $\int \overline{\mathrm{M}}dx$ by Woolhouse's (I), we have

$$\mathring{a}_x^{(m)} = \frac{1}{m\mathrm{D}_x}\left\{\mathrm{D}_{x+\frac{1}{m}}+\ldots\right\}+\frac{1}{\mathrm{D}_x}\left\{\frac{1}{m}\left(\overline{\mathrm{M}}_{x+\frac{1}{m}}+\overline{\mathrm{M}}_{x+\frac{2}{m}}+\ldots\right)\right.$$
$$\left.-\frac{1}{2m}\left(\overline{\mathrm{M}}_x\right)_x^\infty-\frac{1}{12m^2}\left(\frac{d}{dx}\overline{\mathrm{M}}_x\right)_x^\infty-\frac{1}{m}\left(\overline{\mathrm{M}}_{x+\frac{1}{m}}+\overline{\mathrm{M}}_{x+\frac{2}{m}}+\ldots\right)\right\}$$

$$= a_x^{(m)}+\frac{\overline{\mathrm{M}}_x}{2m\mathrm{D}_x}+\frac{1}{12m^2\mathrm{D}_x}\left(\frac{d}{dx}\overline{\mathrm{M}}_x\right)$$

the signs being changed because we are using the lower limit in above equation, the functions vanishing at the upper limit.

It only remains to value

$$\frac{d}{dx}\overline{\mathrm{M}}_x=\frac{d}{dx}\left(\mathrm{D}_x\overline{\mathrm{A}}_x\right)=\frac{d}{dx}\left(\mathrm{D}_x\overline{1-\delta\bar{a}_x}\right)$$
$$=\frac{d}{dx}\left(\mathrm{D}_x-\delta\int_0^\infty \mathrm{D}_{x+t}dt\right)$$
$$=-\mathrm{D}_x(\mu_x+\delta)-\delta\frac{d}{dx}\int_0^\infty \mathrm{D}_{x+t}dt$$
$$=-\mathrm{D}_x(\mu_x+\delta)+\delta\mathrm{D}_x$$

because we again only use the lower limit, the function vanishing at the upper limit.

Hence,
$$\mathring{a}_x^{(m)}=a_x^{(m)}+\frac{1}{2m}\overline{\mathrm{A}}_x-\frac{\mu_x}{12m^2}$$

formula (12) of Chap. XI.

6. Eliminating $(\mathrm{D}-\overline{\mathrm{M}})$ by Woolhouse (I), we have,

$$\mathring{a}_x^{(m)}=\frac{1}{m\mathrm{D}_x}\left\{(\mathrm{D}-\overline{\mathrm{M}})_{x+\frac{1}{m}}+(\mathrm{D}-\overline{\mathrm{M}})_{x+\frac{2}{m}}+\ldots\right\}+\frac{1}{\mathrm{D}_x}\int_x^\infty \overline{\mathrm{M}}_x dx$$

$$=\frac{1}{\mathrm{D}_x}\left[\int_x^\infty(\mathrm{D}-\overline{\mathrm{M}})_x dx+\frac{1}{2m}\left(\mathrm{D}_x-\overline{\mathrm{M}}_x\right)_x^\infty+\frac{1}{12m^2}\left\{\frac{d}{dx}(\mathrm{D}_x-\overline{\mathrm{M}}_x)\right\}_x^\infty\right.$$
$$\left.+\int_x^\infty \overline{\mathrm{M}}_x dx\right]$$

$$=\frac{1}{\mathrm{D}_x}\int_x^\infty \mathrm{D}_x dx-\frac{1}{2m}\delta\bar{a}_x-\frac{1}{12m^2\mathrm{D}_x}\left\{\frac{d}{dx}(\mathrm{D}_x-\overline{\mathrm{M}}_x)\right\}$$

$$=\bar{a}_x\left(1-\frac{\delta}{2m}\right)+\frac{\delta}{12m^2}\quad \ldots\ldots\ldots \quad \text{formula (13) of Chap. XI.}$$

by remembering that $\dfrac{\mathrm{D}_x-\overline{\mathrm{M}}_x}{\mathrm{D}_x}=1-\overline{\mathrm{A}}_x=\delta\bar{a}_x$, and that the signs are changed because we are using the lower limit only.

7. Again, taking Lubbock, we get the following by eliminating the integral

$$\bar{a}_x^{(m)} = \frac{1}{m\mathrm{D}_x}\left\{\mathrm{D}_{x+\frac{1}{m}} + \ldots\right\} - \frac{1}{\mathrm{D}_x}\left\{\frac{1}{2m}\left(\overline{\mathrm{M}}_x\right)_x^\infty + \frac{1}{12m^2}\left(\Delta\overline{\mathrm{M}}_x\right)_x^\infty - \frac{1}{24m^2}\left(\Delta^2\overline{\mathrm{M}}_x\right)_x^\infty, \text{ \&c.}\right\}$$

$$= a_x^{(m)} + \frac{1}{2m}\overline{\mathrm{A}}_x + \frac{1}{12m^2}\frac{\Delta\overline{\mathrm{M}}_x}{\mathrm{D}_x}, \text{ \&c.}$$

$$= a_x^{(m)} + \frac{1}{2m}\overline{\mathrm{A}}_x - \frac{1}{12m^2}\overline{\mathrm{A}}^1_{x\overline{1}|} \quad \text{formula (9) of Chap. XI.}$$

8. Finally, eliminating the $(\mathrm{D}-\overline{\mathrm{M}})$ series by Lubbock, we have

$$\bar{a}_x^{(m)} = \bar{a}_x + \frac{1}{\mathrm{D}_x}\left[\frac{1}{2m}\left(\mathrm{D}_x - \overline{\mathrm{M}}_x\right)_x^\infty + \frac{1}{12m^2}\left\{\Delta(\mathrm{D}_x - \overline{\mathrm{M}}_x)\right\}_x - \text{ \&c.}\right]$$

$$= \bar{a}_x\left(1 - \frac{\delta}{2m}\right) + \frac{\delta}{12m^2}(\bar{a}_x - vp_x\bar{a}_{x+1})$$

$$= \bar{a}_x\left(1 - \frac{\delta}{2m}\right) + \frac{\delta}{12m^2}\bar{a}_{x\overline{1}|}$$

9. It will be noticed at once that a further generalization could have been worked almost as easily by taking $\bar{a}_{x\overline{n}|}^{(m)}$ and applying formulas (I) and (II) to the evaluation of this expression. It will also be remarked that the present notes contain nothing really new; my reason for putting them forward is that they give the lines of thought that appealed most readily to me when I first read Chapter XI of the *Text-Book*, and they also seem to have been of some assistance to those to whom I have since shown them.

II.

On the calculation of Limited Payment Life Policy-Values.

By Frederick Alfred Williams, A.I.A., A.A.S., *of "La Nacional" Life Assurance Company of Mexico.*

THE following note on a continuous method of forming policy-values for limited payment life policies is offered in the hope that it may be of some slight interest, as the writer does not recall ever having come across anything in the *Journal* touching upon this particular method.

Having recently had occasion to calculate complete tables of policy-values for various limited payment life policies and endowment assurances, the following relation existing between the two classes was brought out and utilized, it being remembered that in America the popular form of endowment assurance is that maturing at the end of a term of years and not at a given age.

Denoting the value of a t-year limited payment life policy taken out at age x, which has been n years in force, by ${}^t_n\mathrm{V}_x$, we have

${}^t_n\mathrm{V}_x = \mathrm{A}_{x+n} - {}_t\mathrm{P}_x \,.\, \mathbf{a}_{x+n:\overline{t-n}|}$, where ${}_t\mathrm{P}_x$ denotes the net premium

$$= \mathrm{A}_x + \Delta\mathrm{A}^n_x - \mathrm{A}_x \cdot \frac{\mathbf{a}_{x+n\overline{t-n}|}}{\mathbf{a}_{x\overline{t}|}} \quad (\text{putting } \Delta\mathrm{A}^n_x = \mathrm{A}_{x+n} - \mathrm{A}_x)$$

$$= \mathrm{A}_x\left[1 - \frac{\mathbf{a}_{x+n\overline{t-n}|}}{\mathbf{a}_{x\overline{t}|}}\right] + \Delta\mathrm{A}^n_x$$

$$= \mathrm{A}_x \cdot {}_n\mathrm{V}_{x\overline{t}|} + \Delta\mathrm{A}^n_x.$$

This relation lends itself to application by a continuous method for each age at entry, as, having first calculated the policy-values for the endowment assurances by means of the well-known method denoted by the relation ${}_{n+1}\mathrm{V}_{x\overline{t}|} = {}_n\mathrm{V}_{x\overline{t}|} + \dfrac{\Delta\mathbf{a}}{\mathbf{a}_{x\overline{t}|}}$ where $\Delta\mathbf{a}$ is the difference* of the corresponding temporary annuities, we can utilize these latter differences and obtain the limited payment life policy-values thereby with great facility. The only preliminary step required is the preparation of the values $\Delta\mathrm{A}^n$, which can be quickly done on the machine, and, having these, our policy-values are very speedily prepared.

We have as above, ${}^t_n\mathrm{V}_x = \mathrm{A}_x \cdot {}_n\mathrm{V}_{x\overline{t}|} + \Delta\mathrm{A}^n_x$.

The second term of the right hand side of the equation may be disregarded and left to be added by a clerk to the last machine result whilst the next operation is being performed. The operation is easily seen from the following :

Let $\qquad \mathrm{Y}_n = \mathrm{A}_x \cdot {}_n\mathrm{V}_{x\overline{t}|}$.

Similarly $\mathrm{Y}_{n+1} = \mathrm{A}_x \cdot {}_{n+1}\mathrm{V}_{x\overline{t}|} = \mathrm{A}_x\left[{}_n\mathrm{V}_{x\overline{t}|} + \dfrac{\Delta\mathbf{a}}{\mathbf{a}_{x\overline{t}|}}\right] = \mathrm{Y}_n + {}_t\mathrm{P}_x \,.\, \Delta\mathbf{a}.$

Again $\qquad \mathrm{Y}_{n+2} = \mathrm{A}_x \cdot {}_{n+2}\mathrm{V}_{x\overline{t}|} = \mathrm{A}_x\left[{}_{n+1}\mathrm{V}_{x\overline{t}|} + \dfrac{\Delta\mathbf{a}_{+1}}{\mathbf{a}_{x\overline{t}|}}\right]$

$$= \mathrm{Y}_{n+1} + {}_t\mathrm{P}_x \,.\, \Delta\mathbf{a}_{+1}.$$

* Taken negatively, so that $\Delta\mathbf{a} = \mathbf{a}_{x+n.\overline{t-n}|} - \mathbf{a}_{x+n+1.\overline{t-n-1}|}$.—[ED. *J.I.A.*]

and so on, $\Delta \mathbf{a}$ being the difference of the proper temporary annuities.

Thus, starting with ${}_1V_{\overline{xt}}$, this is multiplied by A_x, and the result left on the machine, so that the next operation when performed will be automatically added. Next ${}_tP_x$ is set up, which is a constant for all values of n, and is multiplied successively by the corresponding $\Delta \mathbf{a}$, the result in each case (including the initial step) being called to the clerk, who adds the proper ΔA whilst the next value is being brought out. The machine does not have to be cleared until the next age at entry is dealt with, and the only check required on the work is that on the additions by the clerk, which can be handed out and independently checked. The machine work does not require checking, as, being continuous, an automatic verification occurs with the last policy-value for each age at entry, which should obviously be A_{x+t}.

This note is submitted in the hope that, if the method is not entirely new, it may at least prove of passing interest.

CORRESPONDENCE.

ON THE EFFECT OF A RISE, OR FALL, IN MARKET VALUES OF SECURITIES, ON THE FINANCIAL POSITION AND RESERVES OF A LIFE OFFICE.*

To the Editor of the Journal of the Institute of Actuaries.

DEAR SIR,—The heavy fall in marketable securities during recent years, has brought forward the question, to what extent a depreciation of the life investments means a real loss to a life company, and ought to be charged to the profit and loss account, in case the aggregate market value of the securities should be exceeded by the amount appearing in the balance sheet of the previous year.

The matter might be dealt with in the following way.

A fall of market prices is always accompanied by a rise in the average rate of interest, and if the valuation is made on the corresponding higher percentage basis, then the diminution of the value of the life assets is to a great extent counterbalanced, in some cases even exceeded, by the reduction of the value of the company's liabilities.

Assuming that by the depreciation of the investments the value of the life assets is fallen from B to B′, that the average rate of interest of these assets is risen from i to i', and that the company's liability calculated upon bases i and i' is V and V′, then the value of the company's real loss arising from depreciation of a unit in the market value of the assets is:

$$f = \frac{B - B' - (V - V')}{B - B'}.$$

* See also the remarks on this subject in the President's Address (pp. 16–18 of present volume).

Now, evidently $B = V$,

and, understanding that the amount of interest on the life assets has remained unaltered, then: $Bi = B'i'$,

whence
$$f = \frac{V'i' - Vi}{V(i' - i)}$$

in which
$$V = A - P\mathbf{a},$$

and
$$V' = A' - P\mathbf{a}',$$

A and $\mathbf{a}$ being valued on basis i, A' and $\mathbf{a}'$ on basis i', whilst P denotes in both formulas the same quantity, because the premium payable under the policy is invariable.

Supposing $i = \cdot035$, $i' = \cdot04$ and $P =$ net premium $O^{M(5)}$ $3\frac{1}{2}$ per-cent, I find according to the $O^{M(5)}$ mortality table, and for age 30 at entry:

Years elapsed	Values of f				
	Whole Life	Endowment Assurance			
		30 years	25 years	20 years	15 years
5	−2·11	−1·20	−0·70	−0·18	0·31
10	−0·68	−0·11	0·18	0·49	0·77
15	−0·16	0·32	0·55	0·79	1·00
20	0·13	0·59	0·79	1·00	...
25	0·32	0·80	1·06	...	...
30	0·47	1·00	...	...	...

On the same bases of mortality and interest I find for life annuities:

Age attained	Value of f
	Life Annuity
40	0·53
50	0·62
60	0·71
70	0·79
80	0·87
90	0·93

In these tables f means, as has been said, the real loss to the Company, if the market value of its life securities has decreased by unity.

Using other mortality tables and rates of interest, the figures remain about the same.

It appears from these tables that the gains or losses caused to a life company by a rise or a fall of the market value of its securities are for the greater part imaginary, and have but little influence on the results of the year, provided the liabilities are valued in the manner described.

A valuation on a variable per-cent basis can be practically realised with sufficient accuracy, by making the valuation say, on a 3 and on a 4 per-cent basis, and by calculating the liabilities on the desired per-cent basis by interpolation.

Your obedient servant,

DR. D. P. MOLL.

Actuary of The Netherlands Fire and Life Insurance Company.

The Hague, July 1908.

INSTITUTE TEXT-BOOK, PART I.

To the Editor of the Journal of the Institute of Actuaries.

SIR,—It has been pointed out to me by Mr. H. B. Smither that by application of the general formula (21) of Chapter V the answer to the question discussed in § 30 of that chapter may be accurately and much more simply expressed in the form

$$\frac{:C}{1+g}a'_{\overline{n}|} + \frac{g(1+\sqrt{1+i})}{2i}\left(C - \frac{:C}{1+g}a'_{\overline{n}|}\right)$$

where a' is calculated at the rate given by $v' = v(1+g)$. This result follows at once from

$$K = v{:}C + v^2{:}C(1+g) + v^3{:}C(1+g)^2 + \ldots + v^n{:}C(1+g)^{n-1}$$

Its identity with the present value at rate i of the varying annuity of which the rth payment is

$$(g + :)C + g\frac{C}{2}(\sqrt{1+i} - 1)(1 - :s_{\overline{r-1}|}),$$

where $s_{\overline{r-1}|}$ is calculated at rate g, can of course be established by algebraical transformations.

I shall be much obliged if you can find space for these few lines, as they may save some unnecessary trouble to future students.

I am, Sir,

Your obedient servant,

R. TODHUNTER.

25, *Pall Mall, S.W.*
17 *December* 1908.

THE INSTITUTE OF ACTUARIES.

EXAMINATIONS, 1909.

We are authorized to state, for the information of Candidates entering for the Examinations to be held in 1909, that Candidates who have passed Part I only of any previous Syllabus will be permitted to take Section (3) of the new Part I, as a separate Examination; also, that the paper set in that Section (Compound Interest and Annuities-Certain) will in 1909 be *common* to the old Part II and the new Part I; and that the Board of Examiners in that year will report separately as to the Candidates in Part II who pass in that Section.

The Council have appointed the undermentioned gentlemen to constitute the Board of Examiners until further notification:—Messrs. Thomas G. Ackland (*Chairman*, 1909), Henry J. Baker (*Hon. Secretary*, 1909), Arthur R. Barrand, Joseph Burn, W. Palin Elderton, George King, John Spencer, Robert R. Tilt, Harold M. Trouncer and Alfred W. Watson.

OBITUARY NOTICE.

Dr. Israel C. Pierson.

We regret to announce the death of Dr. Israel C. Pierson, which took place on September 11th last. Dr. Pierson was the first Secretary of the Actuarial Society of America, a post which he held for ten years, during which period he very largely contributed to its growth and development. Those who attended the meetings of the Fourth International Congress of Actuaries held at New York in 1903, will have pleasant recollections of the dignity and courtesy with which Dr. Pierson presided over its sessions, and of his cordial relations with the delegates from Great Britain and other countries, both in the business meetings and in the social engagements of the Congress. He was elected a corresponding member of the French Institute of Actuaries, and of the Belgian Association of Actuaries, and was an honoured Associate of our own Institute. He was a Graduate of the New York University, from which he received the degree of Ph.D. in 1890. He was also a Fellow of the American Statistical Society, the New York Academy of Science, and the New York Mathematical Society. He received his actuarial training in the New York Life Insurance Company, and the Equitable Life Assurance Society of the United States, and was Actuary of the Washington Life Office until his resignation in 1905.

JOURNAL

OF THE

INSTITUTE OF ACTUARIES.

On a New Method of Constructing and of Graduating Mortality and other Tables. By GEORGE KING, F.I.A., F.F.A., *Consulting Actuary.*

[Read before the Institute, 14 December 1908.]

1. THIS is the fourth, and, let it be hoped, the last, of a series of papers on mortality tables in their various aspects which the Institute has honoured me by accepting. The investigation had its origin many years ago in efforts to improve upon the then methods of constructing mortality tables from Census Returns; and, when the enquiry seemed to be nearing its conclusion, the fact became evident that the formulas which had presented themselves could be applied effectively in summation form to the graduation of tables derived from other sources. There was, however, a preliminary objection to be overcome. Summation Formulas of Graduation were under a cloud. Dr. Sprague had condemned them in unmeasured terms, and Mr. Todhunter, in the head-note to his paper (*J.I.A.*, xxxii, 378), had accepted Dr. Sprague's authoritative judgment. Hence my first paper of the series, "On the Error introduced into Mortality Tables by Summation Formulas of Graduation", (*J.I.A.*, xli, 54), wherein it was attempted, and, I think, successfully, to show that that error is generally so minute as, with ordinary care, to be safely negligible in practice.

2. The second paper of the series, "Notes on Summation Formulas of Graduation", (*J.I.A.*, xli, 530), still on a side issue, was a natural sequel to the first. It dealt in a way differing

from those of the late Mr. J. A. Higham, and of our honoured President, Mr. George F. Hardy, with the fascinating problem of the graduation of mortality tables; and in it were applied to the purposes of graduation some of the formulas which had been deduced for the purposes of construction.

3. The main object of the whole investigation as originally contemplated was reached when I had the honour of presenting the third paper of the series, (*J.I.A.*, xlii, 225), "On the "Construction of Mortality Tables from Census Returns and "Records of Deaths", hereinafter frequently quoted, and for the sake of brevity called the "Census Paper." But all along there had been the intention of seeking further developments, and of applying similar principles to the construction of tables derived from sources of various kinds, with the more special object of showing that census tables constructed on these principles are worthy of confidence. It must be confessed that the hopes were but modest of results being achieved in this direction in themselves of practical and important use; but now, in submitting the final paper of the series, I cannot refrain from expressing the conviction that a completeness and success have been reached beyond anything that could have been anticipated. The construction of mortality tables, no matter from what source derived, is brought under one harmonious system; and the tables so produced are as smooth as the very best constructed and graduated in other ways, while at the same time it would appear that they adhere with the most minute fidelity to the original facts. Moreover, the labour involved is reduced to a minimum. There is no need, for instance, to go to the trouble of calculating the unadjusted values of q_x; and the work required to produce directly from the Exposed to Risk and the Deaths a table of exceptional smoothness and fidelity to data is much less than that involved in graduation alone by any of the more satisfactory formulas previously devised. Lastly, the system, sometimes suitably modified to meet special circumstances, is applicable to tables of many and varied kinds. From a not inconsiderable experience, I can bear testimony to the celerity and certainty with which crude statistics can be dealt with by the new method, and to the great saving of labour that ensues.

4. The fundamental processes of the new method are, first, to find graduated quinquennial values of the function to be dealt

with, and secondly, to insert the intervening values by osculatory interpolation, of which Dr. Sprague (*J.I.A.*, xxii, 270) was the inventor. In deducing his formulas he used central differences, and carried them out to the fifth order; but it seemed to me that the introduction of the central difference notation gave needlessly the appearance of intricacy, and fostered the idea of great complexity. Therefore, in the Census Paper an alternative demonstration was given, in which only the notation of ordinary differences was used, and thus Dr. Sprague's 5th difference formula was reproduced; and in *J.I.A.*, xli, 544, I similarly brought out a formula involving three differences only. Of course, seeing that Dr. Sprague and myself use identical differences, and that we both use them centrally, our results are the same, and the question between us is merely one of personal preference for one scheme of notation or another in the analysis.

5. Dr. Buchanan (*J.I.A.*, xlii, 369) favours central difference notation, and uses central differences in the form given to them by Prof. Everett (*J.I.A.*, xxxv, 452); but I can scarcely imagine that anyone reading Dr. Buchanan's paper will think that he has simplified matters. After a most elaborate and difficult investigation, he produces two formulas for osculatory interpolation, which to all intents and purposes are only Dr. Sprague's 5th difference formula and my own with three differences, in a new garb; and it is easy to deduce Dr. Buchanan's expressions from those of Dr. Sprague and myself, or *vice versa*. The arithmetical application of Dr. Buchanan's expressions is not the same as when Dr. Sprague's form is used, but there does not appear to be any gain in brevity—rather the reverse. That, however, is a point of small detail, and each computor should use the form that suits him best. I find that when my 3rd difference formula is used in Dr. Sprague's form, a complete and perfectly smooth mortality table can be constructed from the original data in about three to four hours, and it is not easy to conceive of anything more expeditious.

6. In passing, a protest must be entered against part of Dr. Buchanan's scheme of notation. In actuarial literature the symbol

$$\delta u_0 \text{ represents } u_1 - u_0$$

and

$$\delta^2 u_0 \text{ represents } u_2 - 2u_1 + u_0$$

and so on; but, according to Dr. Buchanan,

$$\delta u_0 \text{ represents } u_{\frac{1}{2}} - u_{-\frac{1}{2}}$$

$$\delta^2 u_0 \text{ represents } u_1 - 2u_0 + u_{-1}$$

and so on. In fact, Dr. Buchanan annexes the symbols of ordinary differences, and applies them unchanged to central differences. Such a procedure is surely indefensible. It leads to confusion, and interposes difficulties and pitfalls in the path of the student which are entirely unnecessary. The central differences and the ordinary differences of a function are so nearly related to each other that they must be clearly distinguished by dissimilarity in notation. Woolhouse and others have recognized this necessity, and have written a_0, b_0, &c., for the central differences of u_0, retaining the symbols δu_0, $\delta^2 u_0$, &c., for the ordinary differences, and their good example is one to be followed. Even if a writer transgress in this respect in his own private notes, he should not do so in the pages of the Institute *Journal*, or otherwise the would-be student will be discouraged, and actuarial research will be retarded.

7. As already remarked, the fundamental processes of the new method are, first, to find graduated quinquennial values of the function to be dealt with, and then to insert the intervening values by osculatory interpolation. In paragraphs 80 and 81 of the Census Paper, formulas are given for finding the graduated quinquennial values. The general symbol u is there used for the function, but it would have been better, and more consistent with the rest of the notation in this series of papers, to use the symbol y, and to treat y as the finite integral of the function u, of which a complete table is required.

8. Let, therefore, y be the finite integral, taken negatively,* of u, and let Δ represent the differences for quinquennial invervals. We shall then have, using four orders of differences, and taking y_0 as the initial term,

$$y_{2+\frac{1}{5}} = y_0 + 2{\cdot}2\Delta y_0 + 1{\cdot}32\Delta^2 y_0 + {\cdot}088\Delta^3 y_0 - {\cdot}0176\Delta^4 y_0,$$

when the intervals are quinquennial. It is, however, convenient to pass to yearly intervals, and to write y_{11} for $y_{2+\frac{1}{5}}$, while we

* The summations are here taken from the oldest age to the youngest, the reverse of the order usual in integrating. This is in accordance with the course followed in the Census Paper, copying the example of the General Register Office. With this explanation no confusion is likely to arise.

still retain the differences for quinquennial intervals, so that now Δy_0 will represent $y_5 - y_0$. In this notation the equation becomes

(i). $$y_{11} = y_0 + 2{\cdot}2\Delta y_0 + 1{\cdot}32\Delta^2 y_0 + {\cdot}088\Delta^3 y_0 - {\cdot}0176\Delta^4 y_0$$

(i*a*). $$-u_{10} = {\cdot}2\Delta y_0 + {\cdot}32\Delta^2 y_0 + {\cdot}088\Delta^3 y_0 - {\cdot}0176\Delta^4 y_0$$

9. Here the five original values used are y_0, y_5, y_{10}, y_{15} and y_{20}, of which y_{10} is the central; and the resulting y_{11} is the value for an age one year older than the central point of age. When we deduct y_{11} from y_{10} we obtain u_{10}, the required graduated value of the function at that point. This was the formula used in the Census Paper to find quinquennial graduated values of L_x and d_x, and thence of q_x; and $\log q_x$ was made the subject of osculatory interpolation. Logarithms were used, so that y_{11} was $\log T_{11}$ for the population, and $\log l_{11}$ for the deaths; and the corresponding numbers had to be extracted before making the subtraction $y_{10} - y_{11}$.

10. It will be observed that the series employed contains twenty values of u. Therefore, there is not a central value, and u_{10} has ten values in front of it, and only nine after it. Consequently, it is not quite central, and thereby a small theoretical error is introduced. In dealing with Census Returns this error is of no importance, and even sometimes it may be an advantage to employ the formula, as its tendency seems to be to minimize the errors in the original data due to misstatements of age. When, however, the formula comes to be applied to the more exact annuity and assurance mortality tables, the error, although very small, is not inappreciable, especially at the higher ages, and a really central formula must be sought for.

11. The data for annuity and assurance mortality tables consist of the Exposed to Risk, E_x, and the Deaths, θ_x, at each age; and the problem is to find, by a central formula, graduated quinquennial values of these functions, whence to derive graduated quinquennial values of q_x. If we have a series of 25 terms of the ungraduated function u, of which u_0 is the first term, we shall have u_{12} as the central term, and the problem, therefore, is to find a graduated value of u_{12}.

12. As before, let y be the finite integral, taken negatively, of u, and let Δy, $\Delta^2 y$, &c., be the differences of y for

quinquennial intervals. Then $u_{12}=y_{12}-y_{13}$. By the ordinary formula of finite differences,

$$y_{12}=y_0+12\frac{\Delta y_0}{5}+42\frac{\Delta^2 y_0}{5^2}+28\frac{\Delta^3 y_0}{5^3}-21\frac{\Delta^4 y_0}{5^4}+33{\cdot}6\frac{\Delta^5 y_0}{5^5}$$

$$y_{13}=y_0+13\frac{\Delta y_0}{5}+52\frac{\Delta^2 y_0}{5^2}+52\frac{\Delta^3 y_0}{5^3}-26\frac{\Delta^4 y_0}{5^4}+36{\cdot}4\frac{\Delta^5 y_0}{5^5}$$

whence

$$-u_{12}=\frac{\Delta y_0}{5}+10\frac{\Delta^2 y_0}{5^2}+24\frac{\Delta^3 y_0}{5^3}-5\frac{\Delta^4 y_0}{5^4}+2{\cdot}8\frac{\Delta^5 y_0}{5^5}.$$

But the first differences for quinquennial intervals of the function y are the sums of five values (with negative sign) of the function u: that is

$$-\Delta y_0=u_0+u_1+\ldots+u_4$$

$$-\Delta y_5=u_5+u_6+\ldots+u_9,\ \&c.$$

It is convenient to treat these quinary groups of u as the primary function, and to write $-\Delta y_0=w_0$, $-\Delta y_5=w_5$, &c., so that $-\Delta^2 y_0=\Delta w_0$, $-\Delta^3 y_0=\Delta^2 w_0$, &c. In this notation the equation becomes

(ii). $$u_{12}=\frac{w_0}{5}+10\frac{\Delta w_0}{5^2}+24\frac{\Delta^2 w_0}{5^3}-5\frac{\Delta^3 w_0}{5^4}+2{\cdot}8\frac{\Delta^4 w_0}{5^5}$$

(ii*a*). $$u_{12}=\frac{w_{10}}{5}-\frac{\Delta^2 w_5}{5^3}+2{\cdot}8\frac{\Delta^4 w_0}{5^5}.$$

13. To apply the formula we sum, from a convenient starting point, in groups of fives, the series to be dealt with for each quinquennial interval of the table, and difference four times. We then modify the resulting function, w, and its differences, by dividing by the proper powers of 5; and, lastly, we work out the formula by means of the numerical coefficients. If the arithmometer be used, we may avoid the trouble of modifying w and its differences, and write

(iii). $u_{12}={\cdot}2w_0+{\cdot}4\Delta w_0+{\cdot}192\Delta^2 w_0-{\cdot}008\Delta^3 w_0+{\cdot}000896\Delta^4 w_0$.

(iii*a*). $u_{12}={\cdot}2w_{10}-{\cdot}008\Delta^2 w_5+{\cdot}000896\Delta^4 w_0$.

14. If, instead of taking five differences of y, we stop at the third, then u_7 will be the central of fifteen terms of

a series of which u_0 is the first, and $u_7 = y_7 - y_8$. Proceeding as before

$$y_7 = y_0 \quad + 7\frac{\Delta y_0}{5} + \ 7\frac{\Delta^2 y_0}{5^2} - 7\frac{\Delta^3 y_0}{5^3}$$

$$y_8 = y_0 \quad + 8\frac{\Delta y_0}{5} + 12\frac{\Delta^2 y_0}{5^2} - 8\frac{\Delta^3 y_0}{5^3}$$

$$-u_7 = \frac{\Delta y_0}{5} + 5\frac{\Delta^2 y_0}{5^2} - \frac{\Delta^3 y_0}{5^3}$$

That is

(iv). $$u_7 = \frac{w_0}{5} + 5\frac{\Delta w_0}{5^2} - \frac{\Delta^2 w_0}{5^3}, \text{ or}$$

(v). $$u_7 = \cdot 2 w_0 + \cdot 2 \Delta w_0 - \cdot 008 \Delta^2 w_0.$$

(va). $$u_7 = \cdot 2 w_5 - \cdot 008 \Delta^2 w_0.$$

Equation (va) is very short to work, either mentally or on the arithmometer.

15. Formulas (ii) and (iv) are really Summation Formulas of Graduation in embryo, formula (ii) being the central grouped curve of the five that are combined in Formula A^{I} of my "Notes on Summation Formulas of Graduation", and (iv) being the central grouped curve in Formula C^{I}. No. (ii) is correct up to and including the fifth differential coefficient, the theoretical error being $+3744 u_{12}^{VI}$; and No. (iv) up to and including the third differential coefficient, the theoretical error being $-33{\cdot}6 u_7^{IV} - 1464 u_7^{VI}$. Although the numerical coefficients of u^{IV} and u^{VI} appear large, the differential coefficients themselves are very small, and the theoretical errors are really insignificant.

16. The foregoing developments have been tested on certain standard mortality tables. Many experiments were tried in a preliminary way on Finlaison's Government Female Annuitants (1883) Ultimate Table, because that is perhaps the most intractable that is available, and presents the greatest difficulties in the way of graduation. The table was first reconstructed from E_x and θ_x strictly on the methods of the Census Paper, formula (*ia*) in paragraph 8 above being employed to find quinquennial

values of E_x and θ_x at ages 30, 35, &c., up to 90; while for ages 25 and 95 the short second difference formula of the same kind given in paragraph 80 of the Census Paper was used. Thence, graduated quinquennial values of q_x were derived, and the intervening values of log q_x were interpolated by osculatory third differences, q_{102} being taken as unity. In regard to smoothness, the resulting table was quite satisfactory, and there was no great discrepancy between the expected and the actual deaths; but still the small error spoken of in paragraph 10 above was apparent. The following is a summary of the results. It has not been thought necessary to burden this paper by reproducing the table in extenso.

Government Female Annuitants (1883) *Ultimate Table, reconstructed strictly on the methods of the Census Paper.*

Age Group	Actual Deaths	Expected Deaths	Deviation	Accumulated Deviation	Age Group
30 to 34	10	11·1	+ 1·1	+ 1·1	30 to 34
35 ,, 39	21	21·8	+ ·8	+ 1·9	35 ,, 39
40 ,, 44	55	50·5	− 4·5	− 2·6	40 ,, 44
45 ,, 49	91	96·0	+ 5·0	+ 2·4	45 ,, 49
50 ,, 54	212	207·6	− 4·4	− 2·0	50 ,, 54
55 ,, 59	439	431·0	− 8·0	− 10·0	55 ,, 59
60 ,, 64	802	815·3	+ 13·3	+ 3·3	60 ,, 64
65 ,, 69	1,525	1499·4	− 25·6	− 22·3	65 ,, 69
70 ,, 74	2,279	2310·5	+ 31·5	+ 9·2	70 ,, 74
75 ,, 79	2,834	2796·6	− 37·4	− 28·2	75 ,, 79
80 ,, 84	2,466	2493·4	+ 27·4	− ·8	80 ,, 84
85 ,, 89	1,463	1435·2	− 27·8	− 28·6	85 ,, 89
90 ,, 94	449	447·7	− 1·3	− 29·9	90 ,, 94
95 ,, 99	74	85·1	+ 11·1	− 18·8	95 ,, 99
100 ,, 102	2	5·9	+ 3·9	− 14·9	100 ,, 102
Total	12,722	12707·1	±203·1	±176·0	Total

17. In his recent paper, already referred to above, Dr. Buchanan suggested that, instead of the ordinary differences of formula (i), an osculatory formula should be employed in finding the graduated quinquennial values of q_x. That plan has been tried, but the results are far from satisfactory. Dr. Buchanan propounded the idea with the view of improving the smoothness of the final table, but the effect in that direction is scarcely, if at all, perceptible, while the divergence between the expected and

the actual deaths is markedly increased. The following is a summary of the results:

Government Female Annuitants (1883) Ultimate Table, reconstructed by the method of the Census Paper, except that the graduated quinquennial values of q_x were derived by an osculatory formula.

Age Group	Expected Deaths	Deviation	Accumulated Deviation	Age Group
30 to 34	13·4	+ 3·4	+ 3·4	30 to 34
35 ,, 39	25·1	+ 4·1	+ 7·5	35 ,, 39
40 ,, 44	53·8	− 1·2	+ 6·3	40 ,, 44
45 ,, 49	87·6	− 3·4	+ 2·9	45 ,, 49
50 ,, 54	205·1	− 6·9	− 4·0	50 ,, 54
55 ,, 59	429·9	− 9·1	− 13·1	55 ,, 59
60 ,, 64	821·3	+ 19·3	+ 6·2	60 ,, 64
65 ,, 69	1496·3	− 28·7	− 22·5	65 ,, 69
70 ,, 74	2368·0	+ 89·0	+ 66·5	70 ,, 74
75 ,, 79	2807·0	− 27·0	+ 39·5	75 ,, 79
80 ,, 84	2457·0	− 9·0	+ 30·5	80 ,, 84
85 ,, 89	1402·7	− 60·3	− 29·8	85 ,, 89
90 ,, 94	386·9	− 62·1	− 91·9	90 ,, 94
95 ,, 99	87·2	+ 13·2	− 78·7	95 ,, 99
100 ,, 102	6·1	+ 4·1	− 74·6	100 ,, 102
Total	12647·4	±340·8	±477·4	Total

18. In Tables 1 and 2 of the Appendix are given the results of two constructions, called A and B respectively, of the Government Female Annuitants (1883) Ultimate Table by the new method, improved so as to remove the small theoretical error inherent in the method of the Census Paper, and it has been thought desirable to give the figures in full as an example of what the new method can achieve in a somewhat difficult case.

19. For Construction A formula (v*a*) above was applied to E_x and θ_x separately, in order to find graduated quinquennial values of q_x, and then log q_x was interpolated by the osculatory 3rd difference formula (vii) given in paragraph 24 below, and the whole work was done on the arithmometer. As the results are of importance in illustrating the new method, explanations of the process in some detail may be permitted.

20. The original data, namely, the exposed to risk, E_x, and the deaths, θ_x, at each age, are to be found on pp. 24 and 25 of the late Mr. A. J. Finlaison's well-known report. To suit formula (v*a*) the sums of E_x and θ_x in quinary groups were taken

at ages 20, 25, &c., to 95, and differenced twice, and the following are the figures :

EXPOSED TO RISK, E_x.				
Age	$\Sigma_x^{x+4} E_x = w_x$	Δw_x	$\Delta^2 w_x$	Age
20	123	+ 311	+ 365	20
25	434	+ 676	+ 353	25
30	1,110	+ 1,029	+1,089	30
35	2,139	+ 2,118	+1,590	35
40	4,257	+ 3.708	+1,782	40
45	7.965	+ 5,490	+3,480	45
50	13,455	+ 8.970	+ 718	50
55	22,425	+ 9.688	−1,774	55
60	32,113	+ 7,914	−7,628	60
65	40,027	+ 286	−9,567	65
70	40.313	− 9,281	−3,724	70
75	31,032	−13,005	+1,968	75
80	18.027	−11.037	+5,582	80
85	6.990	− 5,455	+4,102	85
...	(220,410)	+ 1,412)	(−1,664)	...
90	1,535	− 1.353	...	90
95	182	...	...	95

DEATHS, θ_x.				
Age	$\Sigma_x^{x+4} \theta_x = w_x$	Δw_x	$\Delta^2 w_x$	Age
20	0	+ 9	− 8	20
25	9	+ 1	+ 10	25
30	10	+ 11	+ 23	30
35	21	+ 34	+ 2	35
40	55	+ 36	+ 85	40
45	91	+ 121	+ 106	45
50	212	+ 227	+ 136	50
55	439	+ 363	+ 360	55
60	802	+ 723	+ 31	60
65	1.525	+ 754	− 199	65
70	2,279	+ 555	− 923	70
75	2,834	− 368	− 635	75
80	2.466	− 1,003	− 11	80
85	1,463	− 1,014	+ 639	85
...	12.206)	+ 449)	(− 384)	...
90	449	− 375	...	90
95	74	...	...	95

21. Formula (v*a*) was then applied to the figures of each section of the above statement in order to obtain the graduated quinquennial values of E_x and θ_x for ages 27, 32, &c., up to 92, which is as far as the data treated in this way will permit; and hence the values of q_x at these ages were found. It will be

observed that the sums to age 85 of w_x and of its differences are given above as a check on the calculations. Formula (v) is applied to these sums, and the results should be the sums of the graduated quinquennial values of E_x and θ_x. The following are the detailed figures.

Graduated Values of E_x *and* θ_x *and* q_x.

Age	E_x	θ_x	q_x	Age
27	83·880	1·864	·0222222	27
32	219·176	1·920	·0087601	32
37	419·088	4·016	·0095827	37
42	838·680	10·984	·0130968	42
47	1578·744	17·520	·0110974	47
52	2663·160	41·552	·0156025	52
57	4479·256	86·712	·0193586	57
62	6436·792	157·520	·0244718	62
67	8066·424	304·752	·0377803	67
72	8139·136	457·392	·0561966	72
77	6236·192	574·184	·0920728	77
82	3589·656	498·280	·1388099	82
87	1353·344	292·688	·2162703	87
92	274·184	84·688	·3088729	92
	44377·712	2534·072		
97	...	...	·5317415	97
102	...	...	1·0000000	102

22. Formula (v*a*) gives q_x as far as age 92, and we assume that q_{102} is unity, because that is the final age in the table of original data, the Exposed to Risk at that age being unity. We find q_{97} by means of a third difference constructed from the values for ages 82, 87, 92 and 102 as shown in the Census Paper, paragraph 70. The formula is

$$\Delta^3 q_{82} = \frac{1}{4}\{q_{102} - q_{82} - 4\Delta q_{82} - 6\Delta^2 q_{82}\}$$

23. It may be mentioned that the ages 20, 25, &c., at which the quinquennial values of w_x are taken were not chosen arbitrarily, but of purpose, so that age 102, the final age of the table, might be one of the quinquennial points used for the osculatory interpolation of $\log q_x$. For this reason it will be found convenient that the oldest age for w_x shall always be seven years younger than the age at which q_x is taken as unity.

24. The 3rd difference osculatory formula of interpolation is demonstrated in my Notes on Summation Formulas of Graduation, *J.I.A.*, xli, 530, and repeated in abstract in

paragraph 60 of the Census Paper. When u is the function which is to be the subject of interpolation, and when Δ, as before, represents differences for quinquennial intervals, and δ those for yearly intervals, we have

$$\text{(vi).}\quad \begin{cases} \delta u_5 = \dfrac{\Delta u_0}{5} + 3\dfrac{\Delta^2 u_0}{5^2} - 2\dfrac{\Delta^3 u_0}{5^3} \\ \delta^2 u_5 = \dfrac{\Delta^2 u_0}{5^2} - 2\dfrac{\Delta^3 u_0}{5^3} \\ \delta^3 u_5 = 3\dfrac{\Delta^3 u_0}{5^3} \end{cases}$$

This form is very convenient when an arithmometer is not available, because, when we have modified the differences of u by dividing by the suitable powers of 5, which is done by multiplying mentally by ·2, ·04, and ·008, the numerical coefficients are easy to work. If, however, recourse can be had to an arithmometer, we need not go to the trouble of modifying the differences of u, and the formula may be written

$$\text{(vii).}\quad \begin{cases} \delta u_5 = \cdot 2\Delta u_0 + \cdot 12\Delta^2 u_0 - \cdot 016\Delta^3 u_0 \\ \delta^2 u_5 = \cdot 04\Delta^2 u_0 - \cdot 016\Delta^3 u_0 \\ \delta^3 u_5 = \cdot 024\Delta^3 u_0 \end{cases}$$

25. The following are the values of $\log q_x$, increased by 3 to eliminate negative characteristics, and their differences, which were used to construct the differences required in the osculatory interpolations :

Age	$3 + \log q_x$	$\Delta \log q_x$	$\Delta^2 \log q_x$	$\Delta^3 \log q_x$	Age
27	1·34679	−·40429	+·44327	−·34657	27
32	0·94250	+·03898	+·09670	−·30431	32
37	0·98148	+·13568	−·20761	+·42750	37
42	1·11716	−·07193	+·21989	−·27417	42
47	1·04523	+·14796	−·05428	+·06240	47
52	1·19319	+·09368	+·00812	+·07867	52
57	1·28687	+·10180	+·08679	−·10293	57
62	1·38867	+·18859	−·01614	+·05811	62
67	1·57726	+·17245	+·04197	−·07811	67
72	1·74971	+·21442	−·03614	+·05047	72
77	1·96413	+·17828	+·01433	−·05218	77
82	2·14241	+·19261	−·03785	+·11901	82
87	2·33502	+·15476	+·08116	−·04278	87
92	2·48978	+·23592	+·03838	...	92
97	2·72570	+·27430	...	...	97
102	3·00000	...	...	...	102

26. Those students who may wish to verify the processes in their entirety will be glad to have recorded the osculatory differences themselves, calculated from the above data by the formula of paragraph 24, above. They are as follows, the decimal points being omitted.

Age	δ^3	δ^2	δ	$3+\log q_x$	Age
32	− 83178	+232760	−221204	9425000	32
37	− 73035	+ 87370	+242690	9814800	37
42	+102600	−151414	− 46172	11171600	42
47	− 65802	+131824	+163876	10452300	47
52	− 14976	− 31696	+220800	11931900	52
57	+ 18882	− 9340	+184516	12868700	57
62	− 24702	+ 51184	+321216	13886700	62
67	− 13947	− 15754	+348514	15772600	67
72	− 18747	+ 29286	+407762	17497100	72
77	+ 12114	− 22532	+377396	19641300	77
82	− 12522	+ 11080	+382104	21424100	82
87	+ 28563	− 34182	+320758	23350200	87
92	− 10266	+ 39308	+413756	21897800	92

27. By means of these differences all the values of $\log q_x$ from age 32 to age 97 are obtained by continued addition, but it is necessary to complete the table by supplying the values for ages 27 to 32, and 97 to 102. In the case of this particular table the scantiness of the original data prevents us from proceeding to an earlier age than 27. Using the function q_x itself, we construct a fourth difference by the formula given in paragraph 90 of the Census Paper,

$$\delta^4 u_x = \frac{1}{70}\{u_{x+8} - u_x - 8\delta u_x - 28\delta^2 u_x - 56\delta^3 u_x\}$$

At the beginning of the table the values of q_x used are those for ages 35, 34, 33, 32 and 27, and at the end, those for ages 94, 95, 96, 97 and 102. As an example, the figures for the young ages may be reproduced.

δ^4	δ^3	δ^2	δ	$10^7 q_x$	Age
−348·6	+1900·0	+2800	− 3100	86600	35
...	+1551·4	+4700	− 300	83500	34
.	+1202·8	+6251	− 4400	83200	33
...	+ 854·2	+7454	+10651	87600	32
...	+ 505·6	+8308	+18105	98251	31
...	+ 157·0	+8814	+26413	116356	30
...	...	+8971	+35227	142769	29
...	...	...	+44198	177996	28
...	...	...	...	222200	27

28. Construction B, Table 2 of the Appendix, was effected on precisely the same principles as Construction A, but with greater elaboration of detail. To find the graduated quinquennial values of E_x and θ_x, formula (iii), involving four differences of w_x, was used on the arithmometer for ages 32, 37, &c., up to 87 inclusive, the values for ages 27 and 92 being obtained by the shorter formula (v*a*), and the value for age 97 being added as in paragraph 22; and, having the graduated values of $\log q_x$ at these quinquennial points, the intervening values were interpolated by the 5th difference osculatory formula given in paragraph 57 of the Census Paper. We thus have the values of $\log q_x$ complete for ages 37 to 92 inclusive, and the remaining ten values at each end were supplied by means of the formula given in paragraph 87 of the Census Paper.

29. In Tables 1 and 2 of the Appendix, the third differences of q_x are set forth for Constructions A and B respectively, and it will be seen that there is not much to choose between them, B being a little the better. Both the graduations are smooth, quite smooth enough for practical purposes; and, seeing that Construction A involves only about half the work required in B, it is to be preferred, but it may be improved upon with very little addition to the work which it involves. On seeing an early proof of this paper, Mr. Lidstone suggested that formula (iii) might be contracted into that given as (iii*a*), in the same way that I had already contracted (v) into (v*a*). Combining, therefore, formula (iii*a*) with the 3rd difference osculatory formula, we shall have a construction effected nearly as easily as A, and with all the special fidelity to the facts shown by B.* These graduations may be compared with that effected by Mr. J. Spencer by means of his 21-term formula, and given in *J.I.A.*, xli, 366, 369, and 371. Perhaps the graduation of Mr. Spencer has a trifling advantage in the way of smoothness over the others, but against that it must be remembered that his 21-term formula is comparatively laborious to apply, and that even Construction B could be carried out with much less trouble.

30. In Tables 1 and 2 are also given the expected deaths and the deviations for both constructions; and, again, comparison may be made with the results of Mr. Spencer's graduation shown on the pages of the *Journal* above mentioned. But the relative general effects of the different processes will be better appreciated by means of the following summary:

* See, however, pars. 75 and 80 of the Addendum.—G. K.

Government Female Annuitants (1883) Ultimate Table.

Deviation of the Expected from the Actual Deaths according to various methods of construction.

Age Group	Construction A		Construction B		Spencer's Graduation	
	Deviation	Accum. Deviation	Deviation	Accum. Deviation	Deviation	Accum. Deviation
30 to 34	+ ·2	+ ·2	− ·2	− ·2	+ 1·9	+ 1·9
35 ,, 39	+ ·1	+ ·3	+ ·1	− ·1	+ 3·7	+ 5·6
40 ,, 44	− 1·3	− 1·0	− ·7	− ·8	− 3·2	+ 2·4
45 ,, 49	+ 1·4	+ ·4	+ ·6	− ·2	+ ·7	+ 3·1
50 ,, 54	− 1·4	− 1·0	− ·9	− 1·1	− 8·0	− 4·9
55 ,, 59	− ·7	− 1·7	...	− 1·1	− ·4	− 5·3
60 ,, 64	+ 4·8	+ 3·1	+ 1·8	+ ·7	+ 18·1	+ 12·8
65 ,, 69	− 5·9	− 2·8	− 5·3	− 4·6	− 30·2	− 17·4
70 ,, 74	+ 12·7	+ 9·9	+ 8·0	+ 3·4	+ 36·6	+ 19·2
75 ,, 79	− 15·5	− 5·6	− 9·8	− 6·4	− 17·1	+ 2·1
80 ,, 84	+ 2·8	− 2·8	+ 5·4	− 1·0	+ 17·3	+ 19·4
85 ,, 89	− 11·9	− 14·7	− 4·3	− 5·3	− 13·9	+ 5·5
90 ,, 94	+ 5·0	− 9·7	+ 5·0	− ·3	− 8·2	− 2·7
95 ,, 99	+ 11·7	+ 2·0	+ 6·6	+ 6·3	+ ·1	− 2·6
100 to 102	+ 3·9	+ 5·9	+ 3·6	+ 9·9	+ 3·6	+ 1·0
Total	±79·3	±61·1	±52·3	±41·4	±163·0	±105.9

31. It will be observed that both Constructions A and B adhere to the original facts with remarkable fidelity, B being a trifle the better in this respect. Mr. Spencer's graduation is at least as good as any that have been effected hitherto, and yet it shows deviations twice as great as those of Construction A and three times as great as those of Construction B, while its gain in smoothness is scarcely perceptible.

32. The new method has been applied to construct the British Offices O^{M} Aggregate Table, direct from the Exposed to Risk and the Deaths as given on pages 486 and 487 of the volume of "Unadjusted Data"; and the detailed results appear in Table 3 of the Appendix. Except as regards the youngest ages, the processes followed were precisely the same as in Construction A of the Government Annuitants Table fully explained in paragraphs 19 to 27 above. The function w_x was first formed for both E_x and θ_x at ages 11, 16, &c., up to age 96, these points being chosen in accordance with paragraph 23, because in this case the oldest recorded age is 103, at which q_x is taken as unity, and age 96 is seven years younger. Then formula (*va*) was used to find graduated values of E_x and θ_x,

and thence of q_x, at ages 18, 23, &c., up to 93, and, in accordance with paragraph 22, the value of q_{98} was supplied by a 3rd difference from the values for ages 83, 88, 93, and 103. Having thus q_x at the quinquennial points 18, 23, &c., up to 103, the osculatory 3rd difference formula (vii) was employed to calculate the values of log q_x for ages 23 to 98 inclusive; and the values for ages 98 to 103 were computed by a fourth difference as explained in paragraph 27.

33. For the young ages, 23 down to 10, a double process was employed. The table was first carried to age 18 by means of the formula used at the old ages, the fourth difference being derived from the values for ages 26, 25, 24, 23 and 18. The values for ages 18 to 10 were then found by a third difference by a modification of the method used by Woolhouse for the H[M] Table. For ages 10 to 17 the unadjusted values of q_x were taken, and for ages 18, 19 and 20 the adjusted values derived as above; and, to reduce the number of figures, the corresponding values of colog p_x were extracted. The following is the work in its entirety, the calculations being carried to seven places of decimals.

Age	Colog p_x	Σ colog p_x	$\delta\Sigma$ colog p_x $=$ colog p_{x-1}	$\delta^2\Sigma$ colog p_x	$\delta^3\Sigma$ colog p_x
20	00178	0017800	+16,900	− 600	+329
19	00169	0034700	16,300	− 271	...
18	00163	0051000	16,029	+ 58	...
17	00149	...	16,087	+ 387	...
16	00165	...	16,474	+ 716	...
15	00160	...	17,190	+1,045	...
14	00038	...	18,235	+1,374	...
13	00060	...	19,609	+1,703	...
12	00257	...	21,312	+2,032	...
11	00345	...	23,344	...	...
10	00309	0199300	...	...	...

It will be observed that the column of colog p_x is summed from the top downwards, thus giving the graduated values of Σ cologp_x for ages 20, 19 and 18, and the total at age 10, four values in all. From these a third difference is derived by the formula

$$\delta^3 u_0 = \frac{1}{120}\{u_{10} - u_0 - 10\delta u_0 - 45\delta^2 u_0\},$$

and the column $\delta\Sigma$ cologp_x completed. This consists of the graduated values of colog p_x all shifted up one line in the table,

so that, for instance, the value 16474 (or ·0016474 when the decimal point is replaced) against age 16, is colog p_{15}. The formula secures that the sum of colog p_x for ages 10 to 18 is the same by the graduated and the ungraduated tables, or in other words that ${}_8p_{10}$ remains unaltered by the graduation.

34. This plan of completing the table at the younger ages has been found to work very well in practice, but an unlimited number of plans based upon similar principles could be devised, some of which would give equally good results. Moreover, the formula would have to be varied to meet different conditions. According to the final age of the table at which q_x is unity, there may be any number of terms from seven to eleven to be supplied at the beginning. The following are the formulas for all the possible cases—

$$7 \text{ terms}, \quad \delta^3 u_0 = \frac{1}{84}\{u_9 - u_0 - 9\delta u_0 - 36\delta^2 u_0\}$$

$$8 \text{ terms}, \quad \delta^3 u_0 = \frac{1}{120}\{u_{10} - u_0 - 10\delta u_0 - 45\delta^2 u_0\}$$

$$9 \text{ terms}, \quad \delta^3 u_0 = \frac{1}{165}\{u_{11} - u_0 - 11\delta u_0 - 55\delta^2 u_0\}$$

$$10 \text{ terms}, \quad \delta^3 u_0 = \frac{1}{220}\{u_{12} - u_0 - 12\delta u_0 - 66\delta^2 u_0\}$$

$$11 \text{ terms}, \quad \delta^3 u_0 = \frac{1}{286}\{u_{13} - u_0 - 13\delta u_0 - 78\delta^2 u_0\}$$

After all, it does not much matter how the table is completed at the two ends, so long as we have smooth curves, joining on without break to the rest of the table, and which reproduce the actual deaths without an unreasonable amount of deviation. At each end the observations are always scanty, and not entitled to much confidence. The necessity of completing the ends of the table arises also when summation and other formulas of graduation are used, and is not confined to the new method of construction.

35. In Table 3, and also in Table 4, to be described immediately, there is a falling value of q_x from age 10 to age 17, at which point a minimum occurs, a feature not brought out in Mr. Hardy's table, or in the table as graduated by

Mr. Spencer; but an inspection of the unadjusted values of q_x will show that it is a feature which really exists in the Experience, and which should not be eliminated by the graduation. Even according to Tables 3 and 4, the expected deaths are too few at ages 10, 11, and 12, where q_x is large, and too many at ages 13 to 17, where q_x is small, so that the peculiarity, far from being exaggerated, has been partly smoothed down.

36. From the title of this paper it will be gathered that the new method is applicable to the *graduation* of mortality tables as distinguished from their *construction*. An illustration of graduation pure and simple is given in Table 4 of the Appendix, where a new graduation of the O^M Table is submitted.

37. In every respect the process of graduation is the same as that of construction, except that, instead of operating on E_x and θ_x separately in order to find graduated quinquennial values of q_x, we use $\log(q_x + \cdot 1)$ taken from the unadjusted values of q_x. $\text{Log}(q_x + \cdot 1)$ is employed instead of $\log q_x$, because at a few ages the rough q_x is zero, and its logarithm would be negatively infinite; but the function $\log(q_x + \cdot 1)$ is so very convenient in other respects that it may well be used, no matter what graduation formula be adopted. Except at age 103 the characteristic of $\log(q_x + \cdot 1)$ is -1 throughout the whole table, and may therefore be ignored, and the mantissas of the successive logarithms keep much closer to each other than in the case of $\log q_x$. That is, the curve of the unadjusted $\log(q_x + \cdot 1)$ is, to begin with, of much more easy gradients than that of $\log q_x$, and therefore more amenable to interpolation and graduation.

38. Using, then, $\log(q_x + \cdot 1)$ as the fundamental function, Table 4 of the Appendix was calculated, and completed at the ends, by exactly the same methods as Table 3, and there is no need to repeat the explanations.

39. The official O^M Table is given in the volume of "Graduated Experience", published by the Institute of Actuaries and the Faculty of Actuaries in Scotland; and, as far as about age 75 or 80, it follows accurately Mr. G. F. Hardy's construction, but at the older ages the graduation becomes very defective, because q_x is always made to correspond to the tabulated values of l_x and d_x, and when l_x is kept integral and is cut down to less than five figures, all the original smoothness

in the curve of q_x is lost. This will be seen from the following 3rd differences of the official q_x—

Age	$10^5\delta^3 q_x$	Age	$10^5\delta^3 q_x$
75	+ 14	90	− 545
6	...	1	+ 746
7	+ 1	2	− 955
8	+ 19	3	+ 1255
9	− 10	4	− 1687
80	+ 9	95	+ 81
1	+ 15	6	+ 2367
2	+ 6	7	− 1132
3	− 25	8	+ 5237
4	+ 55	9	+ 13095
85	− 46	...	...
6	+ 10	...	...
7	+ 92	...	...
8	− 211	...	...
9	+ 358	...	...

40. I am not aware that Mr. Hardy's original values of q_x for this part of the table have ever been published, but on p. 153 of the volume of *Principles and Methods*, a complete table of colog p_x is given, from which q_x can be obtained; and that was availed of in preparing Table 5 of the Appendix. There, all Mr. Hardy's values of q_x as far as age 103 will be found with their third differences, and also, for the first time so far as I know, the expected deaths and the deviations age by age.

41. Mr. John Spencer with great success, graduated the O^M Table by his 21-term formula. The first part, as far as age 79, is given in *J.I.A.*, xxxviii, 342, and the remainder in *J.I.A.*, xli, 378. The whole is now brought together in Table 6 of the Appendix, to which columns have been added for the third differences of q_x, the expected deaths, and the deviations.

42. Tables 5 and 6 therefore afford the means of comparing in detail my own O^M Table as constructed in Table 3, and as merely graduated in Table 4, with Mr. Hardy's O^M Table, and with Mr. Spencer's.

43. Examining first the Tables as to graduation, it will be seen that all four run with phenomenal smoothness, and that no one of them can claim in this respect appreciable superiority over the others, except that at the older ages Mr. Hardy's third differences remain comparatively small. This advantage, if it can be called such, is, however, secured by sacrificing to a certain extent fidelity to the original facts, as will presently appear (*see*

par. 52, and the table in par. 55.) In Mr. Spencer's table there is a break between ages 95 and 99, due to his method of completing the end, but that is not of the slightest consequence.

44. In *constructing* a mortality table by the new method, graduated quinquennial values of E_x and θ_x are obtained by means of a central formula, and it should be noted that all of the original values of these functions appear in the results exactly the same number of times, except in the case of twelve values at the end of the table, and of from ten to fourteen values according to circumstances at the beginning; but at these extremes another method of construction is substituted. Therefore, throughout all that portion of the table to which the formulas apply, each value of E_x and θ_x influences the results in proportion to its magnitude, and effect is given to the weight of the observations. It might therefore be expected that a table so constructed would adhere closely to the original data, and an examination of Table 3 shows that that is so. (*See* also Tables 1 and 2, and the analysis thereof in par. 30.)

45. The actual deaths are 140,888 in number, and the expected deaths 140,892·5, the difference being only + 4·5. The sum of all the positive deviations is 1163·3, and of the negative 1158·8, and the sum of the deviation irrespective of sign is $\pm$ 2322·1, and these are very small quantities in view of the magnitude of the observations. In the column of deviations there are 49 changes of sign. The largest individual deviation is + 136·2 at age 58, where the actual deaths are 3,045, and there are only two other deviations of over 100, namely −124·3 at age 64, with 3,615 actual deaths, and − 107·6 at age 68, with 3,665 actual deaths.

46. If we look at the column of accumulated deviations, we shall find that the sum of the positive is 1480·4, and of the negative 866·5; and that the sum irrespective of sign is $\pm$ 2346·9. There are 27 changes of sign, and the largest number in the column is + 106·0 at age 63, and there are none others of over 100. Therefore, both as regards graduation, and fidelity to the original facts, the O^M Table constructed by the new method would seem to be very nearly perfect, but possibly it might be made still more accurate by using formula (iii*a*) instead of (v*a*).

47. When a mortality table is merely *graduated* by the new method, weight according to magnitude is assigned to each value of q_x, although all the values do not carry equal authority. At some ages the number of observations is small, and at these the

unadjusted q_x is not entitled to much confidence; while at other ages the reverse is the case. Nevertheless, in graduation this fact is ignored. Therefore it might be expected that a table merely *graduated* by the new method would have a tendency to show more important deviations than one so *constructed.*

48. In Table 4 of the Appendix the O^M Table, merely graduated by the new method, is given. The expected deaths are 140,906·0 in number, and exceed the actual by 18. This is still a very small difference, but it is larger than in the case of Table 3. The sum of all the positive deviations is 1179·4, and of the negative 1161·4; and the sum irrespective of sign is $\pm$ 2340·8. In the column of deviations there are 47 changes of sign. The largest individual deviation is + 134·9 at age 58, and there are only two others of over 100, namely, − 117·0 at age 64, and − 106·3 at age 68; and it is interesting to note that the ages at which these largest deviations occur are identical with the ages where there are similar figures in the *constructed* Table 3, and also, as will presently appear, in Mr. Spencer's table and, as regards two of them, in Mr. Hardy's. The concurrent testimony of four graduations, made on such widely different principles, proves conclusively that these deviations are entirely due to irregularities in the original data.

49. When we look at the column of accumulated deviations, we find that the sum of the positive is 785·3, and of the negative 1648·7, and that the sum irrespective of sign is $\pm$ 2434·0. There are 25 changes of sign in the column, and the largest number in it is − 122·1 at age 38; and there are three others of over 100. It would therefore appear that the *graduated* Table 4 is very nearly as good as the *constructed* Table 3, and we may gather that, at any rate as regards the O^M Table, very little effect is produced by assigning to each of the original sets of observations its relative weight.

50. We may now examine Mr. Hardy's O^M Table, No. 5 of the Appendix, and it will be well understood that there can be no thought of criticizing adversely that actuarial masterpiece. Nevertheless, it is one of the main objects of the Institute to improve upon the past, no matter how good that past may be; and, as a loyal son of the Institute, I have devoted my efforts for many years towards that end.

51. By Mr. Hardy's table the expected deaths are 140,941·7 in number, and exceed the actual by 53·7. This is a small divergence, but it is larger than that by either of my own tables.

The sum of all the positive deviations is 1326·5, and of the negative 1272·8; and the sum irrespective of sign is ±2599·3, which is about 10 per-cent greater than that of my *constructed* table. In the column of deviations there are 51 changes of sign. The largest individual deviation is +151·5 at age 58, and once more there are only two others of over 100, namely, +113·3 at age 61, and −136·2 at age 68.

52. Passing to the column of accumulated deviations, Mr. Hardy's table appears in a light a trifle less favourable. The sum of the positive quantities is 2619·4, and of the negative 2768·4; and the sum irrespective of sign is ±5387·8, which is more than double that of either of my own tables. There are only 11 changes of sign in the column, and the largest number in it reaches the figure of −218·3, there being fifteen of over 100, of which four are over 200. The explanation of this is that Mr. Hardy's O^M Table was graduated by reference to the $O^{M(5)}$, which strictly follows Makeham's Law, and thereby it has been very slightly distorted towards compliance with that law, and from about age 75 onwards it has been made to comply absolutely with the law. From the practical point of view there is no harm whatever in this, because the accumulated deviations at their largest are very small as compared with the mass of the observations. Nevertheless the point is interesting, because my own tables have been produced with very much less labour; they are of as smooth graduation; and they adhere to the facts more closely.

53. Mr. Spencer's graduation in Table 6 of the Appendix is extremely good. The expected deaths are 140,845·1 in number, and fall short of the actual by 42·9. The sum of all the positive deviations is 1150·9, and of the negative 1193·8; and the sum irrespective of sign is ±2344·7. In the column of deviations there are 48 changes of sign. The largest individual deviation is +120·9 at age 58, and there are only two others of over 100, namely −100·6 at age 64, and −118·9 at age 68.

54. In the column of accumulated deviations the sum of the positive quantities is 601·4, and of the negative 2192·6; and the sum irrespective of sign is ±2794·0. There are 26 changes of sign, and the largest number is −99·6 at age 57, there being none of over 100, but at age 60 we have −95·1, and at age 68, −96·8.

55. Perhaps a better general idea of the relative fidelity of the several tables to the data will be gained by setting out the respective deviations in each quinquennium of age, as in the

following statement. In this statement the figures, as regards Mr. Hardy's Table, differ a little from those given by Mr. Hardy himself on page 152 of *Principles and Methods.* For instance, here the expected deaths for ages 20 to 24 are 776, while according to Mr. Hardy they are 766. I cannot explain the discrepancies. It is evident that, to obtain the expected deaths in an age group, Mr. Hardy did not multiply the numbers at risk at the several ages individually by the corresponding values of q_x, and take the sum of the results.

Age Group	Actual Deaths	Deviation of Expected from Actual Deaths			
		New Construction	New Graduation	Hardy	Spencer
10 to 14	10	+ 3·4	+ 3·2	+ ·8	+ ·3
15 „ 19	97	+ ·7	+ 2·2	+ 1·6	+ ·8
20 „ 24	806	+ 2·0	− 12·6	− 30·0	− 21·1
25 „ 29	2,615	+ 21·7	− 3·2	+ 72·2	+ 26·6
30 „ 34	5,202	− 4·0	− 6·0	− 4·5	− 5·7
35 „ 39	7,557	− 23·7	− 27·3	− 36·2	+ 32·4
40 „ 44	9,731	− 75·9	− 75·9	− 146·4	− 108·2
45 „ 49	11,526	+ 122·7	+ 117·0	+ 80·4	+ 80·8
50 „ 54	13,670	+ 28·3	+ 39·5	− 18·2	+ 37·0
55 „ 59	15,594	− 16·2	− 20·1	+ 29·7	− 51·8
60 „ 64	17,093	− 77·3	− 79·0	+ 175·7	− 48·7
65 „ 69	17,677	+ 43·8	+ 64·0	+ 3·4	+ 42·4
70 „ 74	16,150	− 33·1	− 20·3	− 115·4	− 32·7
75 „ 79	12,197	+ 14·5	+ 37·2	− 25·3	+ 43·6
80 „ 84	7.317	− 44·3	− 20·4	− 116·7	− 42·0
85 „ 89	2,865	− ·3	− 21·9	+ 104·4	− 3·5
90 „ 94	692	+ 28·5	+ 28·9	+ 70·2	+ 2·9
95 „ 99	86	+ 11·8	+ 10·9	+ 7·4	+ 3·0
100 „ 103	3	+ 1·9	+ 1·8	+ ·6	+ 1·0
Totals	140,888	+ 279·3 − 274·8	+ 304·7 − 286·7	+ 546·4 − 492·7	+ 270·8 − 313·7
		± 554·1	± 591·4	± 1039·1	± 584·5

56. In the Census Paper, when dealing with mortality tables based upon census returns and records of deaths, the graduated quinquennial values of q_x were derived by the formula called (i) in the present paper; but, as shown above, that formula is not quite central, and introduces a small theoretical error. That that error cannot be of much importance, especially in the case of mortality tables based on general statistics where the observations are very faulty, is sufficiently evident from the results given in paragraph 16 above of the construction of the

Government Female Annuitants Table in precisely the same way. It will be well, however, to examine to what extent census tables are affected when the more exact central formulas, Nos. (iii) and (v*a*), are employed. In Table 7 of the Appendix, therefore, the English Life Table No. 6 has been recomputed by the short central formula (v*a*), and in Table 8 by the longer formula (iii).

57. Here an apology is called for, and, to make matters clear, a little repetition is unavoidable.

58. The original data of the English Life Table No. 6 are given in paragraph 103 of the Census Paper; and, where the age intervals there are decennial, they were bisected by a formula which is absolutely central, as fully explained in paragraphs 69 and 70 of the Census Paper. The following are the figures thus completed. They have not been given before, but are produced now for the benefit of students, who may naturally wish, as an educational exercise, to recalculate everything for themselves—

English Life Table No. 6.—Males.

Data completed for quinquennial intervals.

Age Interval	Population		Deaths in 10 Years	
	In Interval $T_{x\overline{5}\rvert}$	Total, x and over T_x	In Interval $10l_{x5}$	Total, x and over $10l_x$
(1)	(2)	(3)	(4)	(5)
0 ,, 5	1,809,572	14,833,198	1,134,786	2,865,226
5 ,, 10	1,716,048	13,023,626	73,950	1,730,440
10 ,, 15	1,640,058	11,307,578	40,154	1,656,490
15 ,, 20	1,531,756	9,667,520	58,043	1,616,336
20 ,, 25	1,351,555	8,135,764	68,384	1,558,293
25 ,, 30	1,203,088	6,784,209	71,549	1,489,909
30 ,, 35	1,069,405	5,581,121	81,996	1,418,360
35 ,, 40	938,549	4,511,716	94,997	1,336,364
40 ,, 45	820,760	3,573,167	107,283	1,241,367
45 ,, 50	699,627	2,752,407	114,525	1,134,084
50 ,, 55	586,779	2,052,780	129,199	1,019,559
55 ,, 60	461,967	1,466,001	135,188	890,360
60 ,, 65	371,912	1,004,034	156,242	755,172
65 ,, 70	271,006	632,122	158,022	598,930
70 ,, 75	189,428	361,116	166,059	440,908
75 ,, 80	108,501	171,688	134,612	274,849
80 ,, 85	46,150	63,187	91,361	140,237
85 ,, 90	13,826	17,037	40,214	48,876
90 ,, 95	2,827	3,211	7,599	8,662
95 ,, 100	357	384	957	1,063
100 ,, 105	26	27	96	106
105 & over	1	1	10	10

59. In the foregoing table we have in cols. (2) and (4) the population and the deaths in each quinquennial age period, and, by summation, at quinquennial points in cols. (3) and (5) for age x and over. Using logarithms, the values of these latter functions were found for an age one year older at each quinquennial point; and hence were obtained at quinquennial points the population and the deaths in single years of age, and, therefore, the values of m_x and q_x at these quinquennial points.

60. In the Census Paper it was stated that the table was completed by osculatory interpolation from these quinquennial values of q_x, but there was an unfortunate omission from the explanations, which at the old ages would prevent a student from verifying the results. By a curious accident a page of the original manuscript slipped out unnoticed when a clean copy was being prepared for the press, and the mishap was not discovered until the paper had appeared in the *Journal*; but, except that students may have been a little puzzled, no great harm has been done. It should have been explained that, while the method gave consistent results as far as age 85, yet beyond that age anomalies occurred, and that the values of m_x at ages 90, 95, and 100 came out too small, and had a tendency to decrease with the age. To remedy this, the data must be used in another way. We have as facts the values of m_x at ages 70, 75, 80, and 85, and we may assume, not unreasonably, that, at age 105, $m_x=2$, its limiting value. From these we form a fourth difference, by means of which to interpolate the values for ages 90, 95, and 100, and so to complete the skeleton table.

61. In reconstructing the English Life Table No. 6 by the strictly central method of this paper, we notice that in cols. (2) and (4) of the table in paragraph 58 the functions $T_{\overline{5}|}$ and l_5 are the w_x of paragraph 12, and that we can at once apply to them either formula (iii*a*) or formula (v*a*), without going to the trouble of summing them, or of preparing them in any other way. We thus have an exceedingly expeditious method of constructing mortality tables from general statistics.

62. For Table 7 of the Appendix the short formula (v*a*) was used to find L_x and d_x, and thence m_x and q_x for the quinquennial ages 12, 17, &c., to 87 inclusive; while for Table 8 the longer formula (iii*a*) was employed for ages 17, 22, &c., to 87 inclusive, and formula (v*a*) for age 12. For the reasons explained in paragraph 60, we must find specially the values of q_x for ages 92, 97, and 102, but on account of the quinquennial

age points being now differently situated in the curve, the formula must be suitably modified. We have q_x at ages 72, 77, 82, and 87, and we may assume that at age 105 it is unity. Using these values, we have—

$$q_{105}=q_{72}+6\cdot6\Delta q_{72}+18\cdot48\Delta^2 q_{72}+28\cdot336\Delta^3 q_{72}+25\cdot5024\Delta^4 q_{72}$$

whence

$$\Delta^4 q_{72}=\frac{1}{25\cdot5024}\{q_{105}-q_{72}-6\cdot6\Delta q_{72}-18\cdot48\Delta^2 q_{72}-28\cdot336\Delta^3 q_{72}\}$$

63. Having thus q_x at quinquennial points from age 12 to age 102 inclusive, the intervening values from age 17 to age 97 inclusive were interpolated by the 3rd difference osculatory formula, the function $\log(q_x+\cdot1)$ being used. The results are given in Tables 7 and 8 of the Appendix.

64. It remains to complete the ends of the tables, and the principles already fully explained may be applied, with the details modified to meet the slightly changed conditions. Taking, first, the old ages, the preparation of the data gives us $\log(q_x+\cdot1)$ for ages 97, 102, and 105, and the osculatory interpolation supplies the values for all ages as far as 97. We may, therefore, from the values for ages 95, 96, 97, 102, and 105, form a third and a fourth difference. Writing u_x for $\log(q_x+\cdot1)$, we may take u_0 to represent the function at age 95, and at the other ages named u will have corresponding suffixes, the function being u_7 at age 102, and u_{10} at age 105. There are, thus, two simultaneous equations for the third and the fourth difference, which are the unknown quantities—

$$u_7=u_0+7\delta u_0+21\delta^2 u_0+35\delta^3 u_0+35\delta^4 u_0$$

$$u_{10}=u_0+10\delta u_0+45\delta^2 u_0+120\delta^3 u_0+210\delta^4 u_0$$

whence, by easy transformations,

$$\delta^3 u_0=\frac{1}{90}\{6u_7-u_{10}-5u_0-32\,\delta u_0-81\delta^2 u_0\}$$

$$\delta^4 u_0=\frac{1}{35}\{u_7-u_0-7\delta u_0-21\delta^2 u_0-35\delta^3 u_0\}$$

65. At the young ages more plans than one were tried to carry the table back to age 5, and in the end it was thought sufficient for present purposes to use only a third difference, the

results being fairly satisfactory. The method of fixing on q_5 is discussed in paragraphs 82 and 105 of the Census Paper, to which reference is made. Having, therefore, the values of $\log(q_x + \cdot 1)$ at ages 18, 17, 12, and 5, a second and a third difference were formed from a pair of simultaneous equations by the formulas

$$\delta^2 u_0 = \frac{1}{1365}\{143u_6 - 10u_{13} - 133u_0 - 728\delta u_0\}$$

$$\delta^3 u_0 = \frac{1}{20}\{u_6 - u_0 - 6\delta u_0 - 15\delta^2 u_0\}$$

Here the series is written in reverse order, and age 18 is taken as the origin. In completing the ends, a formula must be adapted to the circumstances of each particular case, but it is always easy to construct one that is suitable.

66. It has been strongly urged upon me that better results will be obtained by using logarithms instead of numbers in calculating from the data the graduated quinquennial values of q_x, but a test case does not lend support to the contention. Both plans were tried in constructing Table 7, and the following are the two sets of quinquennial values—

Age	q_x By logarithms	q_x By numbers	Age	q_x By logarithms	q_x By numbers
12	·00232	·00232	52	·02182	·02182
17	·00380	·00379	57	·02881	·02880
22	·00507	·00507	62	·04128	·04130
27	·00591	·00591	67	·05668	·05673
32	·00763	·00763	72	·08477	·08476
37	·01007	·01008	77	·11835	·11796
42	·01301	·01301	82	·18631	·18508
47	·01620	·01620	87	·25737	·26409

There is no appreciable difference between the two sets of values, except perhaps at ages 82 and 87. At age 82 the difference is ·00123, which is only ·66 per-cent., and at age 87 it is ·00672, or 2·53 per-cent. On the whole the values "by numbers" at these ages fit in better with the rest of the table than those "by logarithms."

67. The use of logarithms greatly increases the work. Cols. (3) and (5) of the table in paragraphs 58 must be operated on instead of cols. (2) and (4), and (*see* par. 14), $\log y_7$ and $\log y_8$ must both be calculated separately, and the corresponding

numbers taken out, before we can obtain u_7. It is therefore in every respect preferable to use numbers and not logarithms.

68. Tables 7 and 8 of the Appendix are hardly distinguishable from each other, and it would therefore appear that there is nothing to be gained by using the longer formula (iii*a*) as in Table 8, instead of the short formula (v*a*) as in Table 7. The remarkable accuracy of the short formula is also proved by Tables 1 and 3, which have already been discussed, and we may, therefore, unhesitatingly adopt in all cases formula (v*a*) in calculating the graduated quinquennial values, and the short 3rd difference osculatory formula for the interpolations; and thus we may bring under one system the construction of mortality tables of every kind, and reduce the work to a very low minimum, while at the same time we secure almost absolute fidelity to the original data.

69. Construction A, in Table 7, of the English Life Table No. 6, interprets the original data with probably the greatest accuracy attainable, and it may be compared with my own former construction, and with that of the Registrar-General, both given in Table VII of the Census Paper, *J.I.A.*, xlii, 274 and 275. It will be seen that from about age 15, where the interpolations begin, to about age 70, all the three constructions give nearly identical results, and that as far as about age 85, my own two constructions keep very close together. From about age 85 onwards, however, my own new construction lies between those of the Census Paper and of the Registrar-General, but at these advanced ages none of the constructions can be thorougly trusted. At the old ages the errors in the original data due to mis-statements of age are very serious, and become more and more so as we pass down to the ultimate ages. Before proceeding to construct a mortality table, the original data should if possible be corrected, at any rate from age 85 onwards; but that is a matter which would require careful study. The population and the deaths for each year of age would have to be supplied, and compared with the similar statistics of preceding censuses. It is to be feared that such an enquiry is not feasible.

70. At the oldest ages the introduction of the exact central method of this paper makes more difference in tables based upon census returns and death records than in tables based upon the experience of assured lives or annuity nominees. By the method of the Census Paper the fundamental quinquennial values of q_x, on which the whole table depends, are those for ages 75, 80, 85,

&c., whereas by the central method they are those for ages 77, 82, 87, &c., or always two years older. This advance of two years carries us further down among the large errors of observation, and hence the remark in paragraph 10, that for general statistics the Census Paper method may possess an advantage over the central. This theory is in a way confirmed by other investigations. Recently it fell to my lot to require a table of the marital state of the male population, based upon the census of 1901, similar to the table in the paper on *Family Annuities, J.I.A.*, xxx, 296, based upon the census of 1881. The method of the Census Paper gave results which were throughout consistent with each other, but, at the older of the ages included in the investigation, the central method evidently exaggerated the proportionate number of bachelors. It is not that the central method fails, but that it interprets only too faithfully defective observations.

71. The heading of this paper speaks of the construction of tables other than those of mortality, and many interesting examples could be given of the successful application of the new method to statistics of very varied and curious kinds. It is, however, better to close the paper here, and not to introduce extraneous matter which might divert attention from the consideration of the new method itself.

TABLE I.

Government Female Annuitants (1883) Ultimate Table.

Construction A.

By the new method, from the Exposed to Risk and the Deaths, without calculating the unadjusted values of q_x.

Third Difference Formulas.

Age	q_x	$\Delta^3 q_x \times 10^5$	Expected Deaths	Deviation	Accumulated Deviation	Age
30	·01164	− 11	1·9	− ·1	− ·1	30
1	·00983	− 16	1·9	+ 1·9	+ 1·8	1
2	·00876	− 19	2·0	...	+ 1·8	2
3	·00832	− 12	2·1	− ·9	+ ·9	3
4	·00835	− 18	2·3	− ·7	+ ·2	4
35	·00866	+ 12	2·8	− 2·2	− 2·0	35
6	·00913	+ 15	3·3	− 1·7	− 3·7	6
7	·00958	− 14	4·0	+ 1·0	− 2·7	7
8	·01013	− 22	4·8	− 1·2	− 3·9	8
9	·01093	− 23	6·2	+ 4·2	+ ·3	9
40	·01184	− 26	7·7	− 5·3	− 5·0	40
1	·01264	+ 16	9·3	+ ·3	− 4·7	1
2	·01310	+ 34	10·9	− 1·1	− 5·8	2
3	·01296	+ 27	12·2	+ 3·2	− 2·6	3
4	·01238	− 25	13·6	+ 1·6	− 1·0	4
45	·01170	+ 9	14·6	− 2·4	− 3·4	45
6	·01119	− 11	15·8	− 5·2	− 8·6	6
7	·01110	− 15	17·5	+ 4·5	− 4·1	7
8	·01152	− 16	20·3	+ 6·3	+ 2·2	8
9	·01234	− 22	24·2	− 1·8	+ ·4	9
50	·01341	− 8	29·0	+ 7·0	+ 7·4	50
1	·01457	+ 12	34·8	− 2·2	+ 5·2	1
2	·01560	+ 5	41·6	− 1·4	+ 3·8	2
3	·01642	+ 7	48·1	− 2·9	+ ·9	3
4	·01715	+ 5	57·1	− 1·9	− 1·0	4
55	·01784	− 4	65·7	+ 8·7	+ 7·7	55
6	·01856	− 5	76·2	− 6·8	+ ·9	6
7	·01936	+ 9	86·6	+ ·6	+ 1·5	7
8	·02020	+ 12	98·7	+ 8·7	+10·2	8
9	·02103	+ 11	111·1	−11·9	− 1·7	9
60	·02194	+ 17	123·8	−14·2	−15·9	60
1	·02305	...	139·2	+23·2	+ 7·3	1
2	·02447	− 7	157·1	+ ·1	+ 7·4	2
3	·02637	− 13	178·3	− 3·7	+ 3·7	3
4	·02875	− 18	208·4	− ·6	+ 3·1	4
65	·03154	− 11	239·7	−18·3	−15·2	65
6	·03461	+ 11	272·1	+18·1	+ 2·9	6
7	·03778	+ 14	304·5	−12·5	− 9·6	7
8	·04094	+ 18	335·6	−17·4	−27·0	8
9	·04420	+ 24	367·2	+24·2	− 2·8	9

TABLE 1—*continued.*

Government Female Annuitants (1883) *Ultimate Table.*

Construction A.

By the new method, from the Exposed to Risk and the Deaths, without calculating the unadjusted values of q_x.

Third Difference Formulas.

Age	q_x	$\Delta^3 q_x \times 10^5$	Expected Deaths	Deviation	Accumulated Deviation	Age
70	·04770	+ 29	399·3	+ 3·3	+ ·5	70
1	·05162	+ 5	428·0	+27·0	+27·5	1
2	·05620	− 12	458·4	+26·4	+53·9	2
3	·06173	− 23	488·0	−30·0	+23·9	3
4	·06826	− 37	518·0	−14·0	+ 9·9	4
75	·07567	− 26	545·9	−46·1	−36·2	75
6	·08373	+ 17	562·1	− 4·9	−41·1	6
7	·09207	+ 26	571·3	+23·3	−17·8	7
8	·10043	+ 33	572·9	+ 7·9	− 9·9	8
9	·10898	+ 49	566·3	+ 4·3	− 5·6	9
80	·11798	+ 44	550·3	+ 1·3	− 4·3	80
1	·12776	+ 1	529·2	−28·8	−33·1	1
2	·13881	− 28	496·4	+17·4	−15·7	2
3	·15157	− 40	465·3	−15·7	−31·4	3
4	·16605	− 66	427·6	+28·6	− 2·8	4
85	·18197	−113	390·5	−13·5	−16·3	85
6	·19893	+ 4	339·4	− 2·6	−18·9	6
7	·21627	+145	288·1	−10·9	−29·8	7
8	·23286	+185	237·1	+27·1	− 2·7	8
9	·24874	+244	196·0	−12·0	−14·7	9
90	·26537	+157	151·0	− 9·0	−23·7	90
1	·28460	− 14	111·3	− 3·7	−27·4	1
2	·30887	+ 46	81·9	+ 9·9	−17·5	2
3	·33975	+ 30	64·2	+ 1·2	−16·3	3
4	·37710	+ 11	45·6	+ 6·6	− 9·7	4
95	·42138	+ 97	32·9	+ 1·9	− 7·8	95
6	·47289	+182	22·2	+10·2	+ 2·4	6
7	·53174	+269	18·1	+ ·1	+ 2·5	7
8	·59890	+354	9·6	+ ·6	+ 3·1	8
9	·67619	+440	2·9	− 1·1	+ 2·0	9
100	·76630	...	2·3	+ 2·3	+ 4·3	100
1	·87277	...	2·6	+ ·6	+ 4·9	1
2	1·00000	...	1·0	+ 1·0	+ 5·9	2

TABLE 2.

Government Female Annuitants (1883) *Ultimate Table.*

Construction B.

By the new method, from the Exposed to Risk and the Deaths, without calculating the unadjusted values of q_x.

Fifth Difference Formulas.

Age	q_x	$\Delta^3 q_x \times 10^5$	Expected Deaths	Deviation	Accumulated Deviation	Age
30	·01058	− 16	1·7	− ·3	− ·3	30
1	·00938	− 9	1·8	+ 1·8	+ 1·5	1
2	·00871	− 7	1·9	− ·1	+ 1·4	2
3	·00841	− 2	2·1	− ·9	+ ·5	3
4	·00839	− 2	2·3	− ·7	− ·2	4
35	·00858	+ 1	2·7	− 2·3	− 2·5	35
6	·00896	+ 1	3·2	− 1·8	− 4·3	6
7	·00951	− 15	4·0	+ 1·0	− 3·3	7
8	·01024	− 25	4·8	− 1·2	− 4·5	8
9	·01116	− 20	6·4	+ 4·4	− ·1	9
40	·01212	− 3	7·9	− 5·1	− 5·2	40
1	·01287	− 8	9·4	+ ·4	− 4·8	1
2	·01321	+ 34	11·0	− 1·0	− 5·8	2
3	·01311	+ 43	12·3	+ 3·3	− 2·5	3
4	·01249	+ 21	13·7	+ 1·7	− ·8	4
45	·01169	− 5	14·6	− 2·4	− 3·2	45
6	·01114	+ 5	15·8	− 5·2	− 8·4	6
7	·01105	− 9	17·4	+ 4·4	− 4·0	7
8	·01137	− 28	20·0	+ 6·0	+ 2·0	8
9	·01215	− 24	23·8	− 2·2	− ·2	9
50	·01330	+ 1	28·7	+ 6·7	+ 6·5	50
1	·01454	− 2	34·7	− 2·3	+ 4·2	1
2	·01563	+ 3	41·7	− 1·3	+ 2·9	2
3	·01658	+ 12	48·2	− 2·8	+ ·1	3
4	·01737	+ 8	57·8	− 1·2	− 1·1	4
55	·01803	− 3	66·4	+ 9·4	+ 8·3	55
6	·01868	− 2	76·7	− 6·3	+ 2·0	6
7	·01940	+ 8	86·8	+ ·8	+ 2·8	7
8	·02016	+ 14	98·5	+ 8·5	+11·3	8
9	·02094	+ 11	110·6	−12·4	− 1·1	9
60	·02182	+ 4	123·2	−14·8	−15·9	60
1	·02294	+ 14	138·5	+22·5	+ 6·6	1
2	·02441	− 8	156·8	− ·2	+ 6·4	2
3	·02627	− 19	177·6	− 4·4	+ 2·0	3
4	·02866	− 19	207·7	− 1·3	+ ·7	4
65	·03150	...	239·4	−18·6	−17·9	65
6	·03460	− 8	272·1	+18·1	+ ·2	6
7	·03777	+ 15	304·5	−12·5	−12·3	7
8	·04101	+ 34	336·2	−16·8	−29·1	8
9	·04424	+ 30	367·5	+24·5	− 4·6	9

TABLE 2—*continued.*

Government Female Annuitants (1883) *Ultimate Table.*

Construction B.

By the new method, from the Exposed to Risk and the Deaths, without calculating the unadjusted values of q_x.

Fifth Difference Formulas.

Age	q_x	$\Delta^3 q_x \times 10^5$	Expected Deaths	Deviation	Accumulated Deviation	Age
70	·04761	+ 10	398·6	+ 2·6	− 2·0	70
1	·05146	+ 18	426·7	+25·7	+23·7	1
2	·05609	− 8	457·5	+25·5	+49·2	2
3	·06160	− 39	486·9	−31·1	+18·1	3
4	·06817	− 37	517·3	−14·7	+ 3·4	4
75	·07572	− 3	546·2	−45·8	−42·4	75
6	·08386	− 19	563·0	− 4·0	−46·4	6
7	·09222	+ 25	572·2	+24·2	−22·2	7
8	·10077	+ 64	574·8	+ 9·8	−12·4	8
9	·10932	+ 57	568·0	+ 6·0	− 6·4	9
80	·11812	+ 13	550·9	+ 1·9	− 4·5	80
1	·12781	+ 36	529·4	−28·6	−33·1	1
2	·13896	− 28	496·9	+17·9	−15·2	2
3	·15170	− 89	465·7	−15·3	−30·5	3
4	·16639	− 79	428·5	+29·5	− 1·0	4
85	·18275	+ 3	392·2	−11·8	−12·8	85
6	·19989	− 11	341·0	− 1·0	−13·8	6
7	·21702	+ 46	289·1	− 9·9	−23·7	7
8	·23417	+ 106	238·4	+28·4	+ 4·7	8
9	·25123	+ 106	198·0	−10·0	− 5·3	9
90	·26866	+ 61	152·9	− 7·1	−12·4	90
1	·28752	+ 79	112·4	− 2·6	−15·0	1
2	·30887	+ 109	81·9	+ 9·9	− 5·1	2
3	·33332	+ 146	63·0	...	− 5·1	3
4	·36166	+ 202	43·8	− 4·8	− ·3	4
95	·39498	+ 275	30·8	− ·2	− ·5	95
6	·43474	+ 388	20·4	+ 8·4	+ 7·9	6
7	·48296	+ 551	16·4	− 1·6	+ 6·3	7
8	·54239	+ 801	8·7	− ·3	+ 6·0	8
9	·61691	+1190	4·3	+ ·3	+ 6·3	9
100	·71203	...	2·1	+ 2·1	+ 8·4	100
1	·83576	...	2·5	+ ·5	+ 8·9	1
2	1·00000	...	1·0	+ 1·0	+ 9·9	2

TABLE 3.

British Offices O^M Aggregate Table.

Constructed by the new method, from the Exposed to Risk and the Deaths, without calculating the unadjusted values of q_x.

Third Difference Formulas.

Age	q_x	$\Delta^3 q_x \times 10^5$	Expected Deaths	Deviation	Accumulated Deviation	Age
10	·00535	...	1·5	− ·5	− ·5	10
1	·00489	+ 2	1·9	− 1·1	− 1·6	1
2	·00450	− 2	2·3	− ·7	− 2·3	2
3	·00418	...	3·1	+ 2·1	− ·2	3
4	·00395	...	4·6	+ 3·6	+ 3·4	4
15	·00379	+ 2	7·2	+ ·2	+ 3·6	15
6	·00370	− 3	10·7	− ·3	+ 3·3	6
7	·00368	+ 1	16·1	+ 1·1	+ 4·4	7
8	·00375	− 5	25·2	− ·8	+ 3·6	8
9	·00388	− 3	38·5	+ ·5	+ 4·1	9
20	·00408	− 2	60·6	− 3·4	+ ·7	20
1	·00430	− 2	101·0	+ 5·0	+ 5·7	1
2	·00451	+ 2	151·2	+ 7·2	+ 12·9	2
3	·00469	+ 1	212·3	+ 21·3	+ 34·2	3
4	·00482	+ 4	282·9	− 28·1	+ 6·1	4
25	·00492	+ 4	360·6	− 9·4	− 3·3	25
6	·00500	+ 2	439·2	+ 36·2	+ 32·9	6
7	·00510	− 2	521·1	+ 11·1	+ 44·0	7
8	·00526	− 2	608·1	+ ·1	+ 44·1	8
9	·00550	− 2	707·7	− 16·3	+ 27·8	9
30	·00580	− 4	820·0	− 10·0	+ 17·8	30
1	·00614	− 1	929·7	+ 28·7	+ 46·5	1
2	·00650	+ 1	1043·3	+ 5·3	+ 51·8	2
3	·00684	+ 2	1153·0	+ 26·0	+ 77·8	3
4	·00715	+ 2	1252·0	− 54·0	+ 23·8	4
35	·00744	+ 1	1343·5	− 29·5	− 5·7	35
6	·00773	+ 1	1427·3	− 9·7	− 15·4	6
7	·00804	...	1506·2	− 45·8	− 61·2	7
8	·00838	− 2	1585·8	− 15·2	− 76·4	8
9	·00876	+ 1	1670·5	+ 76·5	+ ·1	9
40	·00918	− 1	1762·7	+ 3·7	+ 3·8	40
1	·00962	...	1845·9	+ 11·9	+ 15·7	1
2	·01009	...	1932·7	− 10·3	+ 5·4	2
3	·01058	+ 2	2015·1	− 33·9	− 28·5	3
4	·01109	...	2098·7	− 47·3	− 75·8	4
45	·01162	+ 3	2177·0	+ 72·0	− 3·8	45
6	·01219	+ 1	2254·6	+ 10·6	+ 6·8	6
7	·01280	...	2328·4	+ 84·4	+ 91·2	7
8	·01348	...	2405·7	− 66·3	+ 24·9	8
9	·01424	...	2483·0	+ 22·0	+ 46·9	9
50	·01508	− 1	2574·0	− 43·0	+ 3·9	50
1	·01600	+ 2	2656·4	+ 54·4	+ 58·3	1
2	·01700	...	2744·2	+ 18·2	+ 76·5	2
3	·01807	+ 2	2823·2	− 3·8	+ 72·7	3
4	·01923	+ 1	2900·5	+ 2·5	+ 75·2	4
55	·02048	+ 1	2977·9	− 81·1	− 5·9	55
6	·02184	...	3051·9	− 47·1	− 53·0	6
7	·02332	+ 1	3115·5	− 13·5	− 66·5	7
8	·02493	+ 3	3181·2	+ 136·2	+ 69·7	8
9	·02667	+ 6	3251·3	− 10·7	+ 59·0	9

TABLE 3—*continued.*

British Offices O^{M} *Aggregate Table.*

Constructed by the new method, from the Exposed to Risk and the Deaths, without calculating the unadjusted values of q_x.

Third Difference Formulas.

Age	q_x	$\Delta^3 q_x \times 10^5$	Expected Deaths	Deviation	Accumulated Deviation	Age
60	·02855	− 3	3314·0	− 70·0	− 11·0	60
1	·03060	+ 10	3360·9	− 63·9	+ 52·9	1
2	·03288	+ 3	3405·2	− 21·2	+ 74·1	2
3	·03542	− 3	3444·9	+ 31·9	+ 106·0	3
4	·03832	− 5	3490·7	− 124·3	− 18·3	4
65	·04161	− 7	3525·2	+ 33·2	− 14·9	65
6	·04526	− 1	3559·7	− 20·3	− 5·4	6
7	·04922	+ 6	3570·8	− 47·8	− 42·4	7
8	·05342	− 8	3557·4	− 107·6	− 65·2	8
9	·05785	+ 3	3507·7	+ 90·7	+ 25·5	9
70	·06257	− 12	3443·7	− 42·3	− 16·8	70
1	·06766	− 5	3351·7	− 54·7	+ 37·9	1
2	·07315	− 1	3240·5	− 54·5	− 16·6	2
3	·07916	− 5	3114·8	− 27·8	− 11·2	3
4	·08574	− 4	2966·2	− 18·8	− 7·6	4
75	·09290	+ 5	2806·6	− 12·4	− 20·0	75
6	·10069	+ 22	2637·3	− ·7	− 20·7	6
7	·10915	− 13	2445·7	− 8·7	− 12·0	7
8	·11833	− 17	2258·0	+ 15·0	− 3·0	8
9	·12845	− 29	2063·9	− 3·9	− 6 9	9
80	·13964	− 36	1867·0	− 25·0	− 18·1	80
1	·15173	− 51	1630·2	− 3·2	− 14·9	1
2	·16443	+ 8	1446·5	− 15·5	− ·6	2
3	·17738	− 53	1247·7	+ 23·7	− 24·3	3
4	·19007	− 68	1051·3	− 61·7	− 37·4	4
85	·20258	+ 87	852·3	− 1·3	− 36·1	85
6	·21541	− 3	691·8	− 28·8	− 7·3	6
7	·22933	− 35	555·2	− 10·8	− 18·1	7
8	·24512	− 65	434·6	− 21·4	− 39·5	8
9	·26278	+ 84	330·8	+ 1·8	− 37·7	9
90	·28196	− 104	242·2	− 16·2	− 21·5	90
1	·30331	− 120	180·8	− 14·8	− 6·7	1
2	·32767	+ 48	139·3	+ 1·3	− 5·4	2
3	·35608	− 27	95·8	− 5·2	− 10·6	3
4	·38974	+ 5	62·4	+ 1·4	− 9·2	4
95	·42913	− 11	39·9	− 1·1	− 10·3	95
6	·47452	− 24	24·7	− 10·7	− ·4	6
7	·52596	+ 94	18·9	+ 1·9	− 2·3	7
8	·58334	+ 190	11·1	− ·9	− 1·4	8
9	·64690	+ 349	3·2	+ 1·2	− 2·6	9
100	·71758	+ 579	2·2	− ·2	+ 2·8	100
1	·79728	...	·8	− ·8	+ 3·6	1
2	·88949	...	·9	− ·9	+ 4·5	2
3	1·00000	...	1·0	·0	+ 4·5	3

TABLE 4.

British Offices O^M Aggregate Table.

Graduation of the Unadjusted Values of q_x by the new method. Third Difference Formulas.

Age	q_x	$\Delta^3 q_x \times 10^5$	Expected Deaths	Deviation	Accumulated Deviation	Age
10	·00514	+ 2	1·4	− ·6	− ·6	10
1	·00478	+ 2	1·8	− 1·2	− 1·8	1
2	·00446	− 4	2·3	− ·7	− 2·5	2
3	·00420	+ 1	3·1	+ 2·1	− ·4	3
4	·00402	+ 2	4·6	+ 3·6	+ 3·2	4
15	·00388	− 1	7·4	+ ·4	+ 3·6	15
6	·00379	− 1	11·0	...	+ 3·6	6
7	·00377	+ 2	16·5	+ 1·5	+ 5·1	7
8	·00381	− 4	25·6	− ·4	+ 4·7	8
9	·00390	− 3	38·7	+ ·7	+ 5·4	9
20	·00406	− 4	60·3	− 3·7	+ 1·7	20
1	·00425	+ 1	99·9	+ 3·9	+ 5·6	1
2	·00444	+ 1	148·9	+ 4·9	+ 10·5	2
3	·00459	+ 3	207·8	+ 16·8	+ 27·3	3
4	·00471	+ 1	276·5	− 34·5	− 7·2	4
25	·00481	+ 3	352·6	− 17·4	− 24·6	25
6	·00492	+ 3	432·2	+ 29·2	+ 4·6	6
7	·00505	− 3	516·0	+ 6·0	+ 10·6	7
8	·00523	− 1	604·6	− 3·4	+ 7·2	8
9	·00549	− 4	706·4	− 17·6	− 10·4	9
30	·00580	− 2	820·0	− 10·0	− 20·4	30
1	·00615	...	931·2	+ 30·2	+ 9·8	1
2	·00650	...	1043·3	+ 5·3	+ 15·1	2
3	·00683	+ 2	1151·3	+ 24·3	+ 39·4	3
4	·00714	+ 2	1250·2	− 55·8	− 16·4	4
35	·00743	+ 2	1341·7	− 31·3	− 47·7	35
6	·00772	...	1425·5	− 11·5	− 59·2	6
7	·00803	− 2	1504·3	− 47·7	−106·9	7
8	·00838	+ 1	1585·8	− 15·2	−122·1	8
9	·00877	...	1672·4	+ 78·4	− 43·7	9
40	·00918	− 1	1762·7	+ 3·7	− 40·0	40
1	·00962	...	1845·9	+ 11·9	− 28·1	1
2	·01009	− 1	1932·7	− 10·3	− 38·4	2
3	·01058	+ 3	2015·1	− 33·9	− 72·3	3
4	·01109	+ 1	2098·7	− 47·3	−119·6	4
45	·01161	+ 4	2175·1	+ 70·1	− 49·5	45
6	·01217	− 1	2250·9	+ 6·9	− 42·6	6
7	·01278	− 2	2324·8	+ 80·8	+ 38·2	7
8	·01348	+ 1	2405·7	− 66·3	− 28·1	8
9	·01426	+ 1	2486·5	+ 25·5	− 2·6	9
50	·01510	+ 2	2577·4	− 39·6	− 42·2	50
1	·01601	− 3	2658·1	+ 56·1	+ 13·9	1
2	·01700	+ 1	2744·2	+ 18·2	+ 32·1	2
3	·01809	+ 2	2826·3	− ·7	+ 31·4	3
4	·01925	+ 3	2903·5	+ 5·5	+ 36·9	4
55	·02049	+ 2	2979·4	− 79·6	− 42·7	55
6	·02183	− 2	3050·5	− 48·5	− 91·2	6
7	·02330	− 2	3112·8	− 16·2	−107·4	7
8	·02492	+ 6	3179·9	+134·9	+ 27·5	8
9	·02667	+ 8	3251·3	− 10·7	+ 16·8	9

TABLE 4—*continued.*

British Offices O^{M} *Aggregate Table.*

Graduation of the Unadjusted Values of q_x *by the new method.*

Third Difference Formulas.

Age	q_x	$\Delta^3 q_x \times 10^5$	Expected Deaths	Deviation	Accumulated Deviation	Age
60	·02853	+ 7	3311·7	− 72·3	− 55·5	60
1	·03056	+ 4	3356·5	− 59·5	− 4·0	1
2	·03284	− 1	3401·0	+ 17·0	+ 21·0	2
3	·03544	− 3	3446·8	+ 33·8	+ 54·8	3
4	·03840	− 3	3498·0	−117·0	− 62·2	4
65	·04171	− 3	3533·7	+ 41·7	− 20·5	65
6	·04534	− 1	3566·0	− 14·0	− 34·5	6
7	·04926	+ 4	3573·7	+ 50·7	+ 16·2	7
8	·05344	+ 8	3558·7	−106·3	− 90·1	8
9	·05787	+ 7	3508·9	+ 91·9	+ 1·8	9
70	·06259	+ 7	3444·8	− 41·2	− 39·4	70
1	·06768	+ 7	3352·7	+ 55·7	+ 16·3	1
2	·07321	− 2	3243·2	− 51·8	− 35·5	2
3	·07925	+ 7	3118·3	+ 31·3	− 4·2	3
4	·08587	+ 4	2970·7	− 14·3	− 18·5	4
75	·09305	+ 5	2811·1	− 7·9	− 26·4	75
6	·10086	+ 33	2641·7	+ 3·7	− 22·7	6
7	·10934	+ 10	2450·0	+ 13·9	− 9·7	7
8	·11854	− 30	2262·0	+ 19·0	+ 9·3	8
9	·12879	− 38	2069·4	+ 9·4	+ 18·7	9
80	·14019	− 51	1874·3	− 17·7	+ 1·0	80
1	·15244	− 55	1668·0	+ 11·0	+ 12·0	1
2	·16516	+ 22	1452·9	+ 21·9	+ 33·9	2
3	·17784	+ 75	1250·9	+ 26·9	+ 60·8	3
4	·18993	+ 85	1050·5	− 62·5	− 1·7	4
85	·20165	−105	848·3	− 2·7	− 4·4	85
6	·21375	+ 18	686·4	+ 23·4	+ 19·0	6
7	·22708	− 37	549·8	− 16·2	+ 2·8	7
8	·24269	+ 36	430·3	− 25·7	− 22·9	8
9	·26076	+ 48	328·3	− ·7	− 23·6	9
90	·28092	+ 50	241·3	+ 15·3	− 8·3	90
1	·30353	+ 37	180·9	+ 14·9	+ 6·6	1
2	·32907	+ 40	139·9	+ 1·9	+ 8·5	2
3	·35804	+ 79	96·3	− 4·7	+ 3·8	3
4	·39081	+ 94	62·5	+ 1·5	+ 5·3	4
95	·42778	+112	39·8	− 1·2	+ 4·1	95
6	·46974	+129	24·4	+ 10·4	+ 14·5	6
7	·51763	+131	18·6	+ 1·6	+ 16·1	7
8	·57257	+145	10·9	− 1·1	+ 15·0	8
9	·63585	+146	3·2	+ 1·2	+ 16·2	9
100	·70878	+145	2·1	+ ·1	+ 16·3	100
1	·79281	...	·8	+ ·8	+ 17·1	1
2	·88940	...	·9	+ ·9	+ 18·0	2
3	1·00000	...	1·0	...	+ 18·0	3

Table 5.

British Offices O^M *Aggregate Table.*

Constructed and Graduated by Mr. George F. Hardy.

Age	q_x	$\Delta^3 q_x \times 10^5$	Expected Deaths	Deviation	Accumulated Deviation	Age
10	·00338	...	1·0	− 1·0	− 1·0	10
1	·00341	− 2	1·3	− 1·7	− 2·7	1
2	·00345	+ 3	1·8	− 1·2	− 3·9	2
3	·00350	− 1	2·6	+ 1·6	− 2·3	3
4	·00354	...	4·1	+ 3·1	+ ·8	4
15	·00360	− 1	6·9	− ·1	+ ·7	15
6	·00367	+ 1	10·7	− ·3	+ ·4	6
7	·00375	+ 2	16·4	+ 1·4	+ 1·8	7
8	·00383	− 3	25·7	− ·3	+ 1·5	8
9	·00392	+ 3	38·9	+ ·9	+ 2·4	9
20	·00404	− 3	60·0	− 4·0	− 1·6	20
1	·00416	+ 2	97·8	+ 1·8	+ ·2	1
2	·00431	− 1	144·5	+ ·5	+ ·7	2
3	·00446	...	201·9	+ 10·9	+ 11·6	3
4	·00463	+ 3	271·8	− 39·2	− 27·6	4
25	·00481	− 6	352·6	− 17·4	− 45·0	25
6	·00500	+ 6	439·2	+ 36·2	− 8·8	6
7	·00523	− 3	534·4	+ 24·4	+ 15·6	7
8	·00544	− 2	628·9	+ 20·9	+ 36·5	8
9	·00569	+ 4	732·1	+ 8·1	+ 44·6	9
30	·00595	− 2	841·2	+ 11·2	+ 55·8	30
1	·00620	− 1	938·8	+ 37·8	+ 93·6	1
2	·00648	+ 3	1040·1	+ 2·1	+ 95·7	2
3	·00677	− 2	1141·2	+ 14·2	+109·9	3
4	·00706	− 1	1236·2	− 69·8	+ 40·1	4
35	·00738	+ 1	1332·7	− 40·3	− ·2	35
6	·00771	+ 4	1423·7	− 13·3	− 13·5	6
7	·00804	− 6	1506·2	− 45·8	− 59·3	7
8	·00838	+ 4	1585·8	− 15·2	− 74·5	8
9	·00877	+ 1	1672·4	+ 78·4	+ 3·9	9
40	·00915	− 2	1756·9	− 2·1	+ 1·8	40
1	·00956	+ 2	1834·4	+ ·4	+ 2·2	1
2	·01001	− 1	1917·4	− 25·6	− 23·4	2
3	·01048	+ 3	1996·1	− 52·9	− 76·3	3
4	·01099	− 2	2079·8	− 66·2	−142·5	4
45	·01153	...	2160·1	+ 55·1	− 87·4	45
6	·01213	+ 5	2243·5	− ·5	− 87·9	6
7	·01277	− 4	2323·0	+ 79·0	− 8·9	7
8	·01345	+ 4	2400·3	− 71·7	− 80·6	8
9	·01422	− 2	2479·5	+ 18·5	− 62·1	9
50	·01504	+ 1	2567·2	− 49·8	−111·9	50
1	·01595	+ 5	2648·1	+ 46·1	− 65·8	1
2	·01693	− 5	2732·9	+ 6·9	− 58·9	2
3	·01799	+ 4	2810·7	− 16·3	− 75·2	3
4	·01918	+ 3	2892·9	− 5·1	− 80·3	4
55	·02045	− 2	2973·6	− 85·4	−165·7	55
6	·02184	+ 4	3051·9	− 47·1	−212·8	6
7	·02338	− 3	3123·5	− 5·5	−218·3	7
8	·02505	+ 6	3196·5	+151·5	− 66·8	8
9	·02689	+ 1	3278·2	+ 16·2	− 50·6	9

TABLE 5—*continued.*

British Offices O^{M} *Aggregate Table.*

Constructed and Graduated by Mr. George F. Hardy.

Age	q_x	$\Delta^3 q_x \times 10^5$	Expected Deaths	Deviation	Accumulated Deviation	Age
60	·02887	− 1	3351·2	− 32·8	− 83·4	60
1	·03105	− 4	3410·3	+113·3	+ 29·9	1
2	·03344	− 3	3463·1	+ 79·1	+109·0	2
3	·03603	− 1	3504·2	+ 91·2	+200·2	3
4	·03886	+ 6	3539·9	− 75·1	+125·1	4
65	·04196	− 1	3554·9	+ 62·9	+188·0	65
6	·04532	+ 6	3564·5	− 15·5	+172·5	6
7	·04900	− 1	3554·8	+ 31·8	+204·3	7
8	·05299	+ 8	3528·8	−136·2	+ 68·1	8
9	·05735	− 2	3477·4	+ 60·4	+128·5	9
70	·06207	+11	3416·2	− 69·8	+ 58·7	70
1	·06723	− 8	3330·4	+ 33·4	+ 92·1	1
2	·07281	+15	3225·5	− 69·5	+ 22·6	2
3	·07892	− 3	3105·3	+ 18·3	+ 40·9	3
4	·08548	− 3	2957·2	− 27·8	+ 13·1	4
75	·09264	+14	2798·7	− 20·3	− 7·2	75
6	·10043	− 2	2630·5	− 7·5	− 14·7	6
7	·10882	+10	2438·3	+ 1·3	− 13·4	7
6	·11795	− 2	2250·7	+ 7·7	− 5·7	8
9	·12780	+ 6	2053·5	− 6·5	− 12·2	9
80	·13847	+ 7	1842·6	− 49·4	− 61·6	80
1	·14998	+ 3	1641·1	− 15·9	− 77·5	1
2	·16239	+ 9	1428·5	− 2·5	− 80·0	2
3	·17577	...	1236·4	+ 12·4	− 67·6	3
4	·19015	+ 7	1051·7	− 61·3	−128·9	4
85	·20562	+ 5	865·0	+ 14·0	−114·9	85
6	·22218	+ 4	713·4	+ 50·4	− 64·5	6
7	·23990	− 1	580·8	+ 14·8	− 49·7	7
8	·25883	− 6	458·9	+ 2·9	− 46·8	8
9	·27901	...	351·3	+ 22·3	− 24·5	9
90	·30043	− 3	258·1	+ 32·1	+ 7·6	90
1	·32315	− 1	192·6	+ 26·6	+ 34·2	1
2	·34717	− 5	147·5	+ 9·5	+ 43·7	2
3	·37246	− 8	100·2	− ·8	+ 42·9	3
4	·39901	− 7	63·8	+ 2·8	+ 45·7	4
95	·42677	−12	39·7	− 1·3	+ 44·4	95
6	·45566	−18	23·7	+ 9·7	+ 54·1	6
7	·48561	−14	17·5	+ ·5	+ 54·6	7
8	·51650	−23	9·8	− 2·2	+ 52·4	8
9	·54815	−22	2·7	+ ·7	+ 53·1	9
100	·58042	−27	1·7	− ·3	+ 52·8	100
1	·61308	...	·6	+ ·6	+ 53·4	1
2	·64591	...	·6	+ ·6	+ 54·0	2
3	·67864	...	·7	− ·3	+ 53·7	3

TABLE 6.

British Offices O^M *Aggregate Table.*

Graduated by Mr. John Spencer, by his Twenty-one Term Formula.

Age	q_x	$\Delta^3 q_x \times 10^5$	Expected Deaths	Deviation	Accumulated Deviation	Age
10	·00328	− 1	·9	− 1·1	− 1·1	10
1	·00329	+ 1	1·2	− 1·8	− 2·9	1
2	·00332	...	1·7	− 1·3	− 4·2	2
3	·00336	− 1	2·5	+ 1·5	− 2·7	3
4	·00342	...	4·0	+ 3·0	+ ·3	4
15	·00350	...	6·7	− ·3	...	15
6	·00359	+ 1	10·4	− ·6	− ·6	6
7	·00369	− 1	16·2	+ 1·2	+ ·6	7
8	·00380	...	25·5	− ·5	+ ·1	8
9	·00393	− 1	39·0	+ 1·0	+ 1·1	9
20	·00407	...	60·5	− 3·5	− 2·4	20
1	·00422	...	99·2	+ 3 2	+ ·8	1
2	·00437	+ 1	146·5	+ 2·5	+ 3·3	2
3	·00452	+ 2	204·6	+ 13·6	+ 16·9	3
4	·00467	+ 1	274·1	− 36·9	− 20·0	4
25	·00481	...	352·6	− 17·4	− 37·4	25
6	·00496	+ 2	435·7	+ 32·7	− 4·7	6
7	·00513	...	524·2	+ 14·2	+ 9·5	7
8	·00532	+ 1	615·0	+ 7·0	+ 16·5	8
9	·00555	− 3	714·1	− 9 9	+ 6·6	9
30	·00582	− 2	822·8	− 7·2	− ·6	30
1	·00614	...	929·7	+ 28·7	+ 28·1	1
2	·00648	− 1	1040·1	+ 2·1	+ 30·2	2
3	·00682	− 1	1150·0	+ 23·0	+ 53·2	3
4	·00716	+ 3	1253·7	− 52·3	+ ·9	4
35	·00749	− 1	1352·6	− 20·4	− 19·5	35
6	·00780	+ 4	1440·3	+ 3·3	− 16·2	6
7	·00812	− 2	1521·2	− 30·8	− 47·0	7
8	·00844	+ 2	1597·2	− 3·8	− 50·8	8
9	·00880	− 1	1678·1	+ 84·1	+ 33·3	9
40	·00918	...	1762·7	+ 3·7	+ 37·0	40
1	·00960	− 1	1842·1	+ 8·1	+ 45·1	1
2	·01005	+ 2	1925·0	− 18·0	+ 27·1	2
3	·01053	− 1	2005·6	− 43·4	− 16·3	3
4	·01103	+ 1	2087·4	− 58·6	− 74·9	4
45	·01157	+ 4	2167·6	+ 62·6	− 12·3	45
6	·01214	− 1	2245·4	+ 1·4	− 10·9	6
7	·01275	+ 2	2319·3	+ 75·3	+ 64·4	7
8	·01344	...	2398·5	− 73·5	− 9·1	8
9	·01420	− 1	2476·0	+ 15·0	+ 5·9	9
50	·01505	...	2568·9	− 48·1	− 42·2	50
1	·01599	− 1	2654·8	+ 52·8	+ 10·6	1
2	·01701	...	2745·8	+ 19·8	+ 30·4	2
3	·01811	+ 1	2829·5	+ 2·5	+ 32·9	3
4	·01928	+ 3	2908·0	+ 10·0	+ 42·9	4
55	·02052	...	2983·8	− 75·2	− 32·3	55
6	·02184	+ 5	3051·9	− 47·1	− 79·4	6
7	·02327	+ 4	3108·8	− 20·2	− 99·6	7
8	·02481	+ 4	3165·9	+ 120·9	+ 21·3	8
9	·02651	+ 2	3231·8	− 30·2	− 8·9	9

TABLE 6—*continued.*

British Offices O^{M} *Aggregate Table.*

Graduated by Mr. John Spencer, by his Twenty-one Term Formula.

Age	q_x	$\Delta^3 q_x \times 10^5$	Expected Deaths	Deviation	Accumulated Deviation	Age
60	·02841	+ 1	3297·8	− 86·2	−95·1	60
1	·03055	+ 2	3355·4	+ 58·4	−36·7	1
2	·03295	− 1	3412·4	+ 28·4	− 8·3	2
3	·03562	...	3464·3	+ 51·3	+43·0	3
4	·03858	+ 1	3514·4	−100·6	−57·6	4
65	·04182	...	3543·0	+ 51·0	− 6·6	65
6	·04534	+ 6	3566·0	− 14·0	−20·6	6
7	·04915	− 3	3565·7	+ 42·7	+22·1	7
8	·05325	+ 7	3546·1	−118·9	−96·8	8
9	·05770	+ 1	3498·6	+ 81·6	−15·2	9
70	·06247	+ 10	3438·2	− 47·8	−63·0	70
1	·06763	− 1	3350·2	+ 53·2	− 9·8	1
2	·07319	+ 10	3242·3	− 52·7	−62·5	2
3	·07925	+ 10	3118·3	+ 31·3	−31·2	3
4	·08580	+ 13	2968·3	− 16·7	−47·9	4
75	·09294	− 5	2807·8	− 11·2	−59·1	75
6	·10077	+ 4	2639·4	+ 1·4	−57·7	6
7	·10942	− 3	2451·8	+ 14·8	−42·9	7
8	·11884	+ 3	2267·7	+ 24·7	−18·2	8
9	·12907	− 21	2073·9	+ 13·9	− 4·3	9
80	·14008	− 33	1872·9	− 19·1	−23·4	80
1	·15190	− 24	1662·1	+ 5·1	−18·3	1
2	·16432	+ 8	1445·5	+ 14·5	− 3·8	2
3	·17701	+ 26	1245·1	+ 21·1	+17·3	3
4	·18973	+ 29	1049·4	− 63·6	−46·3	4
85	·20256	+ 17	852·2	+ 1·2	−45·1	85
6	·21576	+ 23	692·8	+ 29·8	−15·3	6
7	·22962	+ 13	555·9	− 10·1	−25·4	7
8	·24431	− 7	433·2	− 22·8	−48·2	8
9	·26006	− 10	327·4	− 1·6	−49·8	9
90	·27700	+ 76	237·9	+ 11·9	−37·9	90
1	·29506	+ 159	175·9	+ 9·9	−28·0	1
2	·31414	+ 243	133·5	− 4·5	−32·5	2
3	·33500	+ 276	90·1	− 10·9	−43·4	3
4	·35923	+ 320	57·5	− 3·5	−46·9	4
95	·38926	+ 258	36·2	− 4·8	−51·7	95
6	·42785	+ 123	22·2	+ 8·2	−43·5	6
7	·47820	− 142	17·2	+ ·2	−43·3	7
8	·54289	− 257	10·3	− 1·7	−45·0	8
9	·62315	+5888	3·1	+ 1·1	−43·9	9
100	·71756	...	2·2	+ ·2	−43·7	100
1	·82355	...	·8	+ ·8	−42·9	1
2	1·00000	...	1·0	+ 1·0	−41·9	2
3	...	...	·0	− 1·0	−42·9	3

Table 7.

English Life Table No. 6.—Males.

Construction A.

By the new method, by means of graduated quinquennial values of q_x, calculated by second differences.

Age	q_x	$\delta^3 q_x \times 10^5$	Age	q_x	$\delta^3 q_x \times 10^5$
5	·00503	− 4	55	·02558	+ 8
6	·00398	− 4	6	·02706	− 6
7	·00321	− 2	7	·02880	− 7
8	·00268	− 3	8	·03088	− 6
9	·00235	− 3	9	·03324	− 10
10	·00220	− 3	60	·03581	− 10
1	·00220	− 1	1	·03853	+ 4
2	·00232	− 5	2	·04130	+ 20
3	·00253	− 2	3	·04402	+ 17
4	·00282	− 2	4	·04673	+ 22
15	·00314	− 3	65	·04963	+ 19
6	·00347	+ 3	6	·05289	− 7
7	·00379	...	7	·05673	− 18
8	·00407	...	8	·06134	− 23
9	·00434	− 2	9	·06665	− 25
20	·00460	− 1	70	·07248	− 31
1	·00485	+ 1	1	·07860	+ 13
2	·00507	+ 1	2	·08476	+ 49
3	·00525	+ 5	3	·09065	+ 53
4	·00540	+ 3	4	·09640	+ 62
25	·00553	...	75	·10250	+ 62
6	·00569	− 3	6	·10948	− 13
7	·00591	+ 1	7	·11796	− 50
8	·00619	− 1	8	·12856	− 65
9	·00650	− 1	9	·14115	− 83
30	·00685	+ 1	80	·15523	− 14
1	·00723	+ 1	1	·17015	+ 54
2	·00763	− 1	2	·18508	+ 7
3	·00806	− 1	3	·19988	+ 12
4	·00853	− 1	4	·21509	+ 13
35	·00903	+ 1	85	·23078	− 28
6	·00955	...	6	·24707	− 12
7	·01008	+ 1	7	·26409	+ 51
8	·01063	− 3	8	·28156	+ 57
9	·01120	+ 1	9	·29936	+ 65
40	·01180	− 3	90	·31800	− 2
1	·01240	− 1	1	·33805	− 25
2	·01301	+ 6	2	·36016	+ 62
3	·01360	+ 4	3	·38431	+ 74
4	·01416	+ 6	4	·41025	+ 88
45	·01475	+ 2	95	·43860	+ 39
6	·01541	− 3	6	·47010	+ 46
7	·01620	− 2	7	·50563	+ 54
8	·01714	− 5	8	·54558	+ 64
9	·01820	− 1	9	·59041	+ 74
50	·01936	− 6	100	·64066	+ 90
1	·02057	+ 1	1	·69697	+ 105
2	·02182	+ 10	2	·76008	+ 124
3	·02305	+ 8	3	·83089	...
4	·02427	+ 9	4	·91045	...
			5	1·00000	...

TABLE 8.

English Life Table No. 6.—Males.

Construction B.

By the new method, by means of graduated quinquennial values of q_x, calculated by fourth differences.

Age	q_x	$\delta^3 q_x \times 10^5$	Age	q_x	$\delta^3 q_x \times 10^5$
5	·00503	− 2	55	·02556	+ 7
6	·00398	− 5	6	·02703	− 4
7	·00320	− 2	7	·02877	− 9
8	·00267	− 2	8	·03085	− 7
9	·00234	− 5	9	·03323	− 9
10	·00219	− 1	60	·03582	− 12
1	·00220	− 3	1	·03855	+ 5
2	·00232	− 4	2	·04133	+ 20
3	·00254	− 1	3	·04404	+ 19
4	·00283	− 4	4	·04673	+ 21
15	·00315	− 2	65	·04960	+ 20
6	·00349	+ 3	6	·05284	− 8
7	·00381	− 1	7	·05666	− 18
8	·00409	+ 2	8	·06126	− 23
9	·00436	− 3	9	·06656	− 24
20	·00461	− 1	70	·07238	− 33
1	·00486	...	1	·07849	+ 16
2	·00508	+ 3	2	·08465	+ 47
3	·00526	+ 4	3	·09053	+ 54
4	·00540	+ 2	4	·09629	+ 62
25	·00553	+ 2	75	·10240	+ 66
6	·00569	− 4	6	·10940	− 13
7	·00590	+ 1	7	·11791	− 52
8	·00618	− 1	8	·12859	− 71
9	·00649	...	9	·14131	− 85
30	·00684	+ 1	80	·15555	− 19
1	·00722	+ 2	1	·17060	+ 57
2	·00763	− 1	2	·18561	+ 8
3	·00806	− 1	3	·20039	+ 13
4	·00853	− 1	4	·21551	+ 14
35	·00903	+ 1	85	·23105	− 32
6	·00955	+ 1	6	·24714	− 13
7	·01008	− 2	7	·26392	+ 54
8	·01063	+ 1	8	·28107	+ 59
9	·01121	− 2	9	·29846	+ 75
40	·01180	− 3	90	·31663	− 11
1	·01241	+ 1	1	·33617	− 24
2	·01302	+ 4	2	·35783	+ 67
3	·01360	+ 6	3	·38150	+ 75
4	·01416	+ 5	4	·40694	− 95
45	·01474	+ 3	95	·43482	+ 41
6	·01540	− 4	6	·46589	+ 46
7	·01619	−13	7	·50110	+ 59
8	·01714	+ 7	8	·54086	+ 68
9	·01821	− 3	9	·58563	+ 75
50	·01937	+ 4	100	·63600	+ 96
1	·02059	− 8	1	·69265	+113
2	·02184	+ 9	2	·75633	+130
3	·02306	+10	3	·82800	...
4	·02427	+ 9	4	·90879	...
			105	1·00000	...

Abstract of the Discussion.

Mr. ABRAHAM LEVINE, M.A., said that the question which the author had set himself to answer, namely, what were the best methods of procedure to adopt in order to obtain a smooth and satisfactory mortality table from census or other statistics, was one which could never fail to be of interest to the Institute, and any contribution which threw further light on it deserved their most cordial welcome. The historical summary at the beginning of the paper was very interesting, and showed that the method was devised, in the first instance, for the purpose of dealing with census statistics, for which it appeared to work admirably. Except at the infantile ages, such data usually took the form of certain numbers representing the population and the deaths in quinquennial or decennial groups of ages, and the natural procedure must be to find for each group, either the rate of mortality for the central age of the group, or the force of mortality for the central moment of age of the group: and then, by a suitable method of interpolation, to find similar functions for the intervening ages. Now the two features of the author's method corresponded exactly with those two stages. A simple, elegant and easily-applied formula was given for finding the quinquennial graduated functions, and then the author suggested that the osculatory interpolation should be used, instead of the ordinary method of interpolation, for finding the intervening values.

In applying those two processes to the census statistics, it must be admitted that the operations were extremely rapid and easy, more especially if Karup's method of third differences, to which the author did not refer to in his paper, was used. Starting with the quinquennial graduated values, and using Karup's third differences, he (Mr. Levine), had actually obtained a completely graduated table in rather less than an hour and a half, a feat he thought it would be difficult to beat by any other method. It should be noticed incidentally that, in using Karup's third differences, it was necessary to work with at least eight decimal places, if it were desired to get q accurately to five places. Considering the way in which the osculatory formula was derived, one might expect the final table to be smooth, and also to produce approximate fidelity to the groups. But, remembering that in census statistics there were no data for individual ages, but merely data in groups, it was, of course, impossible to estimate the deviations which might arise at individual ages. That brought him to the distinction between census statisties and the records of a life or annuity office, and there he was sorry that, so far as his investigations had proceeded, he could not say it was possible to predict with the same confidence as in the case of census statistics that the author's method would always work well.

Looking at the system, first of all, merely from a theoretical point of view, and taking the short formula (v*a*) for the purpose of illustration, and considering the application of it

to the Government Female Annuitants Ultimate Table, it would be noticed that q_{27}, q_{32} and q_{37}, and so on, were based on fifteen values of the original data, whereas q_{28} was based on thirty values, and q_{29} also on thirty values, but with different coefficients from those which arose in q_{28}: and q_{30} again had different coefficients: so that there was an absence of homogeneity in the formula which might, theoretically at any rate, be expected to involve some want of smoothness in the final table. Again, notwithstanding the author's explanations in the paper, it appeared to him there was something almost arbitrary in the selection of the quinquennial values chosen for graduation in the first instance. The author, for example, in paragraph 23 said that in the Government Female Annuitants Table the central ages 27, 32, and so on, were selected because 102 was taken as the limit of age. In a previous graduation which the author had made of the same table, he believed 101 was taken as the limit of age, and it seemed to him that the mere existence at age 102 of one person exposed to risk, who if he remembered rightly did not even die in the period under observation, should not have so great an influence on the form of the Ultimate Graduated Table. Theoretically, it seemed to him that the want of uniformity, and the selection of certain quinquennial ages throughout the table for different treatment from the other ages were difficulties that should be got over, and possibly the only logical way of doing so would be to treat all the five different groups of quinquennial ages in the same way by applying the formula five times. He had tried that, however, on one mortality table, the $O^{M(5)}$, with results which were not sufficiently satisfactory.

With regard to this arbitrary grouping, it was true that if they applied the formula five times over, then, if the original data were fairly smooth, all the five curves would run very close to each other; but it was difficult to see why any one of the five should be preferred to the others. This theoretical objection, however, would be unimportant, if the method in practice was found everywhere to work really well. Looking at the summary in paragraph 30, one was at first inclined to say that the method did work extremely well, but he thought in that summary the author had been unconsciously misled into a comparison which was rather less than just to Mr. Spencer. If one considered how the original groups were selected for the quinquennial graduated values of q, and that, by the nature of the osculatory interpolation, its tendency would be to reproduce algebraical sums of deviations in those groups comparatively small, even although the individual deviations might be large, it would be seen that, by the accident that the algebraical sums of the deviations in the summary were taken in groups with the same central ages, 32, 37, and so on, as had been originally selected by the author, those algebraical sums of the deviations by his method had been very much under-estimated. To test that, he had computed the deviations in groups 31 to 35, 36 to 40, and so on, with different central ages, and had found that the

author's 52·3 was replaced by 170; the 41·4 was replaced by 155; and in Mr. Spencer's table the 163 was replaced by 114, and the 105·9 by 96. To test it further, he had applied it to the summary in paragraph 55, where it would be noticed that the groups selected by the author had central ages which did not agree with the central ages selected for the original quinquennial graduated values. If the author had taken the algebraical sums of the deviations in groups which had the same central ages as the central ages selected for the original graduation, that figure of 554·1, shown for the new construction, would have been reduced to 278, practically to one-half. In the face of that he thought the author must admit that he had been rather less than just to Mr. Spencer in his inference in paragraph 31.

Personally, he thought it was necessary, in any proper comparison, to take the deviations at the individual ages, and the same remark applied to the O^M Table, where, as the author himself said, the official graduation could not very well be compared with his own, because in the former a deliberate departure from the original data was made in order to bring the table into line with the $O^{M(5)}$, which again was deliberately dealt with in such a way as to make it correspond to a definite mathematical law. Leaving that, and comparing only Mr. King's and Mr. Spencer's graduations, and looking again at the individual deviations rather than the group deviations, he thought both in the magnitude of the deviations, and in the smoothness of the table, it would be found that Mr. Spencer's was just a little better than the author's. The largest individual deviation in the author's table would be found to be greater than the largest in Mr. Spencer's, and so on. From those considerations, he had been rather led to the view that, if the original data were numerous and already fairly smooth, then, with extraordinarily little work, the author's method would give a really smooth and good graduation; but if the original data were scanty and somewhat irregular, the method might give results which could not, by themselves, be taken with very much confidence. It would be desirable, in his opinion, to test the results by some other method, which, from its mathematical basis, might be expected to give a good table.

To justify this conclusion, he had applied an extreme test, taking the first of Mr. Lidstone's irregular series of numbers in the *Journal*, vol. xli, 360. He had graduated those irregular numbers by applying Mr. King's method twice, the first time by summing the numbers in fives from the beginning, and secondly by summing the numbers in fives from the third term, and, as the result, he had obtained two graduations which not only differed widely from the type of curve produced by Mr. Lidstone, by the use of Mr. Spencer's and the Friendly Society formulas, but also differed widely from each other. Which of those two curves, or which of the other three that might have been obtained by summing in fives, was the one to be selected? That was the difficulty, in considering the applicability of the author's method to cases where the data were irregular and scanty.

Proceeding to refer to some minor points, first of all he entirely agreed with the author that his method might be expected to give better results, if applied to construction than to graduation, partly for the reason that was given, and also because, by applying it to the construction, to some extent the preliminary irregularites were removed, both in the exposed to risk and in the deaths, and better quinquennial graduated values were therefore obtained to start with. He did not quite understand why log $(q+\cdot 1)$ should be graduated, but he quite agreed that one should not graduate log q, because q might sometimes be zero. He did not see, however, why one should not graduate the unadjusted values of q themselves. With the assistance of Mr. H. E. Melville, he had applied the author's method to the unadjusted values of q in the $O^{M(5)}$ Table, and also in the Government Female Annuitants Table, using central ages 31, 36, &c., and to portions of the $O^{[am]}$ Table, and he found in every case the results were pretty satisfactory; they were almost as good as the results obtained by graduating log $(q+\cdot 1)$, and of course involved very much less labour.

Finally, owing again to the fact that the deviations were given in groups rather than individually, he was not quite certain whether the author was right in saying in paragraph 17, that the osculatory method, as suggested by Dr. Buchanan, would not give smoother results.

Mr. J. E. FAULKS (Hon. Secretary) read the following communication which had been received from Dr. BUCHANAN:—

In thanking Mr. King for his reference to my paper, I should like to say that it was in no spirit of criticism that that paper was written, but in the belief that I was paying the highest tribute to his work by taking it up immediately, and attempting to develop it. I venture now to draw attention to one or two points on which I am unable to agree with him, and with regard to which my silence might be interpreted as concurrence. He remarks that my two formulas are just Dr. Sprague's and his own in a different garb. I have explicitly pointed this out, and have shown how they may be derived; but they are distinct, in exactly the same sense as Stirling's or Everett's formula is distinct from the standard central difference formula of the text-book. Each of these may be derived from the latter, and each has its own special merits; but Everett's is by far the most convenient for the construction of tables by subdivision of intervals. The osculatory formulas being identically equivalent, each process will be found to give identical results, provided that the work be carried out to its full extent, numerically. But, when values are interpolated by a continuous summation process, and the numerical work is not carried out to its full extent, there may be an accumulation of small inaccuracies, so that a certain tabular error is introduced. When each value is obtained separately, as is done by Everett, it is possible to secure accuracy to a given place of decimals, with the use of fewer figures. A perfect verification can be secured by differencing the Everett results and checking the fifth (or third) difference by Karup's process.

On the subject of notation, Mr. King has given me credit which I do not deserve, and which I have never claimed. The notation was introduced by Dr. Sheppard, in his paper published in the *London Mathematical Society's Proceedings* in 1899, and has been adopted in several papers published subsequently in the same *Proceedings*. The only place in which, according to Dr. Sheppard, central difference operators had previously received separate treatment was in a paper by Hansen, published in Germany in 1865; but the notation of that paper has been condemned by Boole and Moulton as unscientific. In Boole's treatise, central differences are dismissed in a few words, and in Markov's "Differenzen Rechnung", the latest German text-book on the subject, they are entirely ignored. I have little doubt that this neglect has been largely due to the lack of a convenient notation. A good notation should be simple and descriptive, and should avoid confusion with existing notations. Perhaps the greatest merit of the Institute notation is that it is descriptive, so that, when the system has been mastered, the interpretation of a symbol is obvious. In Sheppard's notation, δ, μ and σ are appropriated to central differences; but Δ and Σ are left free for advancing differences. It is true that δ has been used to represent subdivided differences, but this difficulty may be met, as is done by Sheppard, by the use of a suffix letter, *e.g.*, δ_n, Δ_n. The notation is simple, because the operators can be combined, according to the fundamental laws of algebra. Woolhouse's notation has always seemed to me to be difficult, because it is not descriptive. The symbols do not suggest their meaning, so that a certain effort of memory is required, and they do not lend themselves to facility of transformation. A careful examination of Sheppard's notation should convince anyone of its symmetry and simplicity, and of its superiority to any hitherto in use.

In paragraph 17 Mr. King refers to a suggestion which was made for dealing with census data, and which was there attended with some success. But a little consideration will show that it cannot be expected to yield similarly good results when applied to the data of the Government Annuitant Experience. In applying the osculatory formula, an error is introduced, which depends on the fifth difference of the data between which the interpolation is made. The census data lie on an approximately smooth curve, their differences decrease with fair rapidity, and the higher differences are small compared with the original numbers. These differences are multiplied by a small numerical factor, so that the osculation error is generally insignificant; but this is not the case with the data of the Government Annuitant Experience. These data run with great irregularity, the differences do not decrease rapidly, and the higher differences are quite comparable with the original numbers. The osculation error is therefore considerable, and the considerably increased deviations between the "actual" and "expected" deaths is just what might have been expected.

The author has modified his former method of obtaining the graduated quinquennial values of q, so as to make them occupy, as nearly as possible, the central position of the series of values from which they are derived, and when the interpolations are made between the numbers living and the deaths, instead of between the logarithms of these numbers, there is some saving of labour. But in the case of census data, it seems unnecessary to devise special formulas for the purpose. If we use the fifth difference osculatory formula, depending on $u_{-2}, u_{-1}, \ldots u_3$, the exactly central value will be $u_{\frac{1}{2}}$. Hence, if we take the interpolated values $u_{.4}$ and $u_{.6}$, equidistant on each side of it, their difference will be exactly central. From the values given in *J.I.A.*, vol. xlii, p. 380, we have at once:

$$u_{.4} - u_{.6} = -\cdot 2\delta u_{\frac{1}{2}} + \cdot 008\delta^3 u_{\frac{1}{2}} - \cdot 0064\delta^5 u_{\frac{1}{2}}$$

or, if we are working to third differences, our formula will be:

$$u_{.4} - u_{.6} = -\cdot 2\delta u_{\frac{1}{2}} + \cdot 024\delta^3 u_{\frac{1}{2}}$$

I have re-calculated the graduated quinquennial values of q, and compared them with the interpolated values for ages 17, 22, 27, given in Table II. of my paper. Throughout the middle period of life, the agreement is extremely close, but at the ends, where the data are more affected by misstatement of age, the differences are more considerable. The effect of the osculatory method is to produce a curve without discontinuities, but showing a series of undulations, whose heights are proportional to the fifth differences of the data, and these may, in a sense, be taken as a measure of the roughness of the values between which the interpolation is made. In the discussion which followed the reading of Mr. King's former paper, Mr. G. F. Hardy stated that the logarithms of the 1901 population figures could be represented with a very small percentage of error by a constant added to the sum of two geometrical series. Does this not suggest a direction in which the results of our interpolation might be improved, namely, by a preliminary smoothing of the data such as would be secured by fitting them to a suitable frequency curve of this kind? Such treatment would be especially advantageous at those points where the census figures are known to be seriously affected by misstatement of age.

Mr. C. W. KENCHINGTON said that he had taken a keen interest in the subject of the construction and graduation of mortality tables for some time past, and had recently been constructing and graduating mortality tables from the official statistics of the British Offices Experience, 1893, Female Whole-Life Participating Assurances. It seemed to him that that table had received very little attention in the past, and that there were some features in it which would eventually repay the attention of the members. He had applied Mr. Spencer's 21-term formula, and a new formula of a similar character, but with a range of seventeen

terms only, which he had devised for the purpose, to graduate both q_x and colog p_x. The resulting graduated rates had been deduced from the ultimate data, excluding the first five years of experience and the first ten years of experience respectively, and select data had been used throughout. There were brought into account altogether in the ultimate table, excluding the first five years of assurance, 425,432 years of "exposed to risk", and 14,321 deaths, so that the results were based on reliable statistics. He had not had time to complete the entire tables by Mr. King's Construction A, but from age 24 to age 91, where there were altogether 14,183 deaths, there was a total negative deviation over the whole period, computed age by age, of only $-2{\cdot}3$, and throughout the whole of the table the variations were small. Comparing the deviations as shown in Mr. King's method with those produced by Mr. Spencer's 21-term formula, he found that at the younger ages Mr. Spencer's formula seemed to give slightly better results, but Mr. King's gave better results at the later ages. There was really nothing to choose between the third differences of q_x as produced by the two methods: Mr. Spencer's, if anything, giving slightly better values.

(The speaker then referred at some length to his experimental graduations of the $O^{[F]}$ Table during the select period, based upon certain modifications of Mr. King's formulas, with a view to applying them to functions of two variables (age and duration) in the graduation of select tables; and suggested that these formulas and results, when fully developed, might form the subject of a communication for the *Journal*. He also emphasized the point that Mr. King's method was one in which there was great facility for the verification of the work as it proceeded. It was quite possible, in the application of summation formulas of graduation, to obtain verification of the values by the summation of the successive columns, but allowance had to be made for the initial and final terms, which were not symmetrically involved, and no very ready check was available. In the method under discussion it seemed to him that, in about every half-dozen values, simple means could be devised by which the results could be automatically checked).

Mr. W. A. WORKMAN said that Mr. Levine had referred to the different groupings of the material, whereby different results might be obtained, and Mr. Lidstone had also been making experiments in the same direction. The idea underlying them was that the curve of the exposed to risk showed a maximum at a certain age, and that by grouping the original data quinquennially, so that the complete groups about the maximum came on one or other side of the apex of the curve, it would be consistent to expect smoother and better results, from the graduated quinquennial values of q_x so obtained, than from the values obtained by the grouping adopted by the author, where the maximum value came in the centre of a group. At Mr. Lidstone's suggestion, he made a complete interpolation of the Government Female Annuitants (1883) Table with the new groupings, for ages 65 to 80 inclusive,

which ages embraced the numbers exposed to risk indicated by the apex of the curve. The table taken for comparison was Mr. King's Table I, and it would be noticed that at those ages there were successive groups of four plus or four minus deviations. In the new grouping, taking the deviations from the actual deaths, there were eleven changes of sign, against five changes of sign in the author's grouping. Taking the arithmetical sum of those deviations the following figures were arrived at:

Ages	Mr. King's Method	New Grouping
65–68	66·3	84·6
69–72	80·9	73·6
73–76	95·0	52·3
77–80	36·8	27·3
Totals	279·0	237·8

The differences in the algebraical summations were much greater, owing to the succession of plus and minus deviations in Mr. King's figures already mentioned.

In making these interpolations, and also in making some interpolations a few months ago with other data, he used Professor Everett's and Dr. Buchanan's method of applying the formulas, and he would like to draw attention to the particular advantages of that method, more especially in regard to the third difference formula used by the author in his present paper. Professor Everett, in the *Journal*, vol. xxxv, p. 452, and Dr. Buchanan, in vol. xlii, p. 369, had shown that the osculatory formula, if put in a certain form, might be applied to effect the required interpolation with a very considerable saving in the complexity and extent of the arithmetical work, and Mr. Lidstone referred to that method in his remarks following Mr. King's last census paper (vol. xlii, p. 283). Dr. Buchanan gave a full illustration of Prof. Everett's working process, using a fifth difference formula. The formula in Dr. Buchanan's paper for the interpolated value corresponding to that used by Mr. King was given on p. 378 of vol. xlii, and was, using bu_0 and bu_1 for the second central differences, instead of the notation used by Dr. Buchanan—

$$u_x = \xi u_0 + xu_1 + \xi^2\frac{(\xi-1)}{2}bu_0 + x^2\frac{(x-1)}{2}bu_1$$

and, as the author said, that might easily be shown to be the same as his formula. The preliminary work of obtaining the quinquennial graduated values of the function was the same, whichever method was used, but from that point, in working with Dr. Buchanan's formula, the work was considerably simplified. It would be seen from the formula that, in the first place, it would be unnecessary to calculate the tables of leading differences given in paragraphs 24 and

26 of Mr. King's paper. In place of that they had merely to difference the graduated quinquennial values twice centrally, to obtain the values of b, and if, in doing that, the differences were placed between the lines, b_0 would come opposite u_0, b_1 opposite u_1, and so on, thus avoiding, to a great extent, the chance of using a wrong value of b.

In interpolating between quinquennial values of a function, x took the values ·2, ·4, ·6, and ·8 and $x^2\dfrac{(x-1)}{2}$ took the corresponding values of ·016, ·048, ·072 and ·064, all with the negative sign and all, it would be noticed, having ·016 as a factor, being respectively 1, 3, $4\frac{1}{2}$ and 4 times that figure. If, therefore, the values of b were multiplied by ·016, the figures required to perform the interpolation, namely, $x^2\dfrac{(x-1)}{2}bu_1$ and $\xi^2\dfrac{(\xi-1)}{2}bu_0$, could at once be obtained therefrom by a series of exceedingly simple multiplications, which might be performed mentally, and set down against the proper value of xu_1, ξu_0, and added thereto laterally. Professor Everett and Dr. Buchanan had both shown fully how the resultant terms were made to do double duty, "serving both for the preceding and succeeding interval", to quote Professor Everett's own words, who went on to say that the labour of calculation was only about half what it appeared to be on the face of the formula, and certainly it reduced to a very considerable extent the arithmetical work involved in applying the formula in the form given by Mr. King. The figures to be used were throughout smaller; it was unnecessary to work to more than one place of decimals beyond those required in the result, and the advantages of being able to set out in reverse order a set of terms easily obtained, and merely add them to those same terms was, he thought, obvious, seeing that it could be readily performed by anyone having no actuarial knowledge whatever. Moreover, since the values of x and ξ totalled to 2, and those of $x^2\dfrac{(x-1)}{2}+\xi^2\dfrac{(\xi-1)}{2}$ to ·2, a complete check for each quinquennial group of values could be obtained with the greatest simplicity.

Mr. W. P. Elderton said that some few years ago the author wrote a paper on the errors introduced into mortality tables by the use of summation methods of graduation (*J.I.A.*, vol. xli., p. 54), and it occurred to him when he saw the present paper that it would be rather interesting to see what happened if the method then adopted was applied to formula (*va*). In dealing with the present case one was, however, using a different function from those which Mr. King adopted in his former paper, such as m_x, colog p_x, q_x and similar functions; but now in his construction he operated on E_x and θ_x, the forms of the curves for which were obviously quite different from q_x; and, while there might be no error introduced in the q_x functions, there might be an error, when one comes to a function like E_x and θ_x. He therefore tried by taking a somewhat extreme case, and imagining that the exposed to risk, or the deaths as the case might be, were a very steep symmetrical

curve, he found that, using that perfectly smooth curve and formula (*va*), the results he obtained did not agree with the original figures, *i.e.*, some distortion arose when a steep curve was dealt with. He thought that was rather to be expected, because if formula (*va*) was looked at, it would be seen that negative frequency might exceptionally be obtained: in fact, he managed to manufacture an exposed to risk where it would be obtained: but he admitted it was an extreme case. With regard to the distortion that he had just mentioned, the same thing was met with, when an endeavour was made to fit a parabolic curve to data to which it was not quite suitable. If a curve like $a+bx+cx^2+dx^3$ was fitted to a curve like the normal curve of error, one did not reproduce the original figures. In the same way, if Mr. King's method was applied to a curve like that of the form of the normal curve, some distortion arose.

MR. T. G. ACKLAND said that the author had produced, by his new formulas of graduation, excellent results, with perhaps the minimum of labour. The new method was not, at first, quite easily discriminated from Mr. King's former methods, as set out in his Census Paper; but, on looking into the matter more closely, it would be seen that he had now improved those methods, stating them mathematically and analytically, and had attempted with considerable success to get a formula which was truly central, which was not the case with his earlier formulas. He could not help thinking, in common with speakers who had preceded him, that in Dr. Buchanan's valuable paper, published in the same number of the *Journal* as the author's Census Paper, they had a formula which would give results at least equally as good as the author's. Dr. Buchanan there deduced, from the osculatory formulas given in his paper, an absolutely central formula, which had much fewer terms than the fourth difference formula (ii) in the present paper.

The central difference notation employed by Dr. Buchanan, was introduced by Dr. W. F. Sheppard, who might fairly be spoken of as one of the most eminent mathematicians of the present day. The notation was inserted in the *Journal of the London Mathematical Society*, and, so far as its publication in the *Journal of the Institute* was concerned, he might, as Editor, be content to express the opinion that what was good enough for the *London Mathematical Journal* was certainly good enough for their own *Journal*. Personally, he had always considered that the one thing which Mr. Woolhouse (who was usually so clear and lucid) had contributed to their knowledge which might fairly be characterized as somewhat cloudy, and troublesome to the student, was his demonstration of central difference formulas, and he agreed with Dr. Buchanan that the reason why central difference formulas had been comparatively neglected was that they had not had a ready notation to apply to that particular form of work. Dr. Buchanan did not thrust the notation upon them without due explanation, but set out, in two different schemes or tables, its

precise meaning. For central difference purposes, one could not help thinking that the fundamental principle underlying the new notation, namely, that the suffix referred, not to the originating function, but to the position of the central function, was in the direction of a valuable improvement: and that, in this respect, the notation for advancing differences was not appropriate or convenient for central differences, even if it was the best form for advancing differences themselves.

Then Dr. Buchanan referred to the use of osculatory interpolation methods to interpolate the quinquennial values, but he most distinctly limited his remarks to the particular case of Census data. The application by the author of that suggestion to so intractable an experience as the Government Annuitants Experience seemed to be an unfortunate one, which Dr. Buchanan had, in his notes read that evening, definitely repudiated.

With regard to the O^M Table, the author referred to a fall in the death curve in the early years of life. In the period in question, however, there were very few deaths indeed; and he thought that they should not attempt to adjust the curve in such a way that allowance should be made for such a very small body of facts. Reference was then made to the value of q_x at the later years of life. He entirely agreed with the author that it was unfortunate that the values of q_x tabulated in the published volume were based upon integral values of l_x, deduced from too small a value of the radix, so that there was a distortion of the rates of mortality at the end of life, which had nothing to do with the President's excellent work of graduation, but entirely arose from the way in which the values of q were finally got out. With regard to the curious differences between the expected and actual deaths, shown in the graduated table which the author had reproduced in paragraph 55, he fancied the President would tell them that it would probably be found to have arisen from the fact that those figures, as Mr. King suggested, were not obtained from taking the data at every age and applying the graduated rate of mortality, but probably from some quinary grouping, and taking the value of q at the central age of the group.

Personally, he looked with a great deal of suspicion on the separate graduation of numbers exposed to risk and deaths; and he was glad to hear what Mr. Elderton said in that connection, although there did not seem to be the same objection to their arbitrary grouping. One might graduate the resulting rates of mortality, but to proceed on the lines of adjusting, separately, the numerator and denominator from which those rates were deduced, seemed to him at least questionable. In certain circumstances it would be practically impossible, for instance, if they were graduating endowment assurance data, where the number exposed to risk would show sudden falls at quinquennial ages of maturity. The question the paper raised was, as it seemed to him, was it everything to obtain a closer adherence to the facts?

The author had succeeded most admirably in doing that, but a great deal more than that was required, as he submitted, to be done in graduation. The remarks of previous speakers had sufficiently indicated that they shared this view; and there seemed to be much cogency in what Mr. Levine had said, as to the arbitrary selection of a particular group of quinary values for the graduation. He was particularly interested in the author's remarks as to other statistical tables, and especially with regard to the table showing the proportions of married men, widowers, and bachelors in the 1901 census, because he himself made an attempt to deal with those figures, in connection with a reference recently made to him. It would be exceedingly interesting to hear what the author had to say on that subject, and he hoped they would have the benefit of a further contribution, dealing with the application of Mr. King's formulas to other statistical data.

Mr. H. W. MANLY said that summation formulas for the graduation of mortality tables had certainly up to the present time been somewhat discredited, and he thought justly so, where the data was not sufficient in itself to give a fairly orderly progressive series of values. In that respect he agreed with Mr. A. J. Finlaison in his report upon the Government Female Annuitants, and also with Dr. Sprague in his classical paper upon Graphic Graduation. Mr. Finlaison shortly referred to it, stating that "The most satisfactory system of adjustment of tables of mortality at present known, when a sufficiently large number of facts have been dealt with, is that devised by Mr. Woolhouse"—perhaps they should substitute now "that devised by Mr. King"—"but the method becomes inapplicable unless the numbers are so large as to themselves produce a series which demands no more than a slight rectification of its progress. Dr. Sprague said—"I entirely accept the tests of a good graduation described by Mr. Finlaison. In the first place the adjusted facts must form an orderly progressive series that exhibits no breaks or irregularities, and, secondly, the graduated series must in the aggregate yield the same results as the ungraduated, for instance the same number of deaths when the numbers exposed to risk are identical."

He thought there were three advantages in the author's formula for graduation. First, weight was given according to the magnitude of the data to the values of q_x, although, as the author stated, those values did not carry equal authority. Secondly, it did produce an orderly series, particularly when the data was sufficiently large to form a fairly regular series. Thirdly, it was exceedingly simple to apply, and in these days of stress and strain that was a special advantage. The objections that he saw to it were, first, that it followed almost too closely the facts, and, secondly, that it could not be applied to a small experience. In that respect he did not think it would displace Dr. Sprague's method of graphic graduation. He felt it was not desirable to keep too closely to the facts, if to do so they had to make breaks in the curve representing the progression, and it would be even better to sacrifice something in

order to get a really smooth and progressive curve throughout the table. He felt justified in that by Dr. Sprague's remarks and his own observations. If another set of observations were obtained on a similar body of lives, they would no doubt find that they exhibited similar irregularities, but not at the same ages, and if observations on a sufficiently large number of lives could be obtained, those irregularities would either wholly disappear, or be reduced to insignificant proportions. He therefore thought there was no necessity whatever to keep so closely to the facts, if by so doing smoothness and regularity were sacrificed.

With regard to the application of the author's formula to a small number of facts, Mr. Elderton had stated that he had tried to apply it to a series which rapidly decreased, and that he thought it was possible that in another set of figures he might get a negative value. Personally, he had tried it on a very good experience, although the facts were few, and the irregularities in the probabilities of dying very great. It did not look to lend itself to any mathematical graduation, but on trying the author's plan he actually obtained a negative q_x, and consequently did not go any further with it. He thought it right to tell the members, however, that it was a female experience, and, therefore, somewhat exceptional.

Several speakers had referred to the deviations at individual ages, in various groups of ages. Mr. Levine preferred to look at the deviations at individual ages. When he (Mr. Manly) looked at graduations he never looked at the deviations at individual ages; he wanted to see what the total result had been. He therefore looked at the accumulated deviations, and, if he found that they very often produced zero, or somewhere about zero, or that the variations in the signs showed that there was an intermediate age when the accumulated deviations would be zero, he thought that was all that was necessary. Taking Table 1, the accumulated deviations at age 34 amounted to ·2, showing that from 30 to 34 the graduated values of q_x very closely represented the actual mortality. If the unadjusted and adjusted values were represented diagrammatically, the curve representing the graduated rate of mortality would be found to run through the centre of the ungraduated, so that the weight of the deviations on one side of the curve would be the same as the weight of the deviations on the other. Again, at 39, the accumulated deviations amounted to ·2, showing that between 34 and 39 there was a very good graduation, and so they went on to ·4 at 49. Curiously, the ages that he was reading out were apparently quinquennial—34, 39, 44 and 49, which probably arose from Mr. King's formula being based on groups of five ages. That, to him, was the test of the graduation, and the deviations alone had no meaning to him.

To sum up, he thought they had, in the formulas which the author had given, a ready method of graduating a series of facts, if they were sufficiently numerous to present a fairly regular series; but he did not think it was likely to displace other methods altogether, and certainly for experiences with a small number of

facts, where the deaths were very irregular, they would still have to use the graphic method, or some method which graduated larger groups than five ages at a time.

The President asked the members to express their thanks to the author for the able paper he had presented. It contained a great many points which had given rise to a very interesting discussion, and many things which he (the President) might have felt inclined to say in closing the discussion had already been expressed with great ability by speakers who had preceded him: in fact he thought the author might take it as a compliment and as a recognition of the value of his paper that so many points had been taken up in discussion and the whole subject thoroughly threshed out. It was natural, of course, that speakers should refer specially to those points on which they differed from Mr. King, but he thought there was a unanimous feeling, both on the part of those who had criticized the paper and those who had listened to it, that the author had produced a paper of great interest and ability, and that he had set a worthy finish to the series of papers on this subject which he had given from time to time.

The resolution of thanks to Mr. King was then put, and carried with acclamation.

Mr. King, in reply, thanked the members very sincerely for the reception they had given to his paper, and especially those speakers who had examined carefully into various points in it and who had thrown out ideas which might be useful in considering the question further. In submitting such a subject for the first time, he could scarcely expect it to meet with universal acceptance, nor think that it had necessarily been put forward in the best way, and that it could not be improved upon. He, therefore, particularly welcomed the criticisms, which he would rather have than fulsome praise. Mr. Levine brought out some interesting facts with regard to taking the summaries of the tables of deviations given in the paper in different ways. He, for instance, showed that by taking different age groupings from those given in the table in paragraph 30, different comparative results between the various methods of graduation were brought out. That was very interesting, and showed that his (Mr. King's) distrust of that summary method was well founded, and that to get a complete comparison it was necessary to go to the full tables at the end. He wished to point out, however, that, whatever groupings were taken, the total deviations were very small, and that any one of the methods of graduation illustrated gave very good results. It was certainly far from his wish to disparage Mr. Spencer's 21-term formula; the claim that he had over Mr. Spencer in the matter was not that he produced something better, but that he produced something as good very much more quickly and easily, and in these strenuous days he thought that was a claim which, if it could be substantiated, was of some importance.

He wished to remark that it was only a question of convenience which quinquennial points were adopted to start from, in getting the

graduated quinquennial values of the function to be dealt with. As a matter of fact, it would be seen that he spoke of many experiments having been tried upon the Government Experience Table. One of those experiments was to construct from different starting points, and he found that the tables that were produced by starting from various points were almost identical, differing only in the most minute degree, and therefore he came to the conclusion that it was quite unimportant, from the point of view of the final results, what starting point was selected, and that one was perfectly at liberty to select that which was the most convenient for completing the ends of the table. That was the only reason that he gave for selecting seven years from the oldest recorded age as the best for the oldest age for w_x.

As to applying the method to limited experiences, frankly, if the experience was so small that the method failed, then he thought it scarcely worth graduating at all. Such an experience did not throw any real light upon the subject, and it would be equally futile to use any other method of graduation. There were, for instance, in the Government Female Annuitants Table, in fact, in all annuity tables, so very few deaths at the youngest ages, below age 30, that he deprecated any attempts to graduate the tables below that age, as in his opinion, no matter how they were graduated, they told the actuary nothing, and were of no use. When one death more or less would make 25 per-cent difference in the rate of mortality over a series of ages, he did not think that rate of mortality was worth talking about at all. Of course the method was not universally applicable, and there were some kinds of curves it would not graduate. In the Census Paper, he pointed out that ages below five must be ignored. He quite agreed with Mr. Elderton that it was possible to imagine a curve of exposed to risk that it was quite unsuitable for. For instance, he did not think it would meet the case of the early portion of Select Tables, where the rate of mortality was rapidly changing, the rate of change itself becoming rapidly less; but that was not an objection to it any more than to other methods of graduation, except, perhaps, the graphic method.

As to the remarks of Mr. Elderton, with regard to anomalous results, even if a method brought out anomalies, yet, if the deaths corresponded in any way to the exposed to risk, it brought out corresponding anomalous results in the deaths: and when the division was performed to obtain q_x, remarkably good results were obtained. As an illustration of that point, he might say that, at the very beginning of the work, he was trying Mr. Berridge's method of interpolation to construct mortality tables from census returns, and he obtained very broken curves of the living and the dying, but when he came to work those curves into the central death rate it was remarkable how little break there was in them.

With regard to Dr. Buchanan, he did not want it to be thought that he depreciated that gentleman's paper. His only idea in connection with it was that, in himself bringing forward a method that was very simple, easy and expeditious, so that it might be adopted,

he was afraid that Dr. Buchanan's paper, however valuable it might be otherwise, looked so complicated that it would drive those who had not plenty of time and plenty of mathematical knowledge away from considering the subject at all. But after the discussion which had taken place that evening, he doubted whether there was any fear of that now taking place.

Mr. Levine had asked why q_x should not be graduated directly? He did know why that should not be done, but he had been following example in graduating log q: and seeing that log $(q+\cdot 1)$ could be graduated better than log q, he proposed to substitute that function. Nevertheless, q itself did extremely well, and he had no objection whatever to using it.

He was afraid there was a fundamental error in using osculatory formulas for finding graduated quinquennial values, perhaps not always of a very great amount, but still it was present. For instance, by the use of an osculatory formula instead of the formula (v*a*) by ordinary differences that was given in the paper, there was an error of $-\cdot 024$ in the second difference of w, and that was too great to be ignored. He had tried it, and obtained results which were quite impossible. He admitted that he did not try it upon a Census table, but he hesitated to apply to such a table a formula which gave hopeless results in another instance. He preferred to have one formula that could be safely trusted in many and various circumstances, and where one would be pretty sure that in any ordinary table they would not be going far wrong. It was not impossible that the error in the osculatory formula might correct some of the errors in age in the Census tables, but on the other hand it might go the other way. All he could say was that he had tried the osculatory method on mortality tables from records of assured lives and annuitants, and it did not work there.

ADDENDUM.

On a New Method of Constructing and of Graduating Mortality and other Tables; with a Further Development of the Method. *By* GEORGE KING, F.I.A., F.F.A., *Consulting Actuary.*

72*. When the new method of treating mortality statistics was submitted to the Institute on 14 December last, a most interesting and instructive discussion took place, which lasted until a late hour, so that but little time was left for an effective

* The numbering of the paragraphs and tables of this Addendum is, for convenience of reference, consecutive with that of the original paper.

reply on the points raised; and, more particularly, it was not possible to bring forward figures in response to the criticisms. I therefore much appreciate the courtesy of the Editor of the *Journal* in granting space for further explanations and illustrations.

73. At the meeting, the main argument urged for the exercise of caution in accepting the new method was, that there are five curves possible according to the points chosen for the quinquennial values; and that, as all these differ between themselves, an element of uncertainty is introduced, so that we cannot say which of the five curves is the best, and therefore which of them should be adopted.

74. The reply is that these five curves do not differ materially from each other. Each gives a smooth graduation, and each reproduces the actual deaths with close accuracy; and, for all practical purposes, any one of them is very good, and it is not of much consequence which of them is selected. A similar objection may be raised against all methods of graduation. No two methods give precisely the same results; and even good methods, of general acceptance, may produce curves as divergent from each other as any two of mine. I had satisfied myself on this point by actual trial before writing the paper, and I now gladly submit some of the figures, so that all who care may examine for themselves, and not take anything for granted merely on authority.

75. The fundamental formulas of the paper are (iii*a*) and (v*a*), the former being true up to and including the 5th difference, and the latter up to and including the 3rd. The latter formula is the more important, because increasing experience shows that but little advantage is secured by using the longer formula (iii*a*). Formula (v*a*) is

$$u_7 = \cdot 2w_5 - \cdot 008\Delta^2 w_0$$

and it is worthy of mention that its arithmetical application may be shortened by multiplying by 5, and writing

$$5u_7 = w_5 - \cdot 04\Delta^2 w_0$$

The factor 5 disappears on dividing θ_x by E_x.

76. If we represent by (u) the graduated value of the function being dealt with, and make the central term the origin, and if we write γ_h for $(u_{-h}+u_{+h})$, formula (v*a*) may be written

$$(u) = \cdot 216u_0 + \cdot 216(\gamma_1+\gamma_2) - \cdot 008(\gamma_3+\gamma_4+\gamma_5+\gamma_6+\gamma_7)$$

This shows that the graduated central value is merely the mean of five values, corrected for the 2nd difference. Although the results produced seem to be remarkably good in all the cases tested, and to leave little to be desired, yet the fear has been expressed that the foundation is narrow on which to build a table intended for practical use, and more especially so if the original observations are scanty. Personally I do not share this fear, because considerable experience has led me to believe that, even with limited data, the new method will give as good results as any other, and with much less labour; and that, only when the facts are so few as not to be reasonably capable of graduation at all, will the new method break down. Seeing, however, that the question has been raised, I have gone into the matter further, and have arrived at a development which, with only a small amount of additional trouble, will give a broader basis for the table, and, it is hoped, will remove the doubts of any who may think the objection has weight.

77. The development consists in duplicating formula (v*a*) by first finding graduated quinquennial values of w_x, from which to find the graduated quinquennial values of u_x, the function to be dealt with, where u_x represents E_x, or θ_x, or $\log(q_x + \cdot 1)$, or any other function, as the case may be.

78. The unadjusted column of u_x is summed in groups of five for each year of age, and the quinary sum is placed against the youngest age in the group. We thus have a complete column of w_x for every value of x. Formula (v*a*) is then applied to this column, and produces graduated quinquennial values of w_x, to which the formula is applied a second time to produce the graduated quinquennial values of u_x.

79. When the form of the duplicated formula is analysed it will be found that

$$\begin{aligned}5^6(u) = 3655u_0 &+ 2870\gamma_1 + 2085\gamma_2 + 1300\gamma_3 + 515\gamma_4 \\ &- 270\gamma_5 - 215\gamma_6 - 160\gamma_7 - 105\gamma_8 - 50\gamma_9 \\ &+ 5\gamma_{10} + 4\gamma_{11} + 3\gamma_{12} + 2\gamma_{13} + \gamma_{14}\end{aligned}$$

The formula includes twenty-nine terms of the unadjusted series, and principal weight is assigned to the nine terms in the centre, while it contains the necessary correction for the second difference.

80. In the discussion, the interesting fact was brought out by Mr. Levine that formulas (iii*a*) and (v*a*) have a slight cyclical

tendency, so that, when the deviations of the expected from the actual deaths are taken out in groups of five ages, the result has the best appearance when the central age of each group is that assumed for the corresponding graduated quinquennial value of q_x. This had escaped my notice, and thereby an unintentional injustice was done to Mr. Spencer's 21-term formula, and undue efficacy was assigned to formula (iii*a*) as compared with (v*a*). The cyclical tendency is, however, very slight. It is revealed only by what may be called the balance of the deviations, and in practice is of no importance. From the nature of the case it probably does not exist in the duplicated formula.

81. In order to illustrate the points mentioned above, it is now proposed to continue the use of the Government Female Annuitants (1883) Ultimate Table, and in Tables 9, 10, and 11, attached hereto, three constructions are given, additional to those given in the paper.

82. *Construction* A.—In Table I of the paper itself, we had this construction. It was effected by one application of formula (v*a*), and the values of w_x were taken at ages 20, 25, &c., up to 95 inclusive. This gave graduated values of E_x, θ_x, and q_x at ages 27, 32, &c., up to 92 inclusive. Then q_{102} was assumed to be unity, and q_{97} was inserted by a third difference. Hence, q_x was found for ages 32 to 97 inclusive, by osculatory interpolation of log q_x, and the remaining five values at each end were added by a 4th difference, ordinary numbers and not logarithms being used.

83. *Construction* $A^{(a)}$.—Table 9 of this Addendum was also effected by one application of formula (v*a*), and the values of w_x were taken at ages 22, 27, &c., up to 97 inclusive. This gave graduated values of E_x, θ_x, and q_x, at ages 29, 34, &c., up to 94 inclusive; and hence q_x was found by osculatory interpolation of log q_x for ages 34 to 89 inclusive, and five values were added at the beginning, exactly as in Construction A. At the old ages in this case, q_{101} was assumed to be unity, and the values for ages 90 to 101, twelve in all, were found by 3rd and 4th differences, derived from the values at ages 87, 88, 89, 94, and 101, ordinary numbers and not logarithms being used.

84. It will be observed that the quinquennial points used in Construction $A^{(a)}$ are as far removed as can be from those used in Construction A, and, therefore, presumably these two curves differ as much from each other as any pair of the five possible curves. It will also be observed that different processes were

followed in these two constructions at the old ages, and that different points were assumed for the final age. This is in accordance with paragraph 34 of the paper. Throughout the investigation, many experiments were tried in completing the ends of the tables, and in each particular case that method was adopted which appeared to be the most convenient in view of the number of terms to be added, and of the number and position on the curve of those graduated values already available.

85. *Construction* C″, Table 10 of this Addendum, was effected by duplication of formula (v*a*). The values of w_x were taken for each age from 14 to 103 inclusive, it being assumed that E_x and θ_x from 14 to 18 and from 103 to 107, all inclusive, were zero. Then the column of w_x was summed in fives at the points 14, 19, &c., up to 99 inclusive, and hence were found by formula (v*a*) graduated values of w_x at ages 21, 26, &c., up to 96 inclusive; and from these were found, by a second application of the formula, graduated quinquennial values of E_x, θ_x, and q_x, at ages 28, 33, to 93 inclusive. It was then assumed that q_{103} is unity, and q_{98} was inserted as in Construction A. The values of $\log(q_x+\cdot 1)$ were then interpolated for ages 33 to 98 inclusive, and the remaining five values at each end were added by a 4th difference, the function $\log(q_x+\cdot 1)$ being used.

86. Another construction, called C in my private notes, was also effected, but it need not be reproduced. In all respects it is the same as C″, except that in the osculatory interpolations the function q_x, and not $\log(q_x+\cdot 1)$ was used. As far as about age 60 the two tables are practically indistinguishable, but at the older ages C″ has much the smoother graduation. Evidently, therefore, it is wise to go to the trouble of operating on $\log(q_x+\cdot 1)$ instead of q_x.

87. *Construction* C^b, Table 11 of this Addendum, was effected much as C″, except that from the complete column of w_x the quinary sums were formed for ages 16, 21, &c., up to 101 inclusive, and this involves the assumption that E_x, and θ_x, from age 103 to age 109 inclusive, are both zero. Hence graduated values of w_x were obtained for ages 23, 28, &c., up to 98 inclusive, and graduated values of E_x, θ_x, and q_x, for ages 30, 35, &c., up to 95 inclusive. It was then assumed that q_{103} is unity, and q_{100} was found by a 3rd difference. Then $\log(q_x+\cdot 1)$ for ages 35 to 95 inclusive was interpolated, and five values of this function were supplied at the younger ages by a

3rd difference. At the older ages eight values were supplied by 3rd and 4th differences formed from the values at ages 93, 94, 95, 100, and 103.

88. It will be observed that in Constructions $C^{(a)}$ and $C^{(b)}$ a certain amount of liberty has been taken with the original facts. The actual observations commence at age 19, and close at age 102: and it has been assumed that from 14 to 18, and from 103 to 109, all inclusive, both E_x and θ_x are zero. In the present case the assumption seems to be justified by results, and it is convenient as facilitating the completion of the ends of the tables. Nevertheless it is put forward only tentatively, and further investigation is desirable. It had not been thought of when Constructions A and $A^{(a)}$ were prepared.

89. The three new constructions are given in extenso in in Tables 9, 10, and 11, appended, and Construction A will be found in Table 1 of the paper itself. The 3rd differences of q_x show that in each case there is a smooth graduation throughout, but that in this respect the duplication of formula (va) gives the best results. In fact, Constructions $C^{(a)}$ and $C^{(b)}$ have the smoothest graduations of all the versions of this particular mortality table that have yet been produced; and, had the osculatory interpolations been performed by 5th instead of only 3rd differences, probably all irregularities would have been removed. It must be remembered, however, that in these constructions $\log(q_x + \cdot 1)$ was made the subject of the interpolations, and not $\log q_x$ as in A and $A^{(a)}$, and as in Mr. Spencer's graduation; and this may have increased slightly the smoothness at the older ages.

90. In Table 13 the deviations are given age by age for all the constructions, so that they can be examined, and grouped in any way desired; and in Table 14 they have been grouped into age-sections, according to the plan of Mr. Spencer, *J.I.A.*, xli, 361, so that a broader comparison may be secured, and that the effect of any possible cyclical tendency may be eliminated; and in this comparison Mr. Spencer's graduation has been included. It will be seen how very close all the five tables keep to each other. Clearly any one of them would be as good for practical purposes as any of the others, because as regards adherence to the original facts there is nothing to choose between them. Therefore we may fairly say that it does not matter what quinquennial points of age are taken for w_x in the constructions.

91. In Table 12 the values of q_x for all the five tables are

set side by side for the purpose of easy comparison; but it should be remarked that below age 40 they possess but little authority. The experience below that age comprises only 40 deaths, a number too small to be of much use; and, moreover, in my own four constructions the methods followed for completing this portion of the tables differ materially from each other, and therefore the figures are given without endorsation. Similar remarks, but perhaps not with the same emphasis, apply to the ages over 90.

92. The deviations age by age for each of the five tables, given in Table 13, form a very instructive subject for study. There are seventy-three ages in all, and all the five graduations have deviations of the same sign in fifty-seven cases, and one or other has a deviation of opposite sign in only sixteen cases; but at almost every point where the signs differ the deviations are extremely small. The only points where this rule does not hold are at ages 70 and 74, where the extreme divergences of the graduations from each other are

At age 70—

Construction $C^{(a)}$, deviation $= +\ 6{\cdot}3$

" $C^{(b)}$, " $= -\ 4{\cdot}2$

At age 74—

Construction A, deviation $= -14{\cdot}2$

" $A^{(a)}$, " $= +\ 2{\cdot}3$

At age 70 the actual deaths were 396, and at age 74 they were 532. In each of these cases the extreme divergence is only about 3 per-cent of the actual deaths.

93. Comparing Constructions A and $A^{(a)}$, which are similar, but with different starting points for w_x, and, for reasons given above, considering only ages 40 to 89, fifty ages in all, the deviations have the same sign in forty-five cases, and opposite signs in only five cases. In Constructions $C^{(a)}$ and $C^{(b)}$, which are similarly related to each other, the signs are the same in forty-six cases, and differ in only four cases.

94. Table 13 therefore proves that the deviations are not due to vagaries in the methods of graduation, but are due entirely to roughness in the original data; and probably the total amount is a minimum, irreducible if smoothness of graduation is to be secured.

TABLE 9.

Government Female Annuitants (1883) *Ultimate Table.*

Construction $A^{(a)}$.

By the new method from the Exposed to Risk and the Deaths. Formula (va) *applied once. Graduated values of* q_x *at the points* 29, 34, 39, *&c.*

Age	q_x	$\Delta^3 q_x \times 10^5$	Expected Deaths	Deviation	Accumulated Deviation	Age
30	·01089	− 2	1·8	− ·2	− ·2	30
1	·01158	− 4	2·3	+ 2·3	+ 2·1	1
2	·01213	− 3	2·7	+ ·7	+ 2·8	2
3	·01252	− 3	3·1	+ ·1	+ 2·9	3
4	·01271	+ 15	3·6	+ ·6	+ 3·5	4
35	·01267	+ 17	4·0	− 1·0	+ 2·5	35
6	·01237	+ 16	4·5	− ·5	+ 2·0	6
7	·01196	+ 11	5·0	+ 2·0	+ 4·0	7
8	·01161	− 17	5·5	− ·5	+ 3·5	8
9	·01148	− 8	6·6	+ 4·6	+ 8·1	9
40	·01168	− 16	7·6	− 5·4	+ 2·7	40
1	·01204	− 17	8·8	− ·2	+ 2·5	1
2	·01248	− 18	10·4	− 1·6	+ ·9	2
3	·01284	+ 10	12·1	+ 3·1	+ 4·0	3
4	·01295	+ 28	14·2	+ 2·2	+ 6·2	4
45	·01263	+ 30	15·8	− 1·2	+ 5·0	45
6	·01198	+ 29	17·0	− 4·0	+ 1·0	6
7	·01128	+ 19	17·8	+ 4·8	+ 5·8	7
8	·01083	− 20	19·1	+ 5·1	+ 10·9	8
9	·01092	− 28	21·4	− 4·6	+ 6·3	9
50	·01174	− 29	25·4	+ 3·4	+ 9·7	50
1	·01309	− 29	31·2	− 5·8	+ 3.9	1
2	·01469	− 14	39·2	− 3·8	+ ·1	2
3	·01625	+ 20	47·3	− 3·7	− 3·6	3
4	·01748	+ 20	58·2	− ·8	− 4·4	4
55	·01824	+ 21	67·2	+ 10·2	+ 5·8	55
6	·01873	+ 22	76·9	− 6·1	− ·3	6
7	·01915	− 2	85·7	− ·3	− ·6	7
8	·01971	− 13	96·3	+ 6·3	+ 5·7	8
9	·02063	+ 1	108·9	− 14·1	− 8·4	9
60	·02189	...	123·5	− 14·5	− 22·9	60
1	·02336	+ 3	141·0	+ 25·0	+ 2·1	1
2	·02505	− 12	160·9	+ 3·9	+ 6·0	2
3	·02696	− 1	182·3	+ ·3	+ 6·3	3
4	·02912	+ 18	211·0	+ 2·0	+ 8·3	4
65	·03141	+ 14	238·7	− 19·3	− 11·0	65
6	·03382	+ 18	265·9	+ 11·9	+ ·9	6
7	·03653	+ 2	294·5	− 22·5	− 21·6	7
8	·03968	− 11	325·3	− 27·7	− 49·3	8
9	·04345	− 4	360·9	+ 17·9	− 31·4	9

TABLE 9—*continued.*

Government Female Annuitants (1883) *Ultimate Table.*

Construction $A^{(a)}$.

By the new method from the Exposed to Risk and the Deaths. Formula (va) *applied once. Graduated values of* q_x *at the points* 29, 34, 39, *&c.*

Age	q_x	$\Delta^3 q_x \times 10^5$	Expected Deaths	Deviation	Accumulated Deviation	Age
70	·04786	− 3	400·7	+ 4·7	−26·7	70
1	·05280	− 5	437·8	+36·8	+10·1	1
2	·05823	− 10	474·9	+42·9	+53·0	2
3	·06412	+ 2	506·9	−11·1	+41·9	3
4	·07042	+ 17	534·3	+ 2·3	+44·2	4
75	·07703	+ 16	555·7	−36·3	+ 7·9	75
6	·08397	+ 14	563·7	− 3·3	+ 4·6	6
7	·09141	− 23	567·2	+19·2	+23·8	7
8	·09951	− 11	567·6	+ 2·6	+26·4	8
9	·10843	+ 36	563·4	+ 1·4	+27·8	9
80	·11794	+ 33	550·1	+ 1·1	+28·9	80
1	·12793	+ 36	529·9	−28·1	+ ·8	1
2	·13876	+ 72	496·2	+17·2	+18·0	2
3	·15076	− 23	462·8	−18·2	− ·2	3
4	·16429	−109	423·0	+24·0	+23·8	4
85	·18007	−108	386·4	−17·6	+ 6·2	85
6	·19787	−108	337·6	− 4·4	+ 1·8	6
7	·21660	− 68	288·5	−10·5	− 8·7	7
8	·23518	+ 20	239·4	+29·4	+20·7	8
9	·25253	+109	199·0	− 9·0	+11·7	9
90	·26797	+197	152·5	− 7·5	+ 4·2	90
1	·28170	+290	110·1	− 4·9	− ·7	1
2	·29481	+362	78·1	+ 6·1	+ 5·4	2
3	·30927	+467	58·5	− 4·5	+ ·9	3
4	·32798	+551	39·7	+ ·7	+ 1·6	4
95	·35456	+636	27·7	− 3·3	− 1·7	95
6	·39368	+729	18·5	+ 6·5	+ 4·8	6
7	·45085	+812	15·3	− 2·7	+ 2·1	7
8	·53243	+827	8·5	− ·5	+ 1·6	8
9	·64571	...	4·5	+ ·5	+ 2·1	9
100	·79881	...	2·3	+ 2·3	+ 4·4	100
1	1·00000	...	3·0	+ 1·0	+ 5·4	1

Table 10.

Government Female Annuitants (1883) *Ultimate Table.*

Construction $C^{(a)}$.

By the new method from the Exposed to Risk and the Deaths. Formula (va) *duplicated. Graduated values of* w_x *at the points* 21, 26, 31 *&c. Graduated values of* q_x *at the points* 28, 33, 38, *&c.*

Age	q_x	$\Delta^3 q_x \times 10^5$	Expected Deaths	Deviation	Accumulated Deviation	Age
30	·01280	+ 9	2·1	+ ·1	+ ·1	30
1	·01187	...	2·3	+ 2·3	+ 2·4	1
2	·01115	− 3	2·5	+ ·5	+ 2·9	2
3	·01073	− 12	2·7	− ·3	+ 2·6	3
4	·01061	− 12	3·0	...	+ 2·6	4
35	·01076	− 12	3·4	− 1·6	+ 1·0	35
6	·01106	+ 5	3·6	− 1·4	− ·4	6
7	·01139	+ 7	4·7	+ 1·7	+ 1·3	7
8	·01163	− 5	4·8	− 1·2	+ ·1	8
9	·01183	− 6	5·6	+ 3·6	+ 3·7	9
40	·01206	− 5	7·9	− 5·1	− 1·4	40
1	·01227	− 15	9·0	...	− 1·4	1
2	·01240	+ 3	10·3	− 1·7	− 3·1	2
3	·01240	+ 24	11·7	+ 2·7	− ·4	3
4	·01212	+ 22	13·3	+ 1·3	+ ·9	4
45	·01159	+ 22	14·5	− 2·5	− 1·6	45
6	·01105	+ 16	15·6	− 5·4	− 7·0	6
7	·01072	− 14	16·9	+ 3·9	− 3·1	7
8	·01082	− 23	19·1	+ 5·1	+ 2·0	8
9	·01151	− 22	22·5	− 3·5	− 1·5	9
50	·01265	− 24	27·3	+ 5·3	+ 3·8	50
1	·01401	− 8	33·4	− 3·6	+ ·2	1
2	·01537	+ 13	41·0	− 2·0	− 1·8	2
3	·01649	+ 15	48·0	− 3·0	− 4·8	3
4	·01729	+ 12	57·6	− 1·4	− 6·2	4
55	·01790	+ 14	65·9	+ 8·9	+ 2·7	55
6	·01847	− 2	75·8	− 7·2	− 4·5	6
7	·01912	− 9	85·5	− ·5	− 5·0	7
8	·01999	+ 4	97·7	+ 7·7	+ 2·7	8
9	·02106	+ 7	111·2	−11·8	− 9·1	9
60	·02224	+ 6	125·5	−12·5	−21·6	60
1	·02357	− 2	142·3	+26·3	+ 4·7	1
2	·02512	...	161·3	+ 4·3	+ 9·0	2
3	·02695	+ 2	182·2	+ ·2	+ 9·2	3
4	·02904	+ 4	210·5	+ 1·5	+10·7	4
65	·03139	+ 4	238·5	−19·5	− 8·8	65
6	·03402	+ 2	267·5	+13·5	+ 4·7	6
7	·03697	+ 1	298·0	−19·0	−14·3	7
8	·04028	+ 3	330·2	−22·8	−37·1	8
9	·04397	+ 5	365·3	+22·3	−14·8	9

TABLE 10—*continued.*

Government Female Annuitants (1883) Ultimate Table.

Construction $C^{(a)}$.

By the new method from the Exposed to Risk and the Deaths. Formula (va) duplicated. Graduated values of w_x *at the points* 21, 26, 31, *&c. Graduated values of* q_x *at the points* 28, 33, 38, *&c.*

Age	q_x	$\Delta^3 q_x \times 10^5$	Expected Deaths	Deviation	Accumulated Deviation	Age
70	·04805	+ 4	402·3	+ 6·3	− 8·5	70
1	·05255	+ 10	435·7	+ 34·7	+ 26·2	1
2	·05752	+ 7	469·1	+ 37·1	+ 63·3	2
3	·06300	− 7	498·0	− 20·0	+ 43·3	3
4	·06909	− 4	524·3	− 7·7	+ 35·6	4
75	·07586	− 6	547·3	− 44·7	− 9·1	75
6	·08324	...	558·8	− 8·2	− 17·3	6
7	·09119	+ 11	565·8	+ 17·8	+ ·5	7
8	·09965	+ 11	568·4	+ 3·4	+ 3·9	8
9	·10862	+ 12	564·4	+ 2·4	+ 6·3	9
80	·11821	+ 12	551·3	+ 2·3	+ 8·6	80
1	·12853	+ 26	532·4	− 25·6	− 17·0	1
2	·13970	+ 7	499·6	+ 20·6	+ 3·6	2
3	·15184	− 12	466·1	− 14·9	− 11·3	3
4	·16521	− 13	425·4	+ 26·4	+ 15·1	4
85	·17988	− 21	368·0	− 18·0	− 2·9	85
6	·19573	− 75	333·9	− 8·1	− 11·0	6
7	·21263	+ 13	283·2	− 15·8	− 26·8	7
8	·23037	+ 121	234·5	+ 24·5	− 2·3	8
9	·24820	+ 145	195·6	− 12·4	− 14·7	9
90	·26625	+ 184	151·5	− 8·5	− 23·2	90
1	·28573	+ 109	111·7	− 3·3	− 26·5	1
2	·30809	− 8	81·6	+ 9·6	− 16·9	2
3	·33517	+ 36	63·3	+ ·3	− 16·6	3
4	·36806	+ 33	44·5	+ 5·5	− 11·1	4
95	·40668	+ 28	31·7	+ ·7	− 10·4	95
6	·45139	+ 64	21·2	+ 9·2	− 1·2	6
7	·50252	+ 127	17·1	− ·9	− 2·1	7
8	·56035	+ 208	9·0	...	− 2·1	8
9	·62552	+ 336	4·4	+ ·4	− 1·7	9
100	·69930	+ 514	2·1	+ 2·1	+ ·4	100
1	·78377	...	2·4	+ ·4	+ ·8	1
2	·88229	...	·9	+ ·9	+ 1·7	2
3	1·00000	...	...	...	...	3

TABLE 11.

Government Female Annuitants (1883) *Ultimate Table.*

Construction $C^{(b)}$.

By the new method from the Exposed to Risk and the Deaths. Formula (va) *duplicated. Graduated values of* w_x *at the points* 23, 28, 33, *&c. Graduated values of* q_x *at the points,* 30, 35, 40, *&c.*

Age	q_x	$\Delta^3 q_x \times 10^5$	Expected Deaths	Deviation	Accumulated Deviation	Age
30	·01043	+ 18	1·7	− ·3	− ·3	30
1	·01074	+ 10	2·1	+ 2·1	+ 1·8	1
2	·01081	+ 6	2·4	+ ·4	+ 2·2	2
3	·01082	+ 3	2·7	− ·3	+ 1·9	3
4	·01087	− 5	3·0	...	+ 1·9	4
35	·01102	− 6	3·5	− 1·5	+ ·4	35
6	·01130	− 10	4·1	− ·9	− ·5	6
7	·01166	− 8	4·9	+ 1·9	+ 1·4	7
8	·01204	− 8	5·7	− ·3	+ 1·1	8
9	·01234	+ 6	7·0	+ 5·0	+ 6·1	9
40	·01248	+ 12	8·1	− 4·9	+ 1·2	40
1	·01238	+ 12	9·1	+ ·1	+ 1·3	1
2	·01210	+ 12	10·1	− 1·9	− ·6	2
3	·01176	...	11·1	+ 2·1	+ 1·5	3
4	·01148	− 10	12·6	+ ·6	+ 2·1	4
45	·01138	+ 3	14·2	− 2·8	− ·7	45
6	·01146	+ 3	16·2	− 4·8	− 5·5	6
7	·01162	...	18·3	+ 5·3	− ·2	7
8	·01189	+ 9	20·9	+ 6·9	+ 6·7	8
9	·01230	− 2	24·1	− 1·9	+ 4·8	9
50	·01285	− 9	27·8	+ 5·8	+ 10·6	50
1	·01363	− 13	32·5	− 4·5	+ 6·1	1
2	·01462	− 8	39·0	− 4·0	+ 2·1	2
3	·01573	− 13	45·8	− 5·2	− 3·1	3
4	·01683	+ 7	56·0	− 3·0	− 6·1	4
55	·01784	+ 17	65·7	+ 8·7	+ 2·6	55
6	·01863	+ 15	76·5	− 6·5	− 3·9	6
7	·01927	+ 17	86·2	+ ·2	− 3·7	7
8	·01993	+ 5	97·4	+ 7·4	+ 3·7	8
9	·02076	− 10	109·6	− 13·4	− 9·7	9
60	·02193	− 5	123·8	− 14·2	− 23·9	60
1	·02349	− 5	141·8	+ 25·8	+ 1·9	1
2	·02534	− 7	162·7	+ 5·7	+ 7·6	2
3	·02743	− 10	185·5	+ 3·5	+ 11·1	3
4	·02971	+ 4	215·3	+ 6·3	+ 17·4	4
65	·03211	+ 19	244·0	− 14·0	+ 3·4	65
6	·03453	+ 21	271·5	+ 17·5	+ 20·9	6
7	·03701	+ 22	298·3	− 18·7	+ 2·2	7
8	·03974	+ 21	325·7	− 27·3	− 25·1	8
9	·04293	− 6	356·6	+ 13·6	− 11·5	9

TABLE 11—*continued.*

Government Female Annuitants (1883) *Ultimate Table.*

Construction $C^{(b)}$.

By the new method from the Exposed to Risk and the Deaths. Formula (va) *duplicated. Graduated values of* w_x *at the points* 23, 28, 33, *&c. Graduated values of* q_x *at the points* 30, 35, 40, *&c.*

Age	q_x	$\Delta^3 q_x \times 10^5$	Expected Deaths	Deviation	Accumulated Deviation	Age
70	·04680	− 17	391·8	− 4·2	−15·7	70
1	·05156	− 20	427·5	+26·5	+10·8	1
2	·05715	− 23	468·1	+36·1	+46·9	2
3	·06340	− 10	501·2	−16·8	+30·1	3
4	·07011	+ 22	532·0	...	+30·1	4
75	·07706	+ 13	555·9	−36·1	− 6·0	75
6	·08415	+ 17	564·9	− 2·1	− 8·1	6
7	·09160	+ 19	568·4	+20·4	+12·3	7
8	·09954	+ 14	567·8	+ 2·8	+15·1	8
9	·10814	...	561·9	− ·1	+15·0	9
80	·11759	...	548·4	− ·6	+14·4	80
1	·12803	+ 1	530·3	−27·7	−13·3	1
2	·13946	+ 1	498·7	+19·7	+ 6·4	2
3	·15188	+ 24	466·3	−14·7	− 8·3	3
4	·16530	+ 28	425·6	+26·6	+18·3	4
85	·17973	− 22	385·7	−18·3	...	85
6	·19551	− 38	333·5	− 8·5	− 8·5	6
7	·21282	− 43	283·5	−15·5	−24·0	7
8	·23144	− 119	235·6	+25·6	+ 1·6	8
9	·25099	+ 13	197·8	−10·2	− 8·6	9
90	·27104	+ 171	154·2	− 5·8	−14·4	90
1	·29040	+ 206	113·5	− 1·5	−15·9	1
2	·30920	+ 254	81·9	+ 9·9	− 6·0	2
3	·32915	− 7	62·2	− ·8	− 6·8	3
4	·35231	+ 49	42·6	+ 3·6	− 3·2	4
95	.38122	+ 117	29·7	− 1·3	− 4·5	95
6	·41581	+ 218	19·5	+ 7·5	+ 3·0	6
7	·45657	+ 358	15·5	− 2·5	+ ·5	7
8	·50467	+ 572	8·1	− ·9	− ·4	8
9	·56229	+ 909	3·9	− ·1	− ·5	9
100	·63301	+1464	1·9	+ 1·9	+ 1·4	100
1	·72255	...	2·2	+ ·2	+ 1·6	1
2	·84000	...	·8	+ ·8	+ 2·4	2
3	1·00000	...	...	...	...	3

Table 12.

Government Female Annuitants (1883) Ultimate Table.

Comparison of the values of q_x, by various constructions.

Age	Spencer 21-Term Formula	New Method: Construction A	New Method: Construction $A^{(a)}$	New Method: Construction $C^{(a)}$	New Method: Construction $C^{(b)}$	Age
30	·01140	·01164	·01089	·01280	·01043	30
1	·01086	·00983	·01158	·01187	·01074	1
2	·01053	·00876	·01213	·01115	·01081	2
3	·01047	·00832	·01252	·01073	·01082	3
4	·01060	·00835	·01271	·01061	·01087	4
35	·01089	·00866	·01267	·01076	·01102	35
6	·01119	·00913	·01237	·01106	·01130	6
7	·01150	·00958	·01196	·01139	·01166	7
8	·01174	·01013	·01161	·01163	·01204	8
9	·01198	·01093	·01148	·01183	·01234	9
40	·01212	·01184	·01168	·01206	·01248	40
1	·01226	·01264	·01204	·01227	·01238	1
2	·01231	·01310	·01248	·01240	·01210	2
3	·01225	·01296	·01284	·01240	·01176	3
4	·01198	·01238	·01295	·01212	·01148	4
45	·01162	·01170	·01263	·01159	·01138	45
6	·01129	·01119	·01198	·01105	·01146	6
7	·01120	·01110	·01128	·01072	·01162	7
8	·01139	·01152	·01083	·01082	·01189	8
9	·01193	·01234	·01092	·01151	·01230	9
50	·01278	·01341	·01174	·01205	·01285	50
1	·01382	·01457	·01309	·01401	·01363	1
2	·01494	·01560	·01469	·01537	·01462	2
3	·01605	·01642	·01625	·01649	·01573	3
4	·01707	·01715	·01748	·01729	·01683	4
55	·01795	·01784	·01824	·01790	·01784	55
6	·01871	·01856	·01873	·01847	·01863	6
7	·01940	·01936	·01915	·01912	·01927	7
8	·02012	·02020	·01971	·01999	·01993	8
9	·02095	·02103	·02063	·02106	·02076	9
60	·02200	·02194	·02189	·02224	·02193	60
1	·02334	·02305	·02336	·02357	·02349	1
2	·02501	·02447	·02505	·02512	·02534	2
3	·02699	·02637	·02696	·02695	·02743	3
4	·02924	·02875	·02912	·02904	·02971	4
65	·03169	·03154	·03141	·03139	·03211	65
6	·03426	·03461	·03382	·03402	·03453	6
7	·03697	·03778	·03653	·03697	·03701	7
8	·03992	·04094	·03968	·04028	·03974	8
9	·04327	·04420	·04345	·04397	·04293	9

TABLE 12—*continued.*

Government Female Annuitants (1883) Ultimate Table.

Comparison of the values of q_x, by various constructions.

Age	Spencer 21-Term Formula	New Method				Age
		Construction A	Construction A^a	Construction C^a	Construction C^b	
70	·04719	·04770	·04786	·04805	·04680	70
1	·05177	·05162	·05280	·05255	·05156	1
2	·05706	·05620	·05823	·05752	·05715	2
3	·06300	·06173	·06412	·06300	·06340	3
4	·06955	·06826	·07042	·06909	·07011	4
75	·07654	·07567	·07703	·07586	·07706	75
6	·08393	·08373	·08397	·08324	·08415	6
7	·09160	·09207	·09141	·09119	·09160	7
8	·09977	·10043	·09951	·09965	·09954	8
9	·10851	·10898	·10843	·10862	·10814	9
80	·11806	·11798	·11794	·11821	·11759	80
1	·12845	·12776	·12793	·12853	·12803	1
2	·14011	·13881	·13876	·13970	·13946	2
3	·15294	·15157	·15076	·15184	·15188	3
4	·16707	·16605	·16429	·16521	·16530	4
85	·18216	·18197	·18007	·17988	·17973	85
6	·19832	·19893	·19787	·19573	·19551	6
7	·21506	·21627	·21660	·21263	·21282	7
8	·23246	·23286	·23518	·23037	·23144	8
9	·24968	·24874	·25253	·24820	·25099	9
90	·26641	·26537	·26797	·26625	·27104	90
1	·28240	·28460	·28170	·28573	·29040	1
2	·29760	·30887	·29481	·30809	·30920	2
3	·31416	·33975	·30927	·33517	·32915	3
4	·33444	·37710	·32798	·36806	·35231	4
95	·36103	·42138	·35456	·40668	·38122	95
6	·39676	·47289	·39368	·45139	·41581	6
7	·44474	·53174	·45085	·50252	·45657	7
8	·50839	·59890	·53243	·56035	·50467	8
9	·59144	·67619	·64571	·62552	·56229	9
100	·69800	·76630	·79881	·69930	·63301	100
1	·83256	·87277	1·00000	·78377	·72255	1
2	1·00000	1·00000	...	·88229	·84000	2
3	...	...	...	1·00000	1·00000	3

Table 13.

Government Female Annuitants (1883) Ultimate Table.

Comparison of the Deviations of the Expected Deaths from the Actual Deaths, by various Constructions.

Age	Actual Deaths	Deviation: Expected minus Actual Deaths					Age
		Spencer 21-Term Formula	New Method				
			Construction A	Construction A(a)	Construction C(a)	Construction C(b)	
30	2	− ·1	− ·1	− ·2	+ ·1	− ·3	30
1	...	+ 2·1	+ 1·9	+ 2·3	+ 2·3	+ 2·1	1
2	2	+ ·3	...	+ ·7	+ ·5	+ ·4	2
3	3	− ·4	− ·9	+ ·1	− ·3	− ·3	3
4	3	...	− ·7	+ ·6	...	...	4
35	5	− 1·5	− 2·2	− 1·0	− 1·6	− 1·5	35
6	5	− 1·0	− 1·7	− ·5	− 1·4	− ·9	6
7	3	+ 1·8	+ 1·0	+ 2·0	+ 1·7	+ 1·9	7
8	6	− ·4	− 1·2	− ·5	− 1·2	− ·3	8
9	2	+ 4·8	+ 4·2	+ 4·6	+ 3·6	+ 5·0	9
40	13	− 5·1	− 5·3	− 5·4	− 5·1	− 4·9	40
1	9	...	+ ·3	− ·2	...	+ ·1	1
2	12	− 1·7	− 1·1	− 1·6	− 1·7	− 1·9	2
3	9	+ 2·5	+ 3·2	+ 3·1	+ 2·7	+ 2·1	3
4	12	+ 1·1	+ 1·6	+ 2·2	+ 1·3	+ ·6	4
45	17	− 2·5	− 2·4	− 1·2	− 2·5	− 2·8	45
6	21	− 5·0	− 5·2	− 4·0	− 5·4	− 4·8	6
7	13	+ 4·7	+ 4·5	+ 4·8	+ 3·9	+ 5·3	7
8	14	+ 6·1	+ 6·3	+ 5·1	+ 5·1	+ 6·9	8
9	26	− 2·6	− 1·8	− 4·6	− 3·5	− 1·9	9
50	22	+ 5·6	+ 7·0	+ 3·4	+ 5·3	+ 5·8	50
1	37	− 4·0	− 2·2	− 5·8	− 3·6	− 4·5	1
2	43	− 3·1	− 1·4	− 3·8	− 2·0	− 4·0	2
3	51	− 4·3	− 2·9	− 3·7	− 3·0	− 5·2	3
4	59	− 2·2	− 1·9	− ·8	− 1·4	− 3·0	4
55	57	+ 9·1	+ 8·7	+ 10·2	+ 8·9	+ 8·7	55
6	83	− 6·2	− 6·8	− 6·1	− 7·2	− 6·5	6
7	86	+ ·8	+ ·6	− ·3	− ·5	+ ·2	7
8	90	+ 8·3	+ 8·7	+ 6·3	+ 7·7	+ 7·4	8
9	123	−12·4	−11·9	−14·1	−11·8	−13·4	9
60	138	−13·8	−14·2	−14·5	−12·5	−14·2	60
1	116	+24·9	+23·2	+25·0	+26·3	+25·8	1
2	157	+ 3·6	+ ·1	+ 3·9	+ 4·3	+ 5·7	2
3	182	+ ·5	− 3·7	+ ·3	+ ·2	+ 3·5	3
4	209	+ 2·9	− ·6	+ 2·0	+ 1·5	+ 6·3	4
65	258	−17·2	−18·3	−19·3	−19·5	−14·0	65
6	254	+15·4	+18·1	+11·9	+13·5	+17·5	6
7	317	−19·0	−12·5	−22·5	−19·0	−18·7	7
8	353	−25·8	−17·4	−27·7	−22·8	−27·3	8
9	343	+16·4	+24·2	+17·9	+22·3	+13·6	9

TABLE 13—*continued.*

Government Female Annuitants (1883) Ultimate Table.

Comparison of the Deviations of the Expected Deaths from the Actual Deaths, by various Constructions.

Age	Actual Deaths	Deviation: Expected minus Actual Deaths					Age
		Spencer 21-Term Formula	New Method				
			Construction A	Construction A²	Construction C⁴	Construction C⁵	
70	396	− ·9	− 3·3	− 4·7	− 6·3	− 4·2	70
1	401	−28·3	−27·0	−36·8	−34·7	−26·5	1
2	432	−33·4	−26·4	−42·9	−37·1	−36·1	2
3	518	−20·0	−30·0	−11·1	−21·0	−16·8	3
4	532	− 4·2	−14·0	− 2·3	− 7·7	..	4
75	592	−39·8	−46·1	−36·3	−44·7	−36·1	75
6	567	− 3·6	− 4·9	− 3·3	− 8·2	− 2·1	6
7	548	−20·4	−23·3	−19·2	−17·8	−20·4	7
8	565	− 4·1	− 7·9	− 2·6	− 3·4	− 2·8	8
9	562	− 1·8	− 4·3	− 1·4	− 2·4	− ·1	9
80	549	− 1·6	− 1·3	− 1·1	− 2·3	− ·6	80
1	558	−26·0	−28·8	−28·1	−25·6	−27·7	1
2	479	−22·0	−17·4	−17·2	−20·6	−19·7	2
3	481	−11·5	−15·7	−18·2	−14·9	−14·7	3
4	399	−31·2	−28·6	−24·0	−26·4	−26·6	4
85	404	−13·1	−13·5	−17·6	−18·0	−18·3	85
6	342	− 3·7	− 2·6	− 4·4	− 8·1	− 8·5	6
7	299	−12·5	−10·9	−10·5	−15·8	−15·5	7
8	210	−26·6	−27·1	−29·4	−24·5	−25·6	8
9	208	−11·2	−12·0	− 9·0	−12·4	−10·2	9
90	160	− 8·4	− 9·0	− 7·5	− 8·5	− 5·8	90
1	115	− 4·6	− 3·7	− 4·9	− 3·3	− 1·5	1
2	72	− 6·9	− 9·9	− 6·1	− 9·6	− 9·9	2
3	63	− 3·6	− 1·2	− 4·5	− ·3	− ·8	3
4	39	− 1·5	− 6·6	− ·7	− 5·5	− 3·6	4
95	31	− 2·8	− 1·9	− 3·3	− ·7	− 1·3	95
6	12	− 6·6	−10·2	− 6·5	− 9·2	− 7·5	6
7	18	− 2·9	− ·1	− 2·7	− ·9	− 2·5	7
8	9	− ·9	− ·6	− ·5	...	− ·9	8
9	4	− ·1	− 1·1	− ·5	− ·4	− ·1	9
100	...	− 2·1	− 2·3	− 2·3	− 2·1	− 1·9	100
1	2	− ·5	− ·6	− 1·0	− ·4	− ·2	1
2	..	− 1·0	− 1·0	...	− ·9	− ·8	2

TABLE 14.

Government Female Annuitants (1883) *Ultimate Table.*

Comparison, in Age Groups, of the Deviations of the Expected Deaths from the Actual Deaths, by various Constructions.

Age Group	Construction	Actual Deaths	Deviations		
			Positive	Negative	Total
30 to 49	A	177	23·0	22·6	45·6
	$A^{(a)}$	,,	25·5	19·2	44·7
	$C^{(a)}$	,,	21·2	22·7	43·9
	$C^{(b)}$	,,	24·4	19·6	44·0
	Spencer	,,	23·4	20·3	43·7
50 to 69	A	2,978	90·6	93·8	184·4
	$A^{(a)}$	,,	80·9	118·6	199·5
	$C^{(a)}$	,,	90·0	103·3	193·3
	$C^{(b)}$	,,	94·5	110·8	205·3
	Spencer	,,	87·5	108·0	195·5
70 to 90	A	9,202	166·6	187·5	354·1
	$A^{(a)}$	,,	181·6	146·0	327·6
	$C^{(a)}$	,,	175·5	183·9	359·4
	$C^{(b)}$	,,	157·7	160·6	318·3
	Spencer	,,	169·4	154·9	324·3
91 to end	A	365	34·4	4·8	39·2
	$A^{(a)}$	,,	17·1	15·9	33·0
	$C^{(a)}$	,,	29·1	4·2	33·3
	$C^{(b)}$	,,	23·9	7·1	31·0
	Spencer	,,	18·7	14·8	33·5
Whole Table	A	12,722	314·6	308·7	623·3
	$A^{(a)}$	,,	305·1	299·7	604·8
	$C^{(a)}$	,,	315·8	314·1	629·9
	$C^{(b)}$	,,	300·5	298·1	598·6
	Spencer	,,	299·0	298·0	597·0

On an Approximate Method of Valuation of Whole-Life Assurances, grouped according to attained ages, with allowance for Selection, on the basis of $O^{[M]}$ *Mortality. By* E. H. Brown, F.I.A., *of the Prudential Assurance Company, Limited.*

[Read before the Institute, 25 January 1909.]

THE presentation in November 1905, of three papers on "methods of valuation with allowance for selection" (*J.I.A.*, vol. xl, pp. 1, 15, 42) might well be considered to have left room for no further word on the subject. Any attempt to produce a new method merely to achieve an equal degree of accuracy would undoubtedly be presumptuous, for Mr. King's method affords absolute accuracy, Mr. Diver follows with a microscopic error of £9 on a reserve of more than £3,000,000, and Mr. Ackland closes the series with a larger but still relatively insignificant error of £1,183 on a total reserve of upwards of £13,500,000. It is also shown that in both Messrs. Ackland's and Diver's methods, the error may be expected to be always less than ·01 per-cent.

The practical application of all three methods involves, however, a departure from the familiar system of classifying whole-life assurances according to attained ages, with the loss of all the advantages attaching to that system. This will be viewed by many with regret, and was especially referred to by Mr. Lidstone and Mr. R. P. Hardy in the subsequent discussion. Their remarks, together with Mr. Ackland's cordial expression of goodwill to "other workers in this field" led me to consider the subject from the standpoint of a grouping according to attained ages without any sub-grouping according to duration.

The total select reserve at any one attained age may be analyzed as follows:

If we employ the symbol S_t to denote the sum assured on a group of lives at valuation age (y),* the assurances having been effected t years ago at age $(y-t)$, then the total select reserve on

* Note.—It has been found advisable to denote the attained age by y, in order to avoid confusion when referring to Messrs. Ackland's and Diver's papers, where x denotes the entry age.

all policies existing on lives of the same present age (y) may be represented by

$$[S_0\bar{A}_{[y]} + S_1\bar{A}_{[y-1]+1} + \ldots + S_9\bar{A}_{[y-9]+9} + (\sum_{t=10}^{t=y} S_t)\bar{A}_y]$$

$$-[S_0\pi_{[y]}\bar{a}_{[y]} + S_1\pi_{[y-1]}\bar{a}_{[y-1]+1} + \ldots + S_9\pi_{[y-9]}\bar{a}_{[y-9]+9}$$

$$+(\sum_{t=10}^{t=y} S_t\pi_{[y-t]})\bar{a}_y]$$

and equating this expression to

$$(\sum_{t=0}^{t=y} S_t)\bar{A}_{[y-t']+t'} - (\sum_{t=0}^{t=y} S_t\pi_{[y-t]})\bar{a}_{[y-t'']+t''} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

we can then find values for t' and t''. The values thus found will lie between 0 and 10, and will be the weighted average durations of the sums assured and premiums respectively. They will, of course, bear no relation to the exact average durations of the assurances.

The similarity of this problem to that so successfully solved by Mr. Lidstone is well marked, and it would be difficult, even if desirable, to depart from the lines of his investigation. The first step therefore will be to examine the form of the curve for $\bar{a}_{[y-t]+t}$ considered as a function of t.

The curve followed by $\bar{a}_{[y-t]+t}$ is obviously the same as the curve of $\phi_{[y-t]+t}$ (*J.I.A.*, vol. xl, p. 43), since

$$\bar{a}_{[y-t]+t} = \bar{a}_y + \phi_{[y-t]+t}$$

Mr. Ackland has provided tables of $\phi_{[x]+t}$ on the basis of the $O^{[M]}$ Table at $2\frac{1}{2}$ and 3 per-cent interest, and selected values of this function are arranged in the form of $\phi_{[y-t]+t}$ in Table I.

Values of the ratio $\left(\frac{\phi_{[y-t]+t}}{\phi_{[y-1]+1}}\right)$ are given in Table II.

This particular ratio has been selected as it affords a means of comparison with the similar table in Mr. Diver's paper, *J.I.A*, vol. xl, p. 16. It is interesting to note that by the use of attained ages instead of entry ages, the "constancy in the terms corresponding to each value of t greater than 1" disappears, and the ratio decreases as (y) increases for all values of t, thus confirming Mr. Diver's conclusion that "we might more closely

express the relation between the functions by an equation of the form

$$\phi_{[x]+t}=X_t\phi_{[x]+1}+Y_t$$

where X_t and Y_t are independent of x."

I venture to think that the feature noticed by Mr. Diver is accidental and possibly arises from the fact that the ratio

$$\frac{\phi_{[x]+t}}{\phi_{[x]+1}}=\frac{a_{[x]+t}-a_{x+t}}{a_{[x]+1}-a_{x+1}}$$

involves annuity-values at two different ages, whereas in the ratio

$$\frac{\phi_{[y-t]+t}}{\phi_{[y-1]+1}}=\frac{\bar{a}_{[y-t]+t}-\bar{a}_y}{\bar{a}_{[y-1]+1}-\bar{a}_y}$$

all the annuity-values are at the same present age and differ only in the matter of selection.

A consideration of the formula

$$l_{[x]+t}=l_{x+t}10^{-f_t-\beta c^x\psi_t}$$

will make the matter clearer, thus:

$$\frac{l_{[x]+t}-l_{x+t}}{l_{[x]+1}-l_{x+1}}=\frac{l_{x+t}\{10^{-f_t-\beta c^x\psi_t}-1\}}{l_{x+1}\{10^{-f_1-\beta c^x\psi_1}-1\}}\quad . \quad . \quad . \quad . \quad (2)$$

whereas

$$\frac{l_{[y-t]+t}-l_y}{l_{[y-1]+1}-l_y}=\frac{10^{-f_t-\beta c^{y-t}\psi_t}-1}{10^{-f_1-\beta c^{y-1}\psi_1}-1}\quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

It will be seen that formula (2) is influenced by the ratio $\left(\frac{l_{x+t}}{l_{x+1}}\right)$, while formula (3) is free from this disturbing element.

In the course of the present investigation it was noticed that the curve of $\phi_{[y-t]+t}$ bears a striking resemblance to the curve of f_t where $f_t=m(10-t)^2+m'(c')^t$ and is one of the functions employed in the graduation of the O$^{[M]}$ Table.

The values of this function are given in Table III, and are followed by the ratios $f_t\div f_1$. These ratios closely follow the ratios of $\phi_{[y-t]+t}\div\phi_{[y-1]+1}$, and similar results were observed when the divisor was replaced by f_0 and f_2 respectively.

The relation between f_t and $\phi_{[y-t]+t}$ may be more clearly demonstrated by the application of Mr. Ackland's formula

$$\phi_{[x]+t}=A_t+B_tc^x+C_tc^{2x} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

which may be written

$$\phi_{[y-t]+t}=A_t+B'_tc^y+C'_tc^{2y} \quad . \quad . \quad . \quad . \quad . \quad (5)$$

where $B'_t=B_tc^{-t}$ and $C'_t=C_tc^{-2t}$.

Values of the accented functions are given in Table IV, the values of A_t, B_t and C_t being already published in vol. xl at the foot of the tables of $\phi_{[x]+t}$ on pages 60 to 69.

Table V contains the values of the ratios of the accented constants to f_t. It will be seen that $A_t \div f_t$ is, for all practical purposes constant, $B'_t \div f_t$ and $C'_t \div f_t$ are not so regular, but do not vary to any great extent, and it is thought that the variation is sufficiently slight to justify the assumption that

$$\phi_{[y-t]+t}=f_t\{a+\beta c^y+\gamma c^{2y}\} \quad . \quad . \quad . \quad . \quad . \quad (6)$$

where a, β and γ are derived from the values given in Table V. Fairly close results were obtained by the use of the Arithmetic Means of $A_t \div f_t$ and $B'_t \div f_t$ respectively, together with one-half of the Arithmetic Mean of $C'_t \div f_t$, the values being

	a	β	$-\gamma$
$2\frac{1}{2}$ per-cent . .	56·470	·2910	·0001272
3 " . .	50·496	·3125	·0001526

In Table VI is given a comparison of the true values of $\phi_{[y-t]+t}$ with those derived from formulas (5) and (6) respectively.

The three sets of values appear to be somewhat divergent, thus, when $t=0$, the new values lie between the true values and Mr. Ackland's values up to age 40; between ages 40 and 55 they are lowest in value, and at about age 60 they exceed both the true and Mr. Ackland's values. The connection between these sets is, however, well marked. Mr. Ackland points out that the method adopted in the determination of the constants A_t, B_t and C_t is such that his values coincide with the true values at the initial and terminal ages, also that the sum of the two series is equal. Therefore his curve must cross the true curve at some intermediate point or points.

An examination of the tables in vol. xl, p. 60, *et seq.*, shows that his values are slightly in excess up to about age 40, and then drop below the true values until age 60.

The error introduced by the use of formula (6) instead of formula (5) depends on the individual errors caused by the use of $a\times f_t$, $\beta\times f_t$, and $\gamma\times f_t$ instead of A_t, B'_t and C'_t respectively, thus, if

$$E_1=a\times f_t-A_t$$

$$E_2=\beta\times f_t-B'_t$$

$$E_3=\gamma\times f_t-C'_t$$

then the total error at any given age and duration will be

$$E_1+E_2c^y+E_3c^{2y}$$

The special values adopted for A, B and C are such that for each of the first five years of duration, E_1 and E_3 are positive, while E_2 is negative, at both rates of interest. It therefore follows that the total error, which is negative at the youngest age, will increase numerically with the age until

$$\frac{d}{dy}\{E_1+E_2c^y+E_3c^{2y}\}=0$$

i.e., until

$$y=\frac{\log(-E_2)-\log E_3-\log 2}{\log c}$$

The error will then decrease numerically, passing through zero before

$$y=\frac{\log(-E_2)-\log E_3}{\log c}$$

i.e., between 7 and 8 years after reaching its maximum point.

After this point the error becomes positive, and increases, but Table VI shows that it does not attain any great magnitude, even at age 60, which may fairly be treated as the limiting age for new assurances.

The ages at which the maximum negative errors occur, are given in Table VII, together with the errors at those ages.

Since the curve of f_t is practically identical with that of $\phi_{[y-t]+t}$, we possess a system of weights which may be employed

in the determination of the weighted average duration of the sums assured and premiums respectively, in the same way as Mr. Lidstone's Z enables us to determine the average maturity age of a group of endowment assurances.

In its present form f_t is too cumbersome for practical use, but we may equally well employ the values of $\frac{1}{m}f_t$* where $m = \cdot 000{,}040{,}955$. These values are as follows:

t	0	1	2	3	4	5	6	7	8	9	10 and over
$\frac{1}{m}f_t = K_t$	127	87	66	49	36	25	16	9	4	1	0

Dealing with the sums assured under a group of policies at a given attained age (y) as in equation (1), if the sum assured be weighted as suggested, we have

$$\frac{S_0 \times K_0 + S_1 \times K_1 + \ldots + S_9 \times K_9 + (\sum_{t=10}^{t=y} S_t)(\text{zero})}{\sum_{t=0}^{t=y} S_t} = K_{t'}$$

Then, t' having been determined, correct to one place of decimals, by the inverse use of the table of $K_{t+\frac{1}{20}}$ (Table XVI), we may calculate the value of $\bar{A}_{[y-t']+t'}$.

The same course may be followed for the net premiums, and the values of $\bar{A}_{[y-t']+t'}$ and $\bar{a}_{[y-t'']y+t''}$ thus obtained may be used to determine an approximation to the liability. This may be termed Method A.

We may also employ t' for both sums assured and net premiums instead of t' and t'' respectively, calling this Method B.

An alternative method for the determination of the weighted average duration of the sums assured presents itself here, for if we divide the total weighted premiums by the total sum assured, the quotient thus obtained will be $\pi_{[y-t']}K_{t'}$, whence, by inverse entry in tables of $\pi_{[y-t]}K_t$, a different table being used for each attained age, we may determine the weighted average durations of the sums assured.

Thus the weighted premiums may be made to serve a double purpose, being used to ascertain the weighted average duration of

* As $f_t = \log_{10}k_{10} - \log_{10}k_t$, we may replace the symbol $\frac{1}{m}f_t$ by K_t.

both sums assured and premiums. This is practically the same as weighting the sums assured with $\frac{\pi_{[y-t]}}{\pi_{[y-9]}}K_t$ and the premiums with K_t and may be called Method C.

At 3 per-cent $\frac{\pi_{[y]}}{\pi_{[y-9]}}$ varies from 1·25 at age 20 to 1·47 at age 60, thus the curve of K_t is replaced by another curve having higher ordinates throughout, but gradually approaching the original curve as t increases.

The effect is to furnish an average duration for the sums assured, smaller than the true average.

The function K_t is here used as the weight per £1 assured; in practice it may be used as the weight per £100 assured, as in Table XV.

Practical Application of Method.

The choice of data on which to base a valuation, by which to measure the degree of approximation, presented some little difficulty. Mr. King's Model Office is not suitable as it gives the survivors at *all* ages, of entrants at quinary ages; whereas what is wanted is a table of the survivors at *quinary* ages, of entrants at all ages.

It was thought, however, that the "exposed to risk" given in the volume of Unadjusted Data of the O^M Experience would provide data which might be accepted as a substitute for the Model Office No. 2.

Table VIII gives a summary of the data employed. Assuming the sum assured under each policy to be £1, the total sum assured (at age 50, for example) was ascertained by taking the total of

$$E_{[50]} + E_{[49]+1} + E_{[48]+2} + \ldots + E_{[15]+35}$$

Similarly, the total premium was found by summing the products

$$E_{[50]}\pi_{[50]} + E_{[49]+1}\pi_{[49]} + \ldots + E_{[15]+35}\pi_{[15]}$$

In column 4 is given the age distribution per-cent of the policies in force, and in column 6 is given the similar information taken from Table 2 in Mr. King's paper on "Comparative Reserves" (*J.I.A.*, xxxvii, p. 458).

The agreement between the two columns is fairly close, and, it is thought, justifies the course adopted.

In Table IX is given a summary of a detailed valuation on the basis of $O^{[M]}$ Mortality with interest at 3 per-cent. The

values of the sums assured and net premiums are followed by their average durations, ascertained by dividing the value of the sum assured and net premiums by the sums assured and net premiums respectively, thus obtaining $\bar{A}_{[y-t']+t'}$ and $\bar{a}_{[y-t'']+t''}$ from which the values of t' and t'' were readily found.

The effect of weighting the sums assured with K_t and $\pi_{[y-t]}K_t$ and the net premiums with K_t, is shown in Table X. A comparison of column 3 in Table IX with columns 5 and 7 in in Table X, also of column 5 in Table IX with column 9 in Table X, shows that the curve of f_t closely represents the curve of $\bar{a}_{[y-t]+t}$ for all values of y, so far at least as this particular distribution of assurances is concerned.

As might be expected, the error which is practically non-existent up to age 30, increases until about age 55, and then diminishes.

Approximate valuations have been made according to Methods A, B and C, and, as affording an interesting comparison, Mr. Ackland's method has also been applied, the results being given under Method D.

Summaries of these valuations are given in Table XI.

The net liabilities obtained by Methods A and B are in excess of the true liability by about ·05 per-cent and ·17 per-cent. respectively.

Method C furnishes a net liability too small by about ·01 per-cent, a result very similar to that obtained by Method D.

In considering the practical application of Methods A, B and C, it is evident that method B, although the least accurate, is most easy to apply. It is certain to give a net liability in excess of the true liability, even when reversionary bonuses are included in the valuation.

It would serve no good purpose to increase the amount of tabular matter by tables of estimated reversionary bonus, but it may be of interest to state that the inclusion of simple reversionary bonuses at the rate of £1 per-cent per annum, declared quinquennially, reduced the errors under Methods A and B to ·01 per-cent and ·1 per-cent respectively and increased the negative error under Method C to ·04 per-cent.

Mr. Ackland's method has the great advantage of remaining practically unaffected by the inclusion or exclusion of reversionary bonuses.

An examination of Table XII, giving the details of the total error of £687 under Method B, shows that an increase in the

average entry age adversely affects the method and seems to demonstrate that a system of weights based on the value of $\phi_{[y-t]+t}$ at a higher age, say about 40, would furnish the means of effecting a closer approximation to the true liability.

It would, however, depend on the circumstances of any individual Office to determine whether or no such a course would be advisable.

Expected Claims.

It is customary to read in office reports that "The claims by death were well within those expected according to the table of mortality employed in the valuation", but it has been pointed out, I believe, by Mr. Manly, that such a statement conveys no more than an indication that the company in whose report it appears, is transacting an increasing business and is calculating the expected claims on the basis of an aggregate mortality table.

It is possible that such statements would be less frequently made if the expected claims were calculated on a select basis, although it is, of course, obvious that the mortality experience of some offices must be more favourable than the select mortality of the combined experience of the 60 offices which contributed to the data for the $O^{[M]}$ Table.

In the case of a valuation conducted on the lines of methods A and B, we have a ready means of obtaining a very close approximation to the expected claims. Thus, assuming the valuation to be effected as at 31 December, and that, on the average, the policies are issued on the 30 June, then the expected claims will be

$$\mu_{[y-\frac{1}{2}-t]+t}\frac{{}_0S_t+{}_1S_t}{2}$$

where ${}_0S_t$ and ${}_1S_t$ represent the sums assured at the beginning and end of the year respectively.

We have $$\mu_{[y-t]+t}=A_t+B_tc^y$$

also $$\mu_y=A_{10}+B_{10}c^y$$

whence $$c^y=\frac{\mu_y-A_{10}}{B_{10}}$$

therefore $$\mu_{[y-t]+t}=A_t-A_{10}\frac{B_t}{B_{10}}+\frac{B_t}{B_{10}}\mu_y$$

$$=A_t''+B_t''\mu_y$$

The successive values of A''_t and B''_t are given in Table XIII.

Again $$\mu_y - \mu_{[y-t]+t} = (1 - B''_t)\mu_y - A''_t$$

and it was found that

$$\Sigma(\mu_y - \mu_{[y-t]+t}) = G_y \mu_y \Sigma f_t$$

where G_y decreases as the age increases, the values of G_y being as follows:

At age 25	...	...	144.
,, 35	...	...	140.
,, 45	...	...	136.
,, 55	...	...	131.

If, therefore, the sums assured at each successive duration in any one group do not differ greatly from each other, it is possible to calculate the expected claims by the following formula:

$$\text{Expected claims} = \sum_{t=0}^{t=y} S_t \mu_{y-\frac{1}{2}} - mG_y \mu_{y-\frac{1}{2}} \sum_{t=0}^{t=9} K_t S_t$$

That is, first ascertain the expected claims according to the ultimate table. This will be too large and must be corrected by the deduction of the sum of total weights multiplied into a factor dependent on the age attained and independent of the duration.

In the practical application of this formula to the data given in Table VIII it was considered sufficient to use one value throughout for G_y, namely 140, so that $mG_y = \cdot 0057337$.

The expected claims, as estimated by the foregoing method, are given in Table XIV, as are also the true expected claims, based on O^M and $O^{[M]}$ mortality. It will doubtless be considered that the approximate values are sufficiently close to the true expected select claims for all practical purposes.

On the other hand, the expected claims based on O^M aggregate mortality, with an error in excess of $2\frac{3}{4}$ per-cent cannot be considered satisfactory, especially when it is remembered that the data with which we are concerned assumes a stationary rate of new business and is exclusive of reversionary bonus additions. It is therefore safe to assume that the error would be increased in the case of any representative office.

PREPARATION OF DATA FOR VALUATION.

As the method of weighting the sums assured or premiums is such that the weight varies with the duration, it follows that the same policy will not have the same weight at any two valuation periods.

If the period during which the weights are in operation were unlimited we should be faced with an insuperable difficulty, but the limitation of the period to ten years, renders it possible to devise means of adjustment that can be used without entailing too much additional labour.

It will be sufficient to describe the course to be adopted in the case of an annual valuation with a continuous system of classification by means of class books, combined with the card system, as all other systems are but modifications of this.

Whenever a policy is dealt with, either at time of issue or by discontinuance at any subsequent period, the appropriate value of (sums assured) $\times K_t$, *see* Table XV, must be recorded on the card (as the record is merely temporary it may be made in pencil and erased when the card has been dealt with).

Thus, if a policy were discontinued during the first year, the weight to be recorded would be (sum assured) $\times K_0$. If it were revived in the third year of duration, the weight would be (sum assured) $\times K_2$ and not the weight used at date of lapsing.

In order to record the weights in the class books it will be necessary to increase the columns in the existing valuation class books by one to be headed K_t, which will be used only when the particulars of a policy of less than ten years duration are being entered.

Also a new set of class books for weights will be required, one for "weights discontinued", and another for "weights issued and revived." The ruling of the pages as shown below will be the same in both books.

Class Book for Weights, 1909.

Discontinued by Lapsing, Surrender, or Death. *Attained Age......*

Date	Duration										Total Weights
	0	1	2	3	4	5	6	7	8	9	

The cards will first be dealt with in the usual way, the particulars being entered in the valuation class books. They

will then be sorted into two groups—durations 0 to 9, and durations 10 and over.

The group containing durations 0 to 9 will then be sorted according to attained ages, and each sub-group will be finally sorted according to durations, the total weight at each duration being entered in the correct column. These entries will then be cross-cast, the total being entered in the last column headed "total weights."

A check on the accuracy of the work is afforded by periodical comparisons of the totals in the weights class books with the corresponding totals in the valuation class books.

In the valuation the totals of the various columns in the valuation class books, including the column headed K_t, will be entered on the valuation schedules in the usual way, and K_t will be dealt with in exactly the same way as Mr. Lidstone's Z. This stage therefore calls for no further remark.

After the net liability has been ascertained and the heavy work of the valuation has been completed, there will be ample time to adjust the total weights on policies in force at 31 December.

It will be seen that if there were no new business, the total weights would diminish year by year, and would disappear after ten years, thus differing from the Z method, and Mr. Ackland's method, wherein the weights remain constant so long as they are in operation. The annual change can be readily effected by means of the class books for weights and Schedule A.

The first line of Schedule A will have been already copied from the last line of the corresponding Schedule A of the preceding year.

Line 2 will be entered from the class book for weights, "new business and revived."

Line 4 will be entered from the class book for "weights discontinued."

Line 5 contains the weights on policies in force at 31 December, and the entry in the line headed "total weights" must agree with the value of K_t on the "in force" valuation schedule.

The multiplying factors printed on line 6 are the successive values of $K_{t+1} \div K_t$.

These are employed as follows: The figures in line 5, duration 0, represent the (sums assured at duration 0) $\times K_0$; if therefore we divide this amount by K_0 we obtain the sums assured at duration 0, as at 31 December 1908, which become the

SCHEDULE A

Attained Age as at 31 December 1908 } 65

	Duration										
	0	1	2	3	4	5	6	7	8	9	Total Weights
(1) In force, as at 1 Jan. 1908, at Attained Age 65	...	38,106	33,792	30,674	21,876	26,550	17,120	10,107	5,204	1,504	187,933
(2) New Business and Revived, 1908	45,339	...	...	...	...	...	...	.	...	...	45,339
(3) Line (1) + Line (2)	45,339	38,106	33,792	30,674	21,876	26,550	17,120	10,107	5,204	1,504	233,272
(4) Discontinued, 1908	...	1,479	1,650	1,764	1,116	1,150	592	468	244	38	8,501
(5) Line (3) − Line (4). In Force, as at 31 December 1908	45,339	36,627	32,142	28,910	23,760	25,400	16,528	9,639	4,960	1,466	224,771
(6) Multiplying Factors	·685039	·758621	·742424	·734694	·694444	·64	·5625	·444444	·25	zero	...
(7) In force, as at 1 January 1909, to be transferred to 1909 Schedules as age 66		31,059	27,786	23,863	21,240	16,500	16,256	9,297	4,284	1,240	151,525

Total Weight Line (1)	...	187,933
" " Line (5)	...	224,771
Sum ...	...	442,704
		206,352 Mean Weighted Sums Assured.

sums assured at duration 1, as at 1 January 1909, and must therefore be weighted with K_1. It is, of course, unnecessary to perform these operations in detail. We therefore multiply any given weight by the ratio $K_{t+1} \div K_t$, placing the product under the heading of "duration $(t+1)$."

If the valuation were quinquennial instead of annual, the multiplying factor would be $K_{t+5} \div K_t$, the product would be placed under heading "duration $(t+5)$", and all weights of more than five years duration would be written off, in the same way as the weight for duration 9 is written off in the example given.

Line 7 of Schedule A will now be ready to be transferred to line 1 of the succeeding year's Schedule A, at the next higher age.

These schedules are used in determining the expected claims, for the mean values of the weighted sums assured is one-half of the sum of the "total weights" given on lines 1 and 5, as shown at foot of the specimen Schedule A.

The proposed method appears to possess the following advantages:

(1) It affords a means of comparison between the weighted average durations at the successive attained ages. It is also probable that the weighted average duration at any one attained age would not vary to any great extent from the corresponding duration at the preceding valuation. We therefore have a powerful check on the accuracy of the work during the intervaluation period.

(2) The actual work of the valuation is very little more than that of a valuation by an aggregate table.

(3) The weights are so few in number as to be readily recorded on the cards without reference to any table.

(4) It is not necessary to transfer all the particulars of each policy from one duration to the next duration each year. The weights alone requiring adjustment.

(5) The labour involved in the preparation of Schedule A is not heavy and it can be performed at any convenient time.

(6) The fact that the working of the method is arranged on a system of double entry is a sufficient guarantee of accuracy. If, by chance, a small error occurred, it would have but little effect on the average duration, and would be automatically reduced each year, finally disappearing at duration 10.

Table I.

Tables of $\Phi_{[y-t]-t}$. **2½ per-cent.**

y	Duration t: 0	1	2	3	4	5	6	7	8	9	y
30	·3098	·2167	.1623	·1214	·0882	·0611	·0387	·0217	·0097	·0026	30
40	·3496	·2486	·1855	·1376	·0988	·0670	·0422	·0233	·0103	·0025	40
50	·4411	·3211	·2384	·1745	·1233	·0823	·0510	·0274	·0119	·0029	50
60	·5977	·4433	·3294	·2392	·1676	·1115	·0688	·0368	·0161	·0041	60

3 per-cent.

y	Duration t: 0	1	2	3	4	5	6	7	8	9	y
30	·2841	·1989	·1492	·1116	·0815	·0561	·0359	·0204	·0092	·0022	30
40	·3242	·2310	·1724	·1276	·0918	·0627	·0392	·0216	·0096	·0021	40
50	·4144	·3022	·2246	·1645	·1164	·0775	·0485	·0264	·0118	·0026	50
60	·5683	·4219	·3136	·2281	·1598	·1062	·0652	·0352	·0151	·0036	60

Table II.

Tables of $\Phi_{[y-t]-t} \div \Phi_{[y-1]-1}$ **2½ per-cent.**

y	Duration: 0	1	2	3	4	5	6	7	8	9	y
30	1·43	1·	·749	·560	·407	·282	·179	·100	·045	·012	30
40	1·41	1·	·746	·553	·397	·270	·170	·094	·041	·010	40
50	1·37	1·	·742	·543	·384	·256	·159	·085	·037	·009	50
60	1·35	1·	·743	·540	·378	·252	·155	·083	·036	·009	60

3 per-cent.

y	Duration: 0	1	2	3	4	5	6	7	8	9	y
30	1·43	1·	·750	·561	·410	·282	·180	·103	·046	·011	30
40	1·40	1·	·746	·552	·397	·271	·170	·094	·042	·009	40
50	1·37	1·	·743	·544	·385	·256	·160	·087	·039	·009	50
60	1·35	1·	·743	·541	·379	·252	·155	·083	·036	·009	60

TABLE III.

t	$f_t = m(10-t)^2 + m'(c')^t$	$f_t \div f_1$
0	·0052155	1·450
1	·0035862	1·000
2	·0026856	·749
3	·0020223	·564
4	·0014781	·412
5	·0010248	·286
6	·0006555	·183
7	·0003686	·103
8	·0001638	·046
9	·0000410	·011

TABLE IV.

O[M] **$2\frac{1}{2}$ per-cent.**

t	A_t	B'_t	$-C'_t$
0	·2908480	·0017440055	·00000156058
1	·1990825	·0014698117	·00000163852
2	·1490889	·0010777886	·00000116957
3	·1127218	·0007383970	·00000074981
4	·0825622	·0004834054	·00000045558
5	·0570472	·0003047560	·00000027243
6	·0367592	·0001699290	·00000013284
7	·0209957	·0000806211	·00000005150
8	·0095568	·0000278998	·00000001081
9	·0024297	·0000052012	−·00000000007

O[M] **3 per-cent.**

t	A_t	B'_t	$-C'_t$
0	·2620055	·0018212956	·00000192572
1	·1786619	·0015152049	·00000183533
2	·1348070	·0010910731	·00000124567
3	·1020831	·0007499850	·00000079553
4	·0744858	·0005031847	·00000051243
5	·0521077	·0003068858	·00000027924
6	·0334942	·0001753931	·00000014595
7	·0188936	·0000871461	·00000006395
8	·0083601	·0000359022	·00000002508
9	·0020333	·0000087106	·00000000597

TABLE V.

t	$f_t = m(10-t)^2 + m'(c')^t$	$O^{[M]}$ $2\frac{1}{2}$ %			$O^{[M]}$ 3 %			t
		$A_t \div f_t$	$B'_t \div f_t$	$-C'_t \div f_t$	$A_t \div f_t$	$B'_t \div f_t$	$-C'_t \div f_t$	
0	·0052155	55·766	·3344	·0002992	50·236	·3492	·0003692	0
1	·0035862	55·513	·4099	·0004569	49·819	·4225	·0005118	1
2	·0026856	55·514	·4013	·0004355	50·196	·4063	·0004638	2
3	·0020223	55·739	·3651	·0003708	50·479	·3709	·0003934	3
4	·0014781	55·857	·3270	·0003082	50·393	·3404	·0003467	4
5	·0010248	55·667	·2974	·0002658	50·847	·2995	·0002725	5
6	·0006555	56·078	·2592	·0002027	51·097	·2676	·0002227	6
7	·0003686	56·961	·2187	·0001397	51·258	·2364	·0001735	7
8	·0001638	58·344	·1703	·0000669	51·038	·2192	·0001531	8
9	·0000410	59·261	·1269	−·0000017	49·593	·2125	·0001456	9

TABLE VI.

A comparison between (1) *the true values of* $\Phi_{[y-t]+t}$, (2) *those based on Mr. Ackland's formula* $A_t + B'_t c^y + C'_t c^{2y}$, (3) *those based on the proposed formula* $f_t\{\alpha + \beta c^y + \gamma c^{2y}\}$.

$O^{[M]}$ **3 per-cent.**

t	AGE 30			AGE 35			AGE 40			t
	(1)	(2)	(3)	(1)	(2)	(3)	(1)	(2)	(3)	
0	·284	·289	·287	·299	·303	·301	·324	·326	·322	0
1	·199	·201	·198	·211	·213	·207	·231	·231	·221	1
2	·149	·151	·148	·158	·159	·155	·172	·173	·166	2
3	·112	·113	·111	·117	·119	·117	·128	·128	·125	3
4	·082	·082	·081	·085	·086	·085	·092	·092	·091	4
5	·056	·057	·057	·059	·059	·059	·063	·063	·063	5
6	·036	·036	·036	·037	·038	·038	·039	·040	·040	6
7	·020	·020	·020	·021	·021	·021	·022	·022	·023	7
8	·009	·009	·009	·009	·009	·009	·010	·010	·010	8
9	·002	·002	·002	·003	·002	·002	·002	·002	·003	9

t	AGE 45			AGE 50			AGE 55			AGE 60			t
	(1)	(2)	(3)	(1)	(2)	(3)	(1)	(2)	(3)	(1)	(2)	(3)	
0	·362	·359	·356	·414	·409	·402	·484	·479	·475	·568	·568	·582	0
1	·261	·259	·245	·302	·299	·277	·356	·354	·327	·422	·422	·400	1
2	·194	·193	·183	·225	·222	·207	·265	·263	·245	·314	·314	·300	2
3	·144	·142	·138	·165	·163	·156	·192	·191	·184	·228	·228	·226	3
4	·102	·101	·101	·116	·115	·114	·136	·135	·135	·160	·160	·165	4
5	·069	·069	·070	·078	·077	·079	·090	·090	·093	·106	·106	·114	5
6	·043	·043	·045	·049	·048	·051	·056	·055	·060	·065	·065	·073	6
7	·024	·024	·025	·026	·026	·028	·030	·030	·034	·035	·035	·041	7
8	·011	·010	·011	·012	·011	·013	·013	·013	·015	·014	·015	·018	8
9	·003	·003	·003	·003	·003	·003	·003	·003	·004	·004	·004	·005	9

TABLE VII.

Showing the maximum difference between

$$f_t\{\alpha + \beta c^y + \gamma c^{2y}\} \text{ and } \{A_t + B'_t c^y + C'_t c^{2y}\}$$

Also the age at which the maximum error occurs.

t	2½ PER-CENT Age	2½ PER-CENT Difference	3 PER-CENT Age	3 PER-CENT Difference	t
0	53·9	−·01073	49·4	−·00675	0
1	57·9	−·03511	56·0	−·02778	1
2	57·8	−·02404	55·8	−·01816	2
3	56·0	−·00999	53·4	−·00711	3
4	51·3	−·00178	47·6	−·00134	4
5	35·2	+·00074	...	...	5
6	...	...	...	...	6
7	...	...	...	...	7
8	...	..	...	...	8
9	...	...	...	...	9

TABLE VIII.

DATA USED IN VALUATION				MR. KING'S MODEL OFFICE NO. 2	
Attained Age	Sums Assured	Net Premium	Distribution of Sums Assured	Attained Ages	Distribution of Sums Assured
(1)	(2)	(3)	(4)	(5)	(6)
20	14,124	187	·82	21–25	1·38
25	76,475	1,124	4·44	26–30	5·07
30	157,288	2,532	9·13	31–35	8·83
35	212,673	3,725	12·35	36–40	11·26
40	235,021	4,473	13·64	41–45	12·45
45	233,980	4,829	13·58	46–50	12·63
50	214,702	4,798	12·17	51–55	12·08
55	183,289	4,408	10·64	56–60	10·91
60	147,357	3,797	8·56	61–65	9·34
65	109,326	2,989	6·35	66–70	7·39
70	72,351	2,091	4·20	71–75	5·02
75	40,489	1,233	2·35	76–80	2·53
80	18,258	581	1·06	81–85	·88
85	5,849	196	} ·41	86 & over	·23
90	1,254	43			
Total	1,722,436	37,006	100·		100·

Table IX

Exact Valuation Summary.

$O^{[M]}$ 3 per-cent.

Attained Age (y)	Sums Assured: Value	Sums Assured: Average Duration	Net Premiums: Value	Net Premiums: Average Duration	Net Liability	Attained Age (y)
(1)	(2)	(3)	(4)	(5)	(6)	(7)
20	4,607	·9	4,266	·9	341	20
25	27,228	1·6	24,491	1·5	2,737	25
30	61,438	2·8	52,226	2·6	9,212	30
35	91,317	4·1	71,961	3·8	19,356	35
40	110,989	5·1	79,945	4·7	31,044	40
45	121,484	5 9	78,640	5·4	42,844	45
50	122,320	6·6	69,950	5·9	52,370	50
55	114,183	7·2	56,326	6·5	57,857	55
60	99,826	7·8	41,520	6·9	58,306	60
65	79,968	8·5	27,209	7·7	52,759	65
70	56,646	8·9	15,389	8·3	41,257	70
75	33,603	9·5	7,112	8·9	26,491	75
80	15,902	9·9	2,542	9·8	13,360	80
85	5,293	10·	631	10·	4,662	85
90	1,169	10·	101	10·	1,068	90
...	945,973	...	532,309	...	413,664	...

Table X.

Attained Age (y)	Weights: $\Sigma S_t K_t$	Weights: $\Sigma S_t \pi_{y-t} K_t$	Sums Assured: $\dfrac{\Sigma S_t K_t}{\frac{1}{10}\Sigma S_t} = 10K_t$	t	Sums Assured: $\dfrac{\Sigma S_t \pi_{y-t} K_t}{\frac{1}{1000}\Sigma S_t}$	t'	Premiums: $\dfrac{\Sigma S_t \pi_{y-t} K_t}{\frac{1}{10}\Sigma S_t \pi_{y-t}} = K_t$	t''	Attained Age (y)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
20	1,279,418	17,200	906	·9	1,218	·9	920	·9	20
25	5,725,254	86,236	749	1·6	1,128	1·5	767	1·5	25
30	8,228,230	140,420	523	2·8	893	2·7	555	2·6	30
35	7,282,852	142,922	342	4·2	672	4·0	384	3·8	35
40	5,411,273	124,473	230	5·2	530	5·0	278	4·7	40
45	3,649,850	99,553	156	6·1	425	5·8	206	5·5	45
50	2,278,645	75,311	106	6·8	351	6·5	157	6·0	50
55	1,234,735	49,195	67	7·5	268	7·2	112	6·7	55
60	622,462	30,738	42	8·0	209	7·7	81	7·2	60
65	224,771	13,566	21	8·6	124	8·5	45	7·9	65
70	71,691	5,615	10	9·0	78	8·8	27	8·4	70
75	14,594	1,447	4	9·6	36	9·5	12	8.9	75
80	632	79	...	10·	...	10·	...	10·	80
85	...	...	...	10·	...	10·	...	10·	85
90	...	...	...	10·	...	10·	...	10·	90

Table XI.

Showing the effect of using different methods of approximate Valuation.

Basis of Valuation O[M] 3 per-cent	Sums Assured		Net Premiums		Net Liability	
	Value	Error	Value	Error	Amount	Error
True Values	945,973	...	532,309	...	413,664	...
Method A	946,094	+121	532,215	− 94	413,879	+215
,, B	946,094	+121	531,743	−566	414,351	+687
,, C	945,841	−132	532,215	− 94	413,626	− 38
,, D	945,921	− 52	532,341	+ 32	413,580	− 84

Table XII.

Valuation Summary. Approximate Method B.

O[M] **3 per-cent.**

Attained Age (y)	Sums Assured			Net Premiums			Net Liability			Attained Age (y)
	Value	Error	Percentage of Error	Value	Error	Percentage of Error	Amount	Error	Percentage of Error	
20	4,606	− 1	·02	4,266	...	...	340	− 1	·29	20
25	27,230	+ 2	·01	24,485	− 6	·02	2,745	+ 8	·29	25
30	61,434	− 4	·01	52,212	− 14	·03	9,222	+ 10	·11	30
35	91,327	+ 10	·01	71,920	− 41	·06	19,407	+ 51	·26	35
40	111,000	+ 11	·01	79,881	− 64	·08	31,119	+ 75	·24	40
45	121,509	+ 25	·02	78,551	− 89	·11	42,958	+114	·27	45
50	122,348	+ 28	·02	69,846	−104	·15	52,502	+132	·25	50
55	114,207	+ 24	·02	56,234	− 92	·16	57,973	+116	·20	55
60	99,844	+ 18	·02	41,438	− 82	·20	58,406	+100	·17	60
65	79,974	+ 6	·01	27,165	− 44	·16	52,809	+ 50	·09	65
70	56,648	+ 2	...	15,366	− 23	·15	41,282	+ 25	·06	70
75	33,603	...	...	7,105	− 7	·10	26,498	+ 7	·03	75
80	15,902	...	...	2,542	...	...	13,360	...	...	80
85	5,293	...	...	631	...	...	4,662	...	...	85
90	1,169	...	...	101	...	...	1,068	...	...	90
...	946,094	+121	·01	531,743	−566	·11	414,351	+687	·17	...

Table XIII.

Showing the values of the constants in the relation $\mu_{[y-t]+t} = A_t'' + B_t''\mu_y$.

t	A_t''	B_t''	t
0	−·00256	·50075	0
1	−·00011	·58924	1
2	+·00029	·66624	2
3	+·00023	·73307	3
4	+·00011	·79084	4
5	+·00001	·84066	5
6	−·00005	·88348	6
7	−·00009	·92011	7
8	−·00008	·95131	8
9	−·00005	·97775	9

Table XIV.

Expected Claims.

Attained Age (y)	Approximate Method, $O^{[M]}$ Mortality				O^M Mortality		True Value. $O^{[M]}$ Mortality	Attained Age (y)
	$\Sigma S_t\mu_y$	${}_mG_y\mu_y\Sigma K^t$	Estimated Expected Claims	Error	$\Sigma S_t\mu_y$	Error	$\Sigma S_t\mu_{[y-t]+t}$	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	
20	94	49	45	+ 3	56	+ 14	42	20
25	534	229	305	+ 2	362	+ 59	303	25
30	1,184	355	829	− 3	919	+ 87	832	30
35	1,784	350	1,434	+ 1	1,542	+109	1,433	35
40	2,289	302	1,987	...	2,115	+128	1,987	40
45	2,773	248	2,525	− 2	2,649	+122	2,527	45
50	3,255	198	3,057	− 3	3,163	+103	3,060	50
55	3,728	144	3,584	− 6	3,666	+ 76	3,590	55
60	4,195	102	4,093	− 3	4,161	+ 65	4,096	60
65	4,503	53	4,450	− 1	4,505	+ 54	4,451	65
70	4,423	25	4,398	+ 1	4,450	+ 53	4,397	70
75	3,740	8	3,732	...	3,773	+ 41	3,732	75
80	2,580	1	2,579	...	2,606	+ 27	2,579	80
85	1,275	...	1,275	...	1,288	+ 13	1,275	85
90	424	...	424	...	429	+ 5	424	90
	36,781	2,064	34,717	−11	35,684	+956	34,728	

TABLE XV.

K_t 0[M]

t	Sum Assured										t
	100	200	300	400	500	600	700	800	900	1,000	
0	127	254	381	508	635	762	889	1,016	1,143	1,270	0
1	87	174	261	348	435	522	609	696	783	870	1
2	66	132	198	264	330	396	462	528	594	660	2
3	49	98	147	196	245	294	343	392	441	490	3
4	36	72	108	144	180	216	252	288	324	360	4
5	25	50	75	100	125	150	175	200	225	250	5
6	16	32	48	64	80	96	112	128	144	160	6
7	9	18	27	36	45	54	63	72	81	90	7
8	4	8	12	16	20	24	28	32	36	40	8
9	1	2	3	4	5	6	7	8	9	10	9

TABLE XVI.

VALUES OF $K_{t+\frac{1}{20}}$

For differences of tenths of a year of duration (corresponding to sum assured £1,000).

Integral part of Duration	Decimal part of Duration.										Integral part of Duration
	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	
0	1,250	1,210	1,170	1,130	1,090	1,050	1,010	970	930	890	0
1	860	839	818	797	776	755	734	713	692	671	1
2	652	635	618	601	584	567	550	533	516	499	2
3	484	471	458	445	432	419	406	393	380	367	3
4	355	344	333	322	311	300	289	278	267	256	4
5	246	237	228	219	210	201	192	183	174	165	5
6	157	150	143	136	129	122	115	108	101	94	6
7	88	83	78	73	68	63	58	53	48	43	7
8	39	36	33	30	27	24	21	18	15	12	8
9	10	9	8	7	6	5	4	3	2	1	9

ABSTRACT OF THE DISCUSSION.

MR. E. C. THOMAS, in opening the discussion, said that on 27 November 1905, three papers on the subject of Valuations by Select Tables were simultaneously presented to the Institute. Those three papers, with the ensuing discussion, constituted a very full literature of the subject, and one might have supposed

that its possibilities had been exhausted. The author, however, had introduced the members that evening to an entirely new and interesting phase of the question, and he was sure he was merely expressing the general opinion in saying that he had earned not merely the thanks of the meeting for his labours, and the special consideration always so generously accorded to new contributors, but their very hearty appreciation of his paper on its merits. His method in brief was as follows: by a system of weights applied to the sums assured, or premiums, or both, the average duration for policies of each attained age was deduced. The policies were scheduled according to present ages, the select net premiums being recorded, and the valuation then proceeded on ordinary lines. The factors used were those for the correct present age according to the Select Table, but the deduced average duration was employed for the purpose of fixing the period at which selection was assumed to have taken place. In carrying out his principles, the author had made an ingenious use of the function f_t, employed by Mr. G. F. Hardy in the graduation of the O^{M} Table. It had been found by the author that that function followed with remarkable fidelity the curve of $\phi_{[y-r]-t}$ when considered as a function of t. It would be remembered that $\phi_{[y-r]-t}$ was the function dealt with by Mr. Ackland and Mr. Diver as the difference between the ultimate and the select annuity values. Consequently the author had to deal with only ten values of the functions, and the labour was thereby reduced to a minimum.

Of the three previous papers, that of Mr. King stood in a class by itself. Those of Messrs. Ackland and Diver were, however, both based on a similar principle to that underlying the present one, and in any discussion of the latest method it was impossible to ignore the earlier ones. There were three fundamental points in which the author's method differed from that of Mr. Ackland, to mention one of the two authors in the same class. Firstly, that while Mr. Ackland made an exact valuation for all durations over ten years, and also for the main part of the liability for shorter durations, employing his approximate method only in assessing a comparatively small correction to the latter part of his valuation, the author boldly came forward with an approximate means of determining the whole liability. In the second place, Mr. Ackland used individual durations, but the average entry and present ages, while the author used true present ages, with approximate durations, and consequently approximate entry ages. In the third place, both used a similar device to Mr. Lidstone's, of a function progressing by a mathematical law to "weight" the amounts to be valued, but whereas Mr. Ackland's function, like Mr. Lidstone's, was constant for each policy, and was inserted on the card once for all, the author's function varied at each successive valuation. Thus stated, it would appear from theoretical considerations that the advantages were with the former method. In regard to the first point he had mentioned, the majority of actuaries would probably feel themselves on safer ground in confining their

approximate methods to a small correction than in applying them to the whole valuation. On the second point, there was no obviously apparent advantage either way, but as to the use of a constant corrective function as against a variable one, there would probably be no question that, other things being equal, the former was preferable. The author, however, claimed certain advantages for his method, and, if they proved to be real and practicable, the members would not let any *a priori* considerations founded on theory deter them from giving them full weight.

In the second paragraph of the paper the author said: "The practical application of all three methods involves, however, a departure from the familiar system of classifying whole-life assurances according to attained ages, with the loss of all the advantages attaching to that system." He thought that, on further consideration, the author would admit that in that statement he had not been altogether just to Messrs. Ackland and Diver. In both cases, in regard to the main liability, a mixed valuation was made, select premiums being valued by ultimate annuities. It was expressly stated by Mr. Ackland, and commented upon by more than one speaker in the discussion on his paper, that, in regard to the overwhelming proportion of the total liability, there need be no change whatever in the system of classification. It was admittedly a great advantage, particularly where an annual valuation was made, to have the policies scheduled according to attained age. A careful comparison, line by line, of the results of two successive valuations afforded an almost infallible means of detecting large errors, even after all the detail work had been finished. That power was just as great where the basis of valuation was a mixed one, as where it was homogeneous. The author, however, might certainly claim that his method not only retained all the checks associated with an ordinary valuation by aggregate tables, but provided an additional one, in the shape of a comparison of the deduced average durations.

No. 6 on the author's list of advantages concerned the means for ensuring the accurate working out of his formula of approximation, and, where such accuracy affected the whole valuation, it was a much more essential point than where it concerned merely a small correction. The author, therefore, did right to lay stress upon his admirable methods in that connection. Nos. 2, 3, 4 and 5 were all concerned with showing that the labour involved was not unduly great. In regard to that question, the author had been so successful in devising a convenient arrangement of the work that the *a priori* objection to a "variable weight" practically disappeared. By means of the "weights on" and the "weights off" books, combined with Schedule A, he showed that, when once the weight had been computed for an individual policy at the time of its observation, either as a new entrant or a discontinuant, they had no further concern with it, the ensuing adjustments being made only to the totals. That ingenious device at once lifted his proposal into the class of practical ideas. If, therefore, they admitted the validity of the author's method "B" of applying the principle, they

must acknowledge he had made out a good case, on the score of the time and labour involved. It was not quite clear, however, that in that respect there was any advantage over Mr. Ackland's method, because in the previous discussion Mr. Austin pointed out (*J.I.A.*, vol. xl, p. 88) that a separate "weights" book might equally well be used for Mr. Ackland's method, in which it was only necessary to transfer the totals from one duration to the next. It would appear, therefore, that if that suggestion were carried out, the labour involved by Mr. Ackland's method would probably be actually less than by the present one, to the extent of the sixty or seventy Schedules A that would be required by Mr. Brown, one for each attained age. It was true, as the author had pointed out, that those schedules could be worked up any time after the valuation was finished, and therefore the point was not of very much moment.

The question arose, however, could they admit method B? In order to determine that point it was necessary to consider the relative accuracy of the results. In that connection, one naturally had a good deal of sympathy with the remarks of Mr. Warner (*J.I.A.*, vol. xl, p. 91) in which he rather deprecated the idea of select valuations at all, in view of the wide discrepancy between theory and practice, in regard to interest, and premium loading. He thought the reason for the different attitude generally taken up in regard to mortality was, that whereas a margin of so much per-cent. interest, and so much loading reserved in excess of expenses, conveyed a very definite meaning, it was very difficult to say what might be the effect of employing an unsound mortality table. In view of the lessons of past history, when such tables as the Northampton and Carlisle were in vogue, there was a natural feeling in favour of using the most scientific mortality table obtainable. They had, however, not yet reached the stage of condemning an otherwise sound aggregate valuation, and as most valuations concealed a surplus from interest and premium loadings, far in excess of any possible difference on mortality, it was obvious that even a considerable error in assessing that mortality difference could be of no real consequence as a matter of figures. It was important, however, to bear in mind that, while they might be perfectly free within fairly wide limits to select the basis of valuation most suitable to the requirements of the particular case, once that basis had been chosen and published to the world, it became a point of honour to secure that the final results should represent as accurately as was reasonably possible the true liability according to that basis. From that point of view, although the figures themselves might not be of much moment, he could not consider the result shown by Method "B" as altogether satisfactory. On a fund of $13\frac{1}{2}$ millions, as valued by Mr. Ackland, it would mean an error of £23,000, and with other possible methods open he did not think one would be justified in accepting so considerable an inaccuracy.

The author gave two other methods of applying his formula. Method A consisted in deducing separate weighted average durations,

for sums assured and premiums, respectively. In Method C, the author first of all deduced the weighted average duration from the premiums, and then applied the total premium weights to deduce an average π, and from that an average duration applicable to the sums assured. Contrary to expectation, the latter method gave the closest results of the three, the reason being that whereas Method C did, in fact, produce a slightly larger error in the sums assured than Method A, it was in the opposite direction to that appertaining to the premiums, and so reduced the final error in the liability, while by Method A the two errors were cumulative. The question of using the same or different weights for sums assured and premiums respectively was a very interesting one, as applied to all those methods. Mr. Lidstone was in the happy position that, in regard to endowment assurances, the age had so little effect upon the premiums, where the term remained unaltered, that, for practical purposes, it might be considered as constant. An average age, therefore, deduced from the sum assured, was equally applicable to the premiums. That was a great advantage, because, as sums assured were nearly always multiples of 100, the appropriate weight could, in most cases, be very rapidly copied from a prepared table, whereas in regard to premiums it meant a multiplication process for each separate policy. When they were dealing with whole-life policies, the case was quite different, because the premiums varied very rapidly with the age: consequently, an average age, deduced solely from the sums assured, invariably understated the age applicable to the premiums. From his own observations, that understatement was, on the average, about two years, and that was apparently confirmed by Mr. Ackland (*J.I.A.*, vol. xl, p. 55). That gentleman got over the difficulty by using average instead of true premiums, a method which produced strikingly close results, and, at the same time, retained the great advantage of enabling him to base his weighted functions on sums assured alone. The author's method C was based upon a somewhat similar, although not strictly analogous, principle. Instead of using the average age, deduced from the sums assured, to determine an average premium, he used a weighted average premium, to determine the average age for the sum assured. Unfortunately, he was involved thereby in the very labour of computing his constants for the premiums which Mr. Ackland was at such pains to avoid. The author gave an ingenious method for deducing the expected claims, and the results were sufficiently close for practical purposes. Bearing in mind, however, that for the earlier years of assurance the μ_x varied so much more rapidly in a lateral than in a vertical direction, one might reasonably prefer to use an average age rather than an average duration for assessing the expected claims. It was not likely, however, that the author's figures would be criticized very severely on the score of accuracy.

Mr. Brown had presented the Institute with a cleverly constructed tool for doing certain work in a particular manner. Had they no other machinery adapted to the same end, there was no

question that it would come into general use for its particular purpose. Even with others to choose from, some would prefer the author's, if for no other reason, because the work was all brought together into one compartment, and no part was severed from all connection with the remainder. Personally, however, had he to choose between the author's method and, say, Mr. Ackland's, in the present state of his knowledge he would choose the latter, for the reason that, as compared with the author's method B, Mr. Ackland's method gave better results, with no more labour; while, as compared with method C, Mr. Ackland's method appeared to give practically as good results, with much less labour. Whatever might be the final judgment on the practical utility of the various methods, nothing could rob the author of the credit of having made a very valuable discovery, and of having worked up that discovery into a practical means of doing useful work.

MR. H. J. RIETSCHEL said that the author had endeavoured to obtain a formula showing the relationship subsisting between the $O^{[M]}$ select and ultimate annuity-values, which might be of service in lessening the labour of a valuation by the select tables of mortality. From the examples given, the difference between the select and ultimate annuity-values could, it appeared, be approximately expressed by a function dependent on the duration of the assurance only, multiplied into functions which depended solely on the age attained. By means of that formula they might still continue to group whole-life policies, as hitherto, according to years of birth, and, as the author said, by a system of weights (based on the function of the duration) they could ascertain the weighted average durations of the sums assured and premiums, respectively, in each "year of age" group, in the same way as Mr. Lidstone's Z enabled them to determine the maturity age of a group of endowment assurances. In order to record the totals of those constants, the author inserted them, as at date of issue, against each policy in his class books. He summed them according to years of issue, and multiplied them by the ratio of the constant for duration 0 to that for duration 1, in order to pass to the constants for duration 1, and so on. The discontinuances were deducted in the usual manner, care being taken to deduct the constants applicable to the appropriate year of duration. That classification of the constants involved, it seemed to him, a labour which was very little less than that required to classify the business according to years of birth, and subdivide the last ten years' new business contained in these groups according to years of issue. In addition to the classification of the constants, there was the work entailed in calculating the constants. Further, the select classification would enable the office to take account of the bonuses, of which the author's method did not permit.

On the whole, therefore, he thought that, in practice, it would be found that Mr. Brown's method did not avoid the labour involved by the numerous subdivisions required if a select valuation was to be made in the customary way. Even if the labour involved were considerably lessened, he would ask whether it would be advisable

to undertake any extra labour, so long as they adopted the net premium method of valuation. By that method, they valued by a lower rate of interest than was expected to be earned, and relied upon the excess interest realized to provide, not only the bonuses, but also part of the expenses of managing the business; and, if a very low rate of interest was adopted, the interest was looked to to provide the bonuses and all the expenses. Experience had shown that that method brought out the results they desired, and therefore they continued to employ it, not because it was theoretically defensible, but because it was an easy method which experience had proved produced the desired results. Under those circumstances, could it be said that the adoption of select tables of mortality in the valuation would bring them any nearer a scientific method of valuation? If not, the labour was not desirable. At present, the greater part of the new business of an office consisted of endowment assurances, and any method of valuation must therefore also apply to them. In order to show that the effects of selection could not be ignored in the case of endowment assurances, he had taken out the differences between the select and ultimate annuity-values for age at entry 30, term 30 years, and found that the differences were 73 per-cent of the whole-life differences for duration 0, 72 per-cent for duration 1, and so on. The author apparently had not endeavoured to apply his formula to the endowment assurance class.

On page 193 the author advocated the statement of the ratio of the actual to the expected amounts of the claims, calculated on the basis of select tables of mortality. Such a statement was bound to mislead the public, who would naturally consider the difference to constitute an actually realized profit, and for the actuary it had no meaning, unless account was taken of the amount at risk. Under those circumstances, he would deprecate the stating of the ratio of the actual and expected claims, and preferred the statement of the ratio of the actual to the expected number of deaths. For that purpose, the results shown by an aggregate table would probably be sufficiently accurate, as the ratios were not of any practical use.

MR. T. G. ACKLAND said that it was very interesting to notice the evolution of the subject of selection, as of other questions relating to life assurance. A few short years ago, the matter of selection, in its practical applications at any rate, was not thought very much about, because no tools were available for the purpose, but, no sooner were select tables placed in their hands, than it was sought, by more than one writer, to show how valuations could be made with allowance for selection. It was quite conceivable that, in the not very distant future, actuaries might find it their duty to calculate premiums with allowance for selection (which, indeed, most of them did now), to calculate the expected claims with allowance for selection, and to compute the policy liabilities with a similar allowance. The time might come when it would be thought as inappropriate to ignore the element of duration in the valuation of policies, as it was to ignore the age at entry

in the old days long since, when there was a uniform premium for all ages.

A very ingenious and practical paper had been read by the author, further developing the subject which had been dealt with already in three previous papers, of which he (Mr. Ackland) had the honour of submitting one, in 1905. He hardly agreed with Mr. Thomas that the author's method could not be regarded as an improvement on the method which he (Mr. Ackland) then ventured to suggest, and on that suggested by Mr. Diver. The author's method had the great advantage of classifying the facts according to the present ages, and, instead of having, as was necessary by his (Mr. Ackland's) method, to separate the cases classified for an ordinary ultimate valuation into their ten years of entry during the period of selection, he simply had to make an adjustment, in the course of his valuation, in reference to the average duration of the policies during that period. By looking at the matter as one of present age, he had given an interesting alternative method, and had eliminated the question of age at entry. Under the author's method, however, they had, at successive valuations, which might be annual or at longer intervals, to vary the constant, representing the duration, which was recorded on the card. On the other hand, according to the method which he (Mr. Ackland) suggested, the "weighting" had to continue throughout the whole duration of the policy, whereas under the author's method it expired with the period of selection.

The author had shown them how to conduct the practical side of the valuation, which he thought was rather omitted from some previous contributions on the subject, and, in the result, he obtained a singularly close approximation to the true reserves. He agreed with Mr. Thomas that Method B showed a somewhat large deviation from the facts, but, after all, if they got within £687 on a total of £414,000, a very infinitesimal percentage, they might consider they were near enough.

The application of the method to the calculation of the expected claims, appeared to him to be of great practical value. He thought, notwithstanding what Mr. Rietschel had said, that it was important that an actuary should at least have before him the effect of the expected claims, as computed on a select basis. The system of weights, applied independently by the author's method, appeared to be singularly efficacious, and brought out quite a close approximation to the expected claims.

With regard to the theory of the method of valuation, it was very curious to observe the power of the formulas which the President had deduced for the graduation of select tables, and how they could be looked at from different directions, and gave quite simple results in application. The author—he did not know by what process or by what happy chance—discovered the fact that the curve of $\phi_{[y-t]+t}$, representing the difference between the ultimate and the select annuity at a given present age, coincided practically

with the curve of f_t, which was one of the functions employed in the graduation of the select tables. If the table on page 190 was carefully studied, it would be seen that the "weights" read from right to left, were simply the squares of the natural numbers—1, 4, 9, 16, 25, 36, 49. As they approached more closely to the period of entry, where selection was most in evidence, Mr. Hardy introduced a further corrective term, which did not at all affect the later terms just referred to, but which affected the earlier ones somewhat materially, so that instead of 64, 81 and 100, they had 66, 87 and 127; but fundamentally, the method was based on weighting by a curve which was the squares of the natural numbers. He thought a little light would be thrown on that if reference were made to the formulas given on p. 77, vol xl, of the *Journal*, in a supplement to the paper which he read in 1905. Formulas 10 and 11 gave an approximate expression for the difference between the probabilities of living, according to the select table, and according to the ultimate table; and it would be seen that that expression was largely stated in terms of f_t and ψ_t, the arbitrary functions which regulated the values of A_t and B_t respectively in the formula

$$\mu_{[x]+t} = A_t + B_t c^{x+t}.$$

If it were further borne in mind that f_t and ψ_t were both based upon values of $(10-t)^2$, it was not surprising that the squares of the natural numbers formed the basis of the "weights" employed by Mr. Brown. It was, however, a singularly simple result, and one upon the discovery of which the author was heartily to be congratulated.

It would be interesting to investigate the effect of dealing similarly with the Non-participating Whole-Life Assurance $O^{[NM]}$ Table. He had only had time just to glance at the matter, but if he was not in error he thought the President, in getting out the formula of graduation for the non-participating table (*J.I.A.*, vol. xxxviii, p. 508) used a somewhat more complicated expression for the curve during the period of selection, based upon somewhat higher powers of the natural numbers. It would be interesting to find whether, in valuing by a non-participating table with allowance for selection, the weights employed by the author would apply, or in what way they would require to be modified. The question of endowment assurance had been raised. He did not quite gather whether Mr. Rietschel was speaking of figures deduced from endowment assurance data, or whole-life data, in comparing the annuities.

MR. RIETSCHEL said he was speaking with regard to the differences between the temporary annuities, and the whole-life annuities, based on the $O^{[M]}$ Table throughout.

MR. ACKLAND said it would be interesting to ascertain what the effect would be if endowment assurance data were actually used, but he was afraid that that data was so dominated by selec-

tion, as to be practically unserviceable for that purpose. It would be interesting, however, to discover how far that method was at all practicable, or worth following out in the case of endowment assurances.

The PRESIDENT, in proposing a vote of thanks to the author for his exceedingly interesting and able paper, said that the mere fact that the present was the fourth paper, within a comparatively short period of time, on the subject with which the author had dealt, showed it was a question of great importance to the members in practice, and was much thought about by many. In spite of the convenience of using a single table (and in many cases he was very much in favour of the idea of using a single table of valuation), and in spite of the fact that in a large number of cases it was sufficient for all practical purposes to use an aggregate table, there would always be a large number of strong offices to whom the use of a select table in the valuation was more or less necessary, in order that they might keep up the reserves to the high point to which they had been built. In view of that fact, and the difficulty attaching to all the details of the select valuation, it was important that actuaries should have some ready method of shortening the labour of those details, and arriving approximately at the results of the select valuation with something more approaching the amount of labour than attached to a valuation by a single table. The special feature of the paper was that the author had attempted to assimilate the process of a select valuation to that of an ordinary one-table valuation, working with attained ages, and he thought, on the whole, it must be said he had been successful in his aim.

It was very difficult to assess the relative values of the methods which they had before them. He did not suppose they would be able to do that justly until they had been actually used in practice, when, perhaps, in the actual work of valuations, the merits or demerits of the different methods would come out. Mr. Ackland had very handsomely admitted that, in some respects, the author's method was an advance on his own, or, at all events, presented some advantages which his did not. Mr. Thomas thought that in Method B, where the most labour was saved, the error in the attained result was rather greater than they ought to be content with. That, of course, was a matter of opinion. Probably some actuaries would think that an error of a little over one per mille was not of importance: others, perhaps, might sympathise with Mr. Thomas's view that, having stated that the valuation was based upon certain assumptions, they ought to keep as near as possible to the result of those assumptions. But it must be remembered that all their valuations contained a number of approximations, which met them at every turn. It was assumed, for instance, that the lives were at their nearest birthdays, and that the policies had been a certain number of integral years in force. That, to begin with, was an approximation which was a considerable degree from the truth in some cases, because the difference between

the value of a policy which had been six months in force, and another which had been eighteen months in force, both of which might be classed in the same group, was very considerable. When they had regard to those and numerous other approximations which must necessarily enter into their calculations, they were entitled to contend that if they could get within something like 0·1 per-cent of the true result it was near enough.

Mr. Ackland had more than forestalled him in his very lucid explanation why the f_t function with which Mr. Brown dealt was a sufficiently approximate representation of the Φ function which entered into his formula, because it arose from the form of the expression by which the tables were originally graduated. From a practical point of view, it might be worth the author's while to consider the fact that the policies in the different calendar years of assurance as they came before them, would, at the date of assurance, be in force for a fractional period of a year. For instance, they would have policies which had been issued during the preceding year, and which on the whole had been in force for half a-year, and certain policies issued in the calendar year next preceding, which had been in force on the average one and a-half years. That would affect the weights given in the paper, and it might perhaps be more convenient in practice to use the weights for the half years rather than for the integral durations.

The resolution of thanks was carried by acclamation.

Mr. BROWN, in reply, thanked the members most heartily for the kind way in which he had been received. He was quite sure that, when the paper and discussion came to be printed, it would be found that the centre of interest in the evening's proceedings had been considerably shifted, and that it would be found to reside principally in the discussion, although but few members had taken part in it. It was impossible to take up in detail all the points that had been raised; but, in reply to Mr. Thomas, he wished to disclaim all rivalry with previous methods. The paper originated in an attempt to follow out Mr. Lidstone's method of scientific weighting in group valuations, and he hoped it would not be long before the application of that method would be extended. He had a vision of valuing the whole-life assurances by the aggregate H^M Table, in one group. He did not know whether that was very fanciful. He thought they could at least be valued in three groups—up to age 40 by one average attained age; from 40 to 60; and 60 and over, using different weights for each group. Mr. Rietschel, rightly, he thought, took exception to the labour involved in Schedule A. The only defence he had to offer was that the valuation itself was very little longer than an ordinary aggregate table valuation. The additional labour could be spread over the intervening twelve months, or five years, as the case might be, and consequently would not affect the work of the valuation itself. He ventured to differ from Mr. Rietschel when he said that he thought the additional labour would be very similar to that of a detailed valuation.

He had followed Mr. Ackland's speech with exceeding interest, but would not presume to criticize it. Mr. Ackland, however, asked him whether he had applied the method to non-participating assurances? He had attempted to do so, but speedily retired defeated. For some reason, the graduation functions seemed to become so considerably involved that, by the time they arrived at the annuity-values, he had lost all touch with them. The curves at the successive attained ages seemed to him to bear no possible relation to each other at all; it would require the President to disentangle them.

Note on the application of Mr. E. H. Brown's method to Select Valuations by the British Offices Whole-Life Non-Participating $O^{[NM]}$ *Table.*

Mr. G. F. Hardy's general formula of relation for the graduation of Select Mortality Tables is

$$\log_{10} l_{[x]+t} = \log_{10} l_{x+t} - f_t - \beta c^x \psi_t$$

(*See* "Account of Principles and Methods" &c., pp. 135, 158).

The formula employed in the particular case of the $O^{[NM]}$ Table is

$$\log_{10} l_{[x]+t} = \log_{10} l_{x+t} - \text{M}m \int_{t=t}^{t=5} (5-t)^3 dt$$

$$- \text{MB}c^x \int_{t=t}^{t=5} c^t \left[a(5-t)^2 + b(5-t)^3\right] dt$$

(*See J.I.A.*, vol. xxxviii, p. 508.) *

Or, replacing B by its equivalent $\dfrac{\beta}{\text{M}} \cdot \dfrac{\log_e c}{c-1}$

$$= \log_{10} l_{x+t} - \text{M}m \int_{t=t}^{t=5} (5-t)^3 dt$$

$$- \beta \frac{\log_e c}{c-1} c^x \int_{t=t}^{t=5} c^t \left[a(5-t)^2 + b(5-t)^3\right] dt$$

* In formula 9 (*loc. cit.*) the limits for the integral are printed as 0 and *t*, instead of *t* and 5.

Thus we have, for the $O^{[NM]}$ Table,

$$f_t = \mathrm{M}m\int_{t=t}^{t=5}(5-t)^3 dt = \mathrm{M}m\frac{(5-t)^4}{4},$$

and
$$\psi_t = \frac{\log_e c}{c-1}\int_{t=t}^{t=5} c^t\left[a(5-t)^2 + b(5-t)^3\right]dt$$

$$= \frac{1}{c-1}\left[a\left\{\frac{2(c^5-c^t)}{(\log_e c)^2} - \frac{2c^t(5-t)}{\log_e c} - c^t(5-t)^2\right\}\right.$$

$$\left. + b\left\{\frac{6(c^5-c^t)}{(\log_e c)^3} - \frac{6c^t(5-t)}{(\log_e c)^2} - \frac{3c^t(5-t)^2}{\log_e c} - c^t(5-t)^3\right\}\right]$$

It will be seen that the first three powers of $(5-t)$ are involved in ψ_t, and the fourth power in f_t, and that the expressions are in this respect more complex than those employed in the graduation of the $O^{[M]}$ Table, where the values of both f_t and ψ_t are based upon $(10-t)^2$ only.

Dealing now with select valuations by the $O^{[NM]}$ Table, on the lines of the approximate methods developed by Mr. Lidstone (*J.I.A.*, vol. xxxiv, p. 61), and by Mr. E. H. Brown, in the paper preceding the present note, we may assume that $\phi_{[y-t]+t}$ is of the form $(\mathrm{A}+\mathrm{B}c^y+\mathrm{C}c^{2y})f''_t$, where A, B, and C vary at different rates of interest; and, having regard to the formulas for f_t and ψ_t deduced above, that f'_t, for values of t from 0 to 4 inclusive, is of the form

$$[\mathrm{D}+\mathrm{E}(5-t)+\mathrm{F}(5-t)^2+\mathrm{G}(5-t)^3+\mathrm{H}(5-t)^4]$$

for all rates of interest. By a simple application of the "method of moments" to values of $\phi_{[y-t]+t}$, for $y=25$, 30, . . . 60, 65, and $t=0$, 1, 2, 3, 4, the following numerical values of the constants D to H have been deduced: D $=-{\cdot}80502$; E $=1{\cdot}43266$; F $=-1{\cdot}15916$; G $=1{\cdot}70992$; H $=-{\cdot}04703$; whence values of f'_t have been computed as follows:

$$\begin{aligned} f'_0 &= 161{\cdot}727 \\ f'_1 &= 83{\cdot}775 \\ f'_2 &= 35{\cdot}419 \\ f'_3 &= 10{\cdot}351 \\ f'_4 &= 1{\cdot}131 \end{aligned}$$

and (taking 3 per-cent interest) the following values of A, B, and C, have been deduced:

A $={\cdot}00059122$; B $={\cdot}0000072756$; C $=-{\cdot}000000086668$.

The values of f'_t, given above, are approximately in the relation of the successive numbers, 143, 74, 31, 9, 1; and, by "weighting" the sums assured, in successive years during the period of selection, with these several numbers, we can deduce the average period of duration for a given body of data, according to the method suggested and illustrated by Mr. E. H. Brown.

Mr. Brown has kindly made, experimentally, an exact valuation, and an approximate valuation by his method, employing the above "weights", the valuations being by the $O^{[NM]}$ Table at 3 per-cent interest, and the partial data selected comprising all cases included in the O^{M} experience at the two attained ages of 35 and 55, assuming the sum assured under each policy to be £1. The following are the results of these experimental valuations:

Age attained 35.

Exact Valuation.

Duration	Sums Assured	Net Premiums	Value of Sums Assured	Value of Net Premiums	Net Liability
	£	£	£	£	£
0	17,790	393·515	7,794	7,482	312
1	17,195	368·489	7,568	6,981	587
2	17,239	358·226	7,607	6,773	834
3	17,220	347·155	7,607	6,558	1,049
4	17,172	336·056	7,589	6,346	1,243
0–4	86,616	1,803·441	38,165	34,140	4,025
5 & upwards	122,317	2,112·518	54,057	39,893	14,164
Totals	208,933	3,915·959	92,222	74,033	18,189

Approximate Valuation.

(A)	£	£	£	£	£
0–4	...	...	38,170	34,128	4,042
5 & upwards	...	...	54,057	39,893	14,164
Totals	...	...	92,227	74,021	18,206
Error	...	..	+5	−12	+17
(B)					
All durations	...	...	92,233	74,023	18,210
Error	...	...	+11	−10	+21

Age attained, 55.

Exact Valuation.

Duration	Sum Assured	Net Premiums	Value of Sums Assured	Value of Net Premiums	Net Liability
	£		£	£	£
0	2,467	115·900	1,546	1,465	81
1	2,660	119·647	1,674	1,501	173
2	2,816	121·370	1,778	1,514	264
3	3,138	129·662	1,985	1,613	372
4	3,288	130·336	2,081	1,619	462
0–4	14,369	616·915	9,064	7,712	1,352
5 & upwards	168,150	4,067·981	106,436	50,532	55,904
Totals	182,519	4,684·896	115,500	58,244	57,256

Approximate Valuation

(A)			£	£	£
0–4	...	...	9,060	7,715	1,345
5 & upwards		...	106,436	50,532	55,904
Totals	...	...	115,496	58,247	57,249
Error	...		−4	+3	−7
(B)					
All durations	...	...	115,491	58,257	57,234
Error	...		−9	+13	−22

In the approximate valuation marked (A), the average duration was deduced in respect of the period of selection only; in that marked (B), the average duration was deduced in respect of the whole of the assurances valued, whether in the select or ultimate period.

In the following tables, the approximate and true values of $\phi_{[y-t]+t}$, according to the $O^{[NM]}$ Table at 3 per-cent, are compared; also the values of $\phi_{[y-t]+t} \div \phi_{[y-0]+0}$ (at $2\frac{1}{2}$ and 3 per-cent), with $f'_t \div f'_0$.

$O^{[NM]}$ *Values of* $\phi_{[y-t]+t}$ **3 per-cent.**

y	$t=0$	$t=1$	$t=2$	$t=3$	$t=4$
25	·117	·052	·018	·004	·000
35	·130	·062	·024	·006	·000
45	·159	·082	·035	·010	·001
55	·219	·123	·056	·018	·003
65	·309	·183	·087	·029	·004

O[NM] *Values of* $f'_t(\mathrm{A}+\mathrm{B}c^y+\mathrm{C}c^{2y})$ **3 per-cent.**

25	·107	·055	·023	·006	·001
35	·122	·063	·027	·008	·001
45	·158	·081	·035	·010	·001
55	·232	·120	·051	·015	·002
65	·334	·173	·073	·021	·002

O[NM] *Values of* $\dfrac{\phi_{[y-t]+t}}{\phi_{[y-0]+0}}$ **2½ per-cent.**

25	1·000	·450	·163	·047	·008
35	1·000	·479	·193	·057	·007
45	1·000	·518	·224	·065	·006
55	1·000	·559	·258	·079	·009
65	1·000	·590	·283	·096	·016

O[NM] **3 per-cent.**

25	1·000	·444	·154	·034	·000
35	1·000	·477	·184	·046	·000
45	1·000	·516	·220	·063	·006
55	1·000	·562	·256	·082	·014
65	1·000	·592	·282	·094	·013

Values of $\dfrac{f'_t}{f'_0}$

...	1·000	·518	·219	·064	·007

Two remarks may be made on the above investigation:

(1) It will be observed that, at least in the case of the O[NM] Table, the curve of $\phi_{[y-t]+t}$ does not approximately follow that of the graduation function f_t, but rather approximately follows the curve of the particular functions of t entering into both f_t and ψ_t.

(2) If for y, the attained age, we substitute $(x+t)$, the formula $f'_t(\mathrm{A}+\mathrm{B}c^y+\mathrm{C}c^{2y})$ becomes

$$(f'_t\mathrm{A})+(f'_tc^t\mathrm{B})c^x+(f'_tc^{2t}\mathrm{C})c^{2x}$$

which is of the form

$$\mathrm{A}_t+\mathrm{B}_tc^x+\mathrm{C}_tc^{2x}$$

deduced, by the writer of this note (*J.I.A.*, vol. xl, p. 45) for application in the method there advocated for approximate valuations with allowance for selection.

T. G. A.

LEGAL NOTES.

By Arthur Rhys Barrand, F.I.A., *Barrister-at-Law.*

Income Tax on Annuities.

A CASE of considerable interest and importance to life assurance companies, dealing as it does with the question of their liability to income tax in respect of their annuities, is that of *The Lord Advocate* v. *The Edinburgh Life Assurance Company,* which was decided at Edinburgh a few months ago. The case has not, apparently, been reported as yet in any of the recognised Law Reports, and will not, in all probability, reach its final stage for some time to come; but it has been suggested that some account of it would be welcomed by the readers of these Notes, and it has therefore been decided to deal with it briefly at this stage. The following particulars, taken from the judgment of Lord Johnston in the Court of Session, give the material facts. The Company transacts life business only, including annuities, and is assessed to income tax on its interest, and not on its profits. In making the periodical payments to its annuitants from 5 April 1905, to 30 November 1907, the Company, in accordance with section 24 (3) of the Customs and Inland Revenue Act, 1888, deducted income tax in respect of the whole of such payments, but has not accounted for, or paid to the Crown, any portion of the amount so deducted. It was maintained, on behalf of the Crown, that under the provisions of the Act of 1888, the Company having, as they were bound to do, deducted the tax in question, must account to the Commissioners of Inland Revenue for the same. The Assurance Company, on the other hand, contended that the Crown's demand was an attempt to make them pay the same income tax twice over, and that the annuities in question having, in fact, been paid out of profits and gains already brought into charge, they were entitled to retain the whole of the tax deducted. This latter contention was based on the fact that the annuities are charged, indiscriminately with its other obligations, on its whole funds, and that the actual amount on which the Company was assessed to income tax was largely in excess of the amount paid in annuities, the figures for the preceding quinquennium showing £804,731 as interest and other income from invested funds as against £218,463 paid to annuitants. Had the Inland Revenue authorities elected to assess the Company upon its

profits and gains in the trading sense, it would, in the quinquennium referred to, have received tax upon £334,450 of net profits, together with the tax upon £218,463 of annuities, or upon £552,913 in all. It was also claimed on behalf of the Company that the matter in dispute was really covered by the decision in the case of London County Council *v.* Attorney-General [1901] A.C. 26, in which it was held that—as the London County Council paid the interest on its Consolidated Stock to the extent of £100,000 out of income from lands assessed to income tax under Schedule A, to the extent of £500,000 out of interest on the Council's loans to minor local authorities already taxed at its source, and only to the extent of £500,000 from rates—while it was entitled and bound to deduct income tax from all interest payable by it, it was entitled to retain that portion which corresponded to the proportion paid out of its income from land and from interest receivable, and was only bound to account to the Revenue for that portion which corresponded to the proportion paid out of rates.

London County Council v. Attorney-General.

Lord Johnston decided in favour of the assurance company, and, in doing so, said: "My short ground of judgment is that "the Crown claim is only made possible by their reversing their "election to treat the defenders as assessable, not on profits but "on income from invested funds. If they find the latter course "more profitable, they are entitled to take it. But if they do, "they must deal with the defenders consistently on that footing, "and cannot be allowed to try and make the best of both worlds "by assessing the defenders on one basis and calling for them to "account for this particular item on another . . . Looked at "from another point of view, it was recognised in the London "County Council case, *supra*, that it was the duty of the "managers of every commercial concern to follow the business-"like course of keeping down current obligations out of current "income. The Edinburgh Life Assurance Company was therefore "entitled—and, naturally, as a business concern, bound—to keep "down their current obligations for annuities by paying them out "of current income from invested funds. That income was "profit and gains in the wider sense of the 24th section of the "1888 Act; and having paid the tax, the Company were entitled "to treat their current annuities as paid out of their own current "interests and dividends from invested funds, and to deduct and "retain in a question, both with the annuitants and the Revenue.

"If this result altered what the Commissioners conceived it "profitable for the Crown to do, it was open to them to exercise "their option so as to meet the emergency, but not so as to tax "the same income twice. In some circumstances it is possible "that a similar distribution of the tax deducted from the "annuities might have to be made as was done in the London "County Council case, *supra*. But the figures in the present "case to which I have referred make it unnecessary to go into "that point."

Decision on Appeal.

Since the above Note was written, the case referred to has been heard on appeal, and the decision of Lord Johnston varied in an important respect. The Court has now decided that it is not competent for the assurance company to consider all annuity payments as made out of that portion of its income derived from interest, and already taxed; and to consider its other payments as made out of income from premiums not already taxed. It has laid down that enquiry must be made as to what proportion of the company's total income over the period in question was derived from premiums and what from interest; that the income tax deducted from annuity payments must be divided in the same proportion; and that only such portion of the tax as, on this basis, represents the proportion derived from interest may be retained by the company, the remainder having to be accounted for to the Inland Revenue authorities. A report of the case on appeal will be found in *The Scotsman*, for 10 March 1909.

Repayment of deposit made under Life Assurance Companies' Act.

The case of *In re Popular Life Assurance Company, Limited* [1909] 1 Ch. 80, is concerned with the question of the repayment of the deposit of £20,000 made under the Life Assurance Companies Act, 1870, and presents some interesting features. It took the form of an application by the United Provident Assurance Company, Limited, for the transfer or payment out of Court to them of a sum of £21,621. 11*s*. 4*d*. London County Stock, representing the deposit made by the Popular Life Assurance Company, Limited, in accordance with the provisions of section 3 of the Life Assurance Companies Act, 1870. The latter company had, by an agreement dated October 24 1906, arranged to sell and transfer all its business, policies and assets, except its

uncalled capital, to the former company in consideration of certain shares in that company; and in pursuance of this agreement the Popular Company was duly wound-up in accordance with special resolutions passed in December 1906. On March 12 1907, the agreements were sanctioned by the Court, and since that date all the property, goodwill and assets of the vendor company had been transferred to the petitioning company, with the exception of £8,000 in the hands of trustees and the £20,000 on deposit or the investments representing the same. On December 14 1907, an order was made by Parker, J., carrying over the deposited fund, now amounting to £21,621. 11*s.* 4*d.* to the ledger credit of the account of *ex parte* the United Provident Assurance Company, Limited, and that the petition should stand over until after the dissolution of the vendor company. On June 25 1908, a final meeting of the vendor company was held, when the liquidator made his report to the shareholders; and that report and his accounts were adopted and approved. Notice of this meeting and the liquidator's accounts were filed with the Registrar of Joint Stock Companies, on July 2 1908. and accordingly, on October 2 1908, the company became dissolved. The Popular Company had not accumulated any life assurance fund out of premiums, but the United Provident Company had a fund, accumulated out of premiums, amounting to about £70,000, and had also, on starting business, deposited £20,000 which was still in Court.

When the case came before Warrington, J., he ordered the deposit to be paid out to the United Provident Company, and in the course of his judgment, said: "It seems to me that in this "case, no claim having been made in the winding-up by any "policyholder, the winding-up having resulted in the dissolution, "and there, therefore, being no such body in existence as the "original contracting party, all contracts to which that body was "a party must of necessity have came to an end. . . . It seems to "me, therefore, that so far as that is concerned, the position of "the policyholders is exactly the same as if they had released the "Popular Life Assurance Company from its obligations, and not "merely as if the United Provident Assurance Company had "made itself liable to them. That being so, what is the position "with regard to this deposit?"

"The deposit was paid in under section 3 of the Act of 1870. "Now that section (which provides, *inter alia*, for the return of "the deposit when the life assurance fund, accumulated out of

" premiums, shall have amounted to £40,000), did not come into
" operation here because the Popular Life Assurance Company
" did not accumulate out of its premiums a sum of £40,000.
" The United Provident Assurance Company has made an
" accumulation out of its premiums exceeding £40,000. But
" that seems to me quite irrelevant. In my opinion I have
" nothing to do with that at all. I have to consider what is the
" title to this deposit now that all claims against the persons who
" originally made the deposit have come to an end. It seems to
" me that the result at law must be that the persons who made
" the deposit, the purposes for which it was made having come
" entirely to an end, must be entitled to have that deposit paid
" out under, I do not like to call it a resulting trust—it is not
" exactly a resulting trust—but under a principle similar to that
" which regulates the law relating to resulting trusts. That being
" so, that was a right—a right prospective, subject to the
" purposes for which the deposit was made—which belonged to
" the Popular Life Assurance Company. The Popular Life
" Assurance Company assigned that right with all its other assets
" to the United Provident Assurance Company, and that right
" has now passed to the United Provident Assurance Company. . .
" It is said that I cannot properly adopt that view because by
" doing so I shall upset a practice which was established eighteen

Ex parte Scottish Economic Life Assurance Society.

" years ago by Kay, J., in *ex parte* Scottish Economic
" Life Assurance Society (1890) 45 Ch. D. 220. In
" my opinion, with all respect to the argument which
" has been addressed to me, I do not find that Kay, J., laid
" down any such practice at all. The facts in that case were
" quite different to the facts in the present case. The Scottish
" Economic Life Assurance Society had made a deposit, they
" had not accumulated £40,000 of premiums, they sold their
" their business to the Scottish Metropolitan Life Assurance
" Company, and the latter company had accumulated more than
" a £40,000 life assurance fund. The Scottish Economic
" Life Assurance Society was in course of being wound-up, but
" was not dissolved. The petition there was presented by the
" Scottish Economic Life Assurance Society and by the Scottish
" Metropolitan Life Assurance Company, and what Kay, J., said
" in effect was that 'The purposes for which the deposit was
" 'made have not come to an end, because the company which
" 'made the deposit is still under liability to the policyholders
" 'for whose security the deposit was made. I cannot order the

" 'money to be paid out to them, because, where that is the case, " 'the only circumstance which authorizes me to make the pay- " 'ment, the accumulation of premiums, has not occurred; and " 'therefore I cannot make the payment to the Scottish Economic " 'Life Assurance Society at the present time'; and he went on " to say, in accordance with a suggestion which had been made " to him by counsel in the course of the argument, following a " suggestion of his own, that possibly they might be entitled to " have the money paid out if the Scottish Metropolitan Life " Assurance Society accumulated a further sum of £40,000 out " of its own premiums. . . . In the present case I think the " United Provident Assurance Company are entitled to have the " money paid out, and I do not see that I have any discretion in " the matter; the policyholders are now policyholders in the " United Provident Assurance Company and not in the Popular " Life Assurance Company, the purposes for which the deposit " was made have come to an end, and I think under those " circumstances I am bound to order the transfer. I accordingly " do so."

Suicide clause. Interest based on valuable consideration. The question as to what is sufficient to constitute an interest based on valuable consideration, within the meaning of the ordinary suicide clause in life policies, came before the Court recently in the case of *Wigan* v. *The English and Scottish Law Life Assurance Association* [1909] 1 Ch. 291. Here one Hackblock effected a policy on his own life with the defendants in 1902, subject, *inter alia,* to the following provision: "Policies will also be void and all premiums forfeited " if the lives assured die by their own hands, or by duelling, " or by the hands of justice, but without prejudice to the " *bonâ fide* interests of third parties based on valuable " consideration." In August 1906 Hackblock, who was indebted to Sir Frederick Wigan for an amount exceeding £15,000, was being pressed for payment in whole or part, and on 30 August 1906 he wrote to his solicitors, enclosing the policy, and saying "I enclose a £5,000 policy. Get it " assigned to Wigan or yourselves. Please get this done at " once. When I pay off I can get it reassigned to me . . . Wigan " need not know of the policy until it is repaid." The solicitors thereupon drew up an assignment by way of mortgage to Sir Frederick Wigan in ordinary form and reciting that the mortgagee had asked for further security, which was not the

case, although he or his firm had pressed for a reduction of the debt. Hackblock duly executed this mortgage, and sent it to his solicitors with a letter leaving it entirely to them to decide whether they should tell Sir Frederick Wigan of the deed or not. They arranged for further time without producing the assignment, and, shortly afterwards, at Hackblock's request, they destroyed it. No notice of the assignment was given to the assurance society or to Sir Frederick Wigan or his firm during Hackblock's lifetime, and the latter died by his own hand on 19 September 1906. His estate was administered in the Chancery Division, and the facts as to the assignment having come to the knowledge of the Master, he caused the plaintiffs, who were the executors of Sir Frederick Wigan, to be informed of them; and they thereupon gave notice of the assignment to the assurance association, and brought this action against it for payment of the policy moneys.

On behalf of the defendants it was contended (1) That in the circumstances, the deed was only an escrow, depending for its validity upon a condition which, as a matter of fact, was never fulfilled, and (2) That the plaintiffs were not assignees for valuable consideration within the meaning of the clause of the policy quoted above. Joyce, J., delivered judgment in favour of the assurance association, and in doing so, said: "Dealing first "with the second question, I am of opinion that there is no "possible ground for maintaining under the circumstances . . . "that Sir Frederick Wigan, the mortgagee, gave any consideration "at all for the interest which he had acquired, if he did acquire "any interest, under the deed in question. It appears to me to "be reasonably clear that the mere existence of a debt from A "to B is not sufficient valuable consideration for the giving of a "security from A to B to secure that debt. If such a security is "given, it may of course be given upon some express agreement "to give time for the payment of the debt, or to give consideration "for the security in some other way, or if there be no express "agreement the law may very readily imply an agreement to give "time . . . and further than that, if there is no express agreement, "and no agreement can be implied . . . yet if that security be "communicated to a person who could otherwise sue on the debt, "and on the strength of that security he does in fact forbear to "sue on the debt, he does give that time, with the object of "securing which the security is presumably given, and then, "I think, it appears on the cases that there is sufficient

"consideration, though in a sense it is an *ex post facto* "consideration, for the security which is given. On the other "hand it appears to me that where there is no communication "of the security, where there is no express agreement, and "where there are no circumstances from which the Courts "can imply any agreement, then there is no possibility of its "being said, with any justice, that any consideration has been "given at all . . ." It appears to me that in this case there is nothing which can be seized upon as valuable consideration for the interest which the plaintiffs have acquired, if they have acquired an interest, in this policy . . . Therefore I am bound to hold that the deed was purely voluntary, and that the interest which was acquired under it, if they did acquire any interest under it, by the persons in whose favour it was executed, was not an interest acquired for value. That being the case, I do not think it really necessary for me to give a decision on the other point, namely, whether the deed itself was executed upon a condition which was not fulfilled, and is merely an escrow, or whether it was executed unconditionally, to take effect in any event. I do not go into those questions because I am satisfied that in any event, even if I treat the deed as having been delivered unconditionally, the plaintiffs are not assigns for value within the meaning of the exception contained in the policy, and therefore the policy is, as between the insurance society and them, void under the condition. Under the circumstances therefore, this action must be dismissed with costs.

Disease caused by accident.

The case of *In re* an Arbitration between *Etherington* and *The Lancashire and Yorkshire Accident Insurance Company,* was concerned with the question as to whether a disease resulting more or less directly from an accident, could be described as an "accident" within the terms of a policy issued by the insurance company. The facts are given in an earlier portion of these notes (*J.I.A.*, xlii, 478), in a report of the case when it came before the High Court and was decided in favour of the representatives of the assured. The insurance company appealed against this decision, but on the case coming before the Court of Appeal, consisting of Lords Justices Vaughan Williams, Farwell and Kennedy, the appeal was dismissed unanimously, and the previous decision affirmed. A full report of the case will be found in *The Times* for 6 February 1909.

REVIEW.

*Supplement to the 65th Annual Report of the Registrar-General of Births, Deaths and Marriages in England and Wales.**

Although published in accordance with precedent as a Supplement to one of the annual reports issued from the General Register Office, Dr. Tatham's "Letter on the Mortality in England and Wales in the period of Ten Years 1891-1900" may be perhaps more naturally regarded as an Appendix to the General Report on the Census. The Report and the Letter together constitute the latest complete survey of the vital statistics of the country in their various aspects, the one dealing with the constitution and distribution of the population and the other with its mortality. Some of the main features of the Letter were commented on by the President in his address at the beginning of the Session, and one of the principal subjects of Part I, the construction of the new English Life Table, has been very fully discussed by Mr. King in his recent papers.

In the present notice it is proposed to refer more particularly to Part II of the Letter. With reference to Part I, however, it may be permissible to advocate the publication of a new volume of monetary tables based on English Life mortality. Whatever doubts may exist as to the reliability of a general population table at the older ages, there are many purposes for which such a table must almost necessarily be employed, and it can hardly be considered satisfactory that the only monetary tables at present generally available should be those published by Dr. Farr on the basis of English Life No. 3, a table relating to a remote generation, and vitiated, as Mr. King has conclusively shown, by a not unimportant error of constuction. Probably a certain number of monetary tables have already been privately constructed for special purposes on the basis of English Life No 6, and if these could be placed at the disposal of the Institute, they might form at least the nucleus of such a volume as that suggested. Even if nothing further were done officially it would be useful to know whether the official graduation or Mr. King's has been more generally employed in order that duplication of work might be avoided by the adoption of one graduation or the other for any tables that may be constucted in future.

One other point may be mentioned in connection with Part I. A very considerable section of this Part is occupied with a nosological analysis of the mortality of 1891-1900. Much of the information given under this head is of more practical interest to the medical officer of health than to the Actuary, but the mortality from phthisis has such an important bearing on life assurance business that special attention may be directed to the figures relating

* Part I (Cd. 2,618), published in June 1907; Part II (Cd. 2619), in June 1908.

to this cause of death. Dr. Tatham's investigation discloses a continued decline in phthisis mortality at all ages, and the improvement has been most marked between the ages of 25 and 45. This is of course eminently satisfactory from an insurance point of view, but the point to be particularly noticed is that the incidence of the improvement during the last half century has had the effect of steadily raising the age of maximum mortality from phthisis. In 1851-60 the age of maximum mortality was 20-25; in 1871-91 it had moved on to 35-45, and it has now further advanced to 45-55. It seems, therefore, that phthisis can no longer be regarded as mainly a risk of early life.

Part II is devoted entirely to Occupational Mortality. The subject is dealt with on substantially the same lines as in the preceding decennial supplement. That is to say, the mean annual death-rates, from various causes, of males in different occupations have been calculated for successive periods of age from 15 upwards from the deaths registered in 1900-2 and the populations returned in the 1901 Census, and the method of comparative mortality figures is employed as a general basis of comparison. One important change, however, has been made. In the 1901 Census the "retired" at the successive periods of age were, for the first time, classified according to their previous occupations instead of being grouped together. This rendered it possible to investigate the mortality of the "occupied and retired" for each occupation, and although the death-rates and mortality-figures are also given for the "occupied"—this being necessary for comparison with the results of previous investigations—the "occupied and retired" figures constitute the main result of the present investigation, whether as an absolute measure of occupational mortality or as a basis of comparison between different occupations. The extent to which the death-rates are increased by the change may be gathered from the following comparison of the "occupied" and "occupied and retired" rates for all males and for a few occupations:

Occupation	25–34		35–44		45–54		55–64	
	Occupied	Occupied & Retired	Occupied	Occupied & Retired	Occupied	Occupied & Retired	Occupied	Occupied & Retired
All Males	6·01	6·29	10·22	10·87	17·73	18·72	31·01	35·56
Schoolmaster, Teacher .	3·51	3·64	5·15	5·54	11·35	12·77	24·52	27·94
Commercial Clerk, and Insurance Service .	5·78	6·03	9·26	10·15	15·76	16·99	27·57	30·72
Agriculturist . .	3·96	4·15	5·70	6·02	10·13	10·72	19·76	22·06
Shopkeepers . . .	5·40	5·59	8·81	9·45	15·42	16·35	27·33	30·39
Miners	5·04	5·18	7·86	8·18	14·83	15·34	36·27	38·25

The adoption of the "occupied and retired" as the basis of investigation is, no doubt, to a certain extent open to criticism on

the ground that the statistics of the "retired" are not so reliable as those of the "occupied." The General Report on the 1901 Census states that the former callings of the "retired" are generally returned with less precision than are the callings of those still occupied, and this statement, the correctness of which would seem probable from general considerations, is borne out by the evidence of the following figures:

Description	Number of Deaths	Mean Annual Death Rates per 1,000						
		15–19	20–24	25–34	35–44	45–54	55–64	65–
Occupied Males	385,033	2·4	4·4	6·0	10·2	17·7	31·0	88·4
Retired . . .	106,368	79·7	90 6	96·5	94·8	68·3	91·5	148·0
Never Occupied	18,166	19·3	19·9	13·5	15·5	16·1	15·5	20·2
All Males . .	509,567	3·5	4·8	6·4	10·9	18·7	34·8	94·6

The heavy rates among the "never occupied" at the earlier ages may be accounted for by the deaths of the mentally or physically impaired who have been unable to enter any occupation or profession, but the extraordinarily low rates at the older ages are most improbable, and suggest that the proportion of persons returned as living on their own means instead of as retired from some occupation must have been considerably greater in the census than in the death registers. The pronounced discrepancies, however, occur at the youngest and oldest ages, and it seems probable that if the "occupied and retired" figures cannot as a whole be regarded as so reliable as those relating to the "occupied" they constitute an approximately accurate, or at any rate much improved, basis of investigation for the "main working period", *i.e.*, for ages 25-64, to which period the enquiry is practically confined.

It has been mentioned that the method of comparative mortality figures has been employed as a general basis of comparison. By this method, as applied in the present investigation, the mortality of an occupation for the main working period is represented by the number of deaths that would have occurred, according to the observed rates of mortality in that occupation, in a standard population of the same age-distribution (in the four age-periods 25-34, 35-44, 45-54 and 55-64) as the general male population returned at the census, and of such magnitude that the number of deaths according to the observed rates for "all males" would be 1,000. Thus the comparative mortality figure for gamekeepers—586—means that, according to the rates of mortality of gamekeepers in 1900-2, 586 deaths would have occurred in a year among 26,259 gamekeepers aged 25-34, 20,407 aged 35-44, 14,748 aged 45-54 and 9,591 aged 55-64, whereas the corresponding number of deaths among an equal and similarly distributed number of "all males" would have been 1,000.

The mortality figure is a simple and convenient standard of comparison, and it has the advantage of being subdivisible in proportion to the deaths from different causes, so that the mortality in different occupations from a given cause or group of causes may be readily compared. On the other hand it has a magnifying effect which makes differences appear at first sight more significant than they really are, and although it assigns greater weight to the mortality of early life than to that of later life (as an accidental result of the age-distribution of the standard population) it does not accurately represent their relative importance as regards the average duration of life. For some purposes the temporary expectation (which can be easily calculated with approximate accuracy from the mean death-rates for the four age-groups) might be more useful. The following table shows the comparative mortality figures and approximate expectations for a few occupations:

Occupation	Number of Deaths	Mortality figure	${}_{40}\mathring{e}_{25}$ (approximate)
Farmer, Grazier	6,159	596	35·0
Carpenter, Joiner	6,405	820	33·6
Shopkeeper	12,060	872	33·1
Coal Miner	13,128	885	33·4
Commercial Clerk, Insurance Service	6,504	911	32·7
Plumber, Painter, Glazier	6,884	1,114	31·7
Seaman, &c., Merchant Service	5,071	1,646	27·8
Innkeeper, Publican, Spirit, Wine, Beer Dealer	6,732	1,781	27·2

It will be noticed that a difference of over 200 between the mortality figures of the commercial clerk and the plumber, &c., corresponds to a difference of only about a year in the temporary expectation. The more accurate treatment of the incidence of mortality by the temporary expectation method of comparison does not materially affect the order of merit, and this would no doubt be the case generally as regards the total mortality in different occupations, since one occupation seldom if ever exhibits a materially higher mortality than another in an early age-period and a materially lower mortality in a later period. But as regards mortality from particular causes the case is somewhat different, since the incidence of mortality from different causes varies materially. For example, the mortality figure for accidents for all occupied and retired males is the same as that for bronchitis, viz., 58, but the mortality from accidents is much the more serious in its effect on the working lifetime of the population because its incidence is relatively much heavier at the earlier ages. Similarly the importance of the mortality from phthisis is relatively under represented even by its high mortality-figure of 187. In using mortality figures as a basis of comparison the numbers of deaths upon which they depend should be of course kept in view, and

it would be a convenience in any future table giving comparative mortality figures if a column were added showing the total deaths. In some of the minor occupations the numbers are so small as to render the resulting mortality figures of very little value; in the class of gamekeepers, for example, with a total of 324 deaths between 25 and 65, the mortality-figures for 13 out of the 24 causes of death depend on fewer than 10 deaths each.

A practical difficulty which may be noticed in connection with the mortality-figure method as hitherto applied is that for the purpose of comparing the results of successive investigations either the figures for the later investigations must be based on the standard population employed in the first investigation or on each occasion the mortality figures for all previous investigations must be recalculated on the new basis; the latter course has been followed in the 1900-2 investigation, but in a footnote it is stated that "with the further accumulation of records the increase of labour may eventually become so great as to require the adoption of a fixed standard of reference."

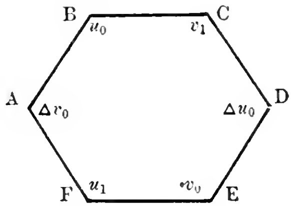
With regard to the scope of the investigation two points may be mentioned. In the first place it deals separately with certain specified occupations (or groups of like occupations) only. These comprise by far the largest part of the data, namely, 26,907,414 years of life out of 30,382,800 and 436,193 deaths out of 491,401, but they do not include (for example) auctioneers, house agents, officers of commercial companies, bankers and bank clerks, these, with a number of other small or ill-defined classes, being grouped as "other occupied males." Secondly, no distinction is made (except so far as the nature of the occupation involves such a distinction) between employers and employed—between persons engaged in administration or supervision and the manual worker; the class of "boot and shoe maker," for instance, comprises 11,380 employers, 143,681 employed, 42,104 "workers on own account" and 1,139 nondescripts, and 53,479 out of the total number were returned as working at home. The general result, consequently, seems to be that the investigation gives the occupational mortality of the principal trades and industries—blended in varying proportions (too small perhaps in most cases to materially affect the figures) with that of a class living under entirely different conditions—and also of certain professions, but that it throws no light on the mortality of "business men" except so far as "commercial clerks" may be regarded as representative of that important class. These limitations are unavoidable owing (mainly) to the unreliable character of the returns as regards status and to the numerous changes of status. None the less they must be borne in mind, and when it is remembered also that different industries (as Dr. Ogle pointed out) "do not start on equal terms as regards the vitality of those who follow them" and that many of those who have broken down physically in some strenuous occupation drift into and die in lighter occupations, it will probably be felt that the results of the investigation are of little value from the life

assurance point of view. They confirm, of course, the evidence from other sources as to the superior vitality of the clergy and the excess mortality of seafaring men and persons engaged in the liquor trade; they may even raise a doubt as to whether the baker ought to pay an extra and the hatter go scot-free, but they can hardly be regarded as affording a reliable basis for the assessment of extra premiums. The accident figures, however, may be regarded with more confidence, and would probably repay further study. It is sufficient to notice here the steady rise in the general rate of accident for all males with the age—from under one-half per 1,000 at ages 15-19 to 1·8 per 1,000 at ages 65 and upwards—and the wide differences for different occupations, the mortality figures varying from 266 for seamen and merchant service (as against 58 for all occupied males) to 9 for the clergy. The experience includes altogether 24,948 deaths from accident.

R. T.

ACTUARIAL NOTES.

I.—*On the Graphic Delineation of Interpolation Formulæ.* By DUNCAN C. FRASER, M.A., F.I.A., *Actuary to the Royal Insurance Company, Limited.*



1. IF the inclined lines in the figure represent the products of the quantities they connect, so that $AB = u_0 \times \Delta v_0$,

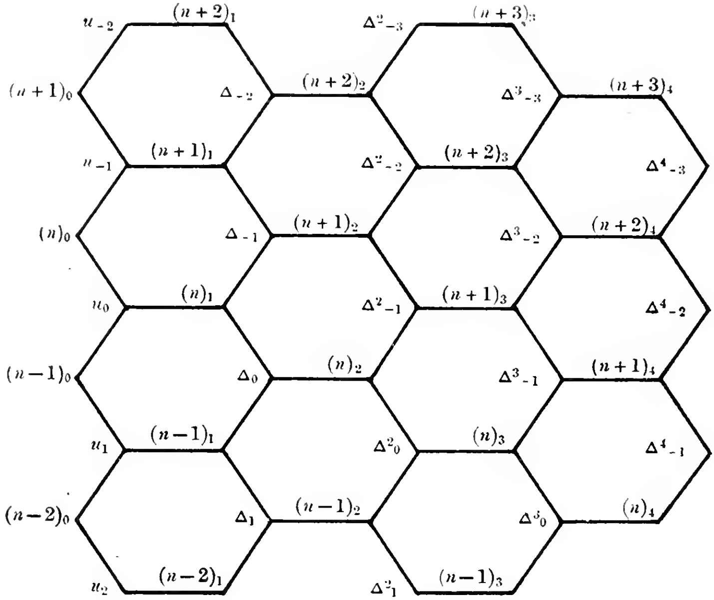
then $$AF - AB = (u_1 - u_0)\Delta v_0 = \Delta u_0(v_1 - v_0) = CD - ED$$

$$\therefore \quad AB + CD = AF + ED$$

Adopt the convention that a line taken rightwards gives the $+ve$ sign to the product, and leftwards the $-ve$ sign; so that $AB = u_0 \times \Delta v_0$, $BA = -u_0 \times \Delta v_0$.

then $$AB + CD + DE + FA = 0$$

Therefore, if we regard the horizontal lines as valueless, we may say that *the circuit of the cell is zero.* It will be noticed that the relative positions of v_0, v_1, Δv_0, are inverted and reversed as compared with the relative positions of u_0, u_1, Δu_0.



2. The 2nd figure is the ordinary scheme of differences with binomial coefficients inserted, the symbol $(n)_r$ being used for

$$\frac{n(n-1)(n-2)\ \ldots\ (n-r+1)}{\underline{|r}}$$

The coefficient $(n)_1$ is placed opposite u_0 and the law followed by the remaining coefficients is obvious.

The coefficients constitute a scheme of differences inverted and reversed—thus $(n+2)_4-(n+1)_4=(n+1)_3$—and each cell has the same properties as that in the first figure.

Consider any group of cells. The circuit of each cell is zero, and therefore the sum of the circuits of all the cells is zero. But the products represented by internal lines all cancel out, since each is taken once positively and once negatively. There remain only the external lines, and therefore *the circuit of any group of cells is zero.*

It follows that if we pass from any chosen coefficient, as $(n)_1$, to any chosen difference, as $\Delta^4 u_{-2}$, the result is identically the same, whatever route be taken. For example

$$(n)_1\Delta u_{-1}+(n+1)_2\Delta^2 u_{-1}+(n+1)_3\Delta^3 u_{-2}+(n+2)_4\Delta^4 u_{-2}$$

$$=(n)_1\Delta u_0+(n)_2\Delta^2 u_0+(n)_3\Delta^3 u_{-1}+(n+1)_4\Delta^4 u_{-2}$$

3. In the second figure suppose fourth differences to be constant. Then every route which extends right across the figure will give the same result. For example, the uppermost route

$$(n+1)_0u_{-2}+(n+2)_1\Delta u_{-2}+(n+2)_2\Delta^2u_{-3}+(n+3)_3\Delta^3u_{-3}+(n+3)_4\Delta^4u_{-3}$$

will give the same result as the lowermost route

$$(n-2)_0u_2+(n-2)_1\Delta u_1+(n-1)_2\Delta^2u_1+(n-1)_3\Delta^3u_0+(n)_4\Delta^4u_{-1}$$

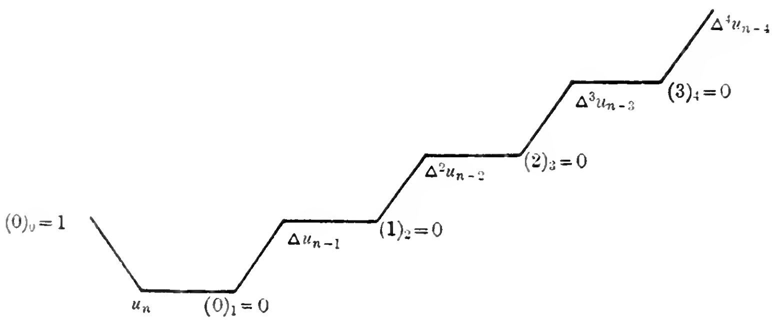
For, the circuit of the whole group of cells is zero. But the lines on the left of the figure between $(n+1)_0$ and $(n-2)_0$ cancel one another in pairs since $(n+1)_0u_{-1}-(n)_0u_{-1}=u_{-1}-u_{-1}=0$, &c. And the lines on the right of the figure between Δ^4u_{-3} and Δ^4u_{-1} cancel one another in pairs, since $(n+2)_4(\Delta^4u_{-3}-\Delta^4u_{-2})=0$, &c., fourth differences being constant.

There are left only the upper route and the lower route, taken in opposite senses and cancelling one another. The two routes taken in the same sense are therefore equivalent.

By adding cells to the figure, or taking cells from it, any route can be made the upper route or the lower route.

Thus every route across the diagram, however twisted or circuitous its course, if it ends at constant differences gives the same result.

4. Suppose n integral, and continue the diagram downwards until the term u_n is reached. The lower boundary of the figure will then be as follows—



Taking now the lowest route across the figure, the coefficient of u_n is unity and the coefficient of every other term is zero, so that the result is simply u_n.

But every route across the diagram gives the same result, and therefore every route gives a formula for u_n.

Taking the inclined lines as representing the sign of multiplication and the horizontal lines the sign of addition, the second diagram is a compendium of formulas for u_n.

5. Formulas which proceed to constant differences are exact, and are true for all values of n whether integral or fractional.

Formulas which stop short of constant differences are approximations.

Approximate formulas which terminate with the same difference are identically equal.

Approximate formulas which terminate with distinct differences of the same order are not identical. The difference between them is expressed by the chain of lines necessary to complete the circuit. Thus,

$$(\text{Any route ending at } \Delta^3 u_0) - (\text{Any route ending at } \Delta^3 u_{-3})$$
$$= (n+2)_3(\Delta^3 u_{-2} - \Delta^3 u_{-3}) + (n+1)_3(\Delta^3 u_{-1} - \Delta^3 u_{-2}) + (n)_3(\Delta^3 u_0 - \Delta^3 u_{-1})$$
$$= (n+2)_3 \Delta^4 u_{-3} + (n+1)_3 \Delta^4 u_{-2} + (n)_3 \Delta^4 u_{-1}$$

= a series of horizontal diagonals.

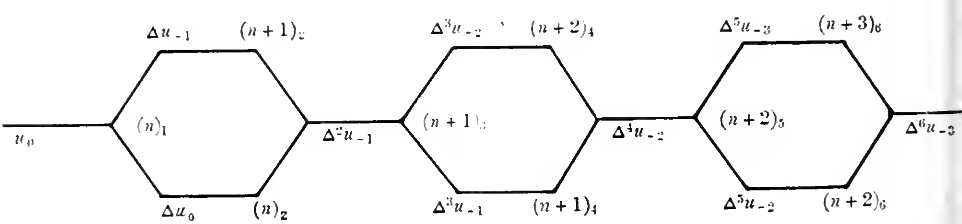
6. *Example of Formulas.*—The route through u_0 and its differences gives the familiar fundamental formula

$$u_n = u_0 + (n)_1 \Delta u_0 + (n)_2 \Delta^2 u_0 + (n)_3 \Delta^3 u_0 + \&c. \quad . \quad . \quad (\mathrm{I})$$

The corresponding route upwards to the right gives Everett's formula for ascending differences

$$u_n = u_0 + (n)_1 \Delta u_{-1} + (n+1)_2 \Delta^2 u_{-2} + (n+2)_3 \Delta^3 u_{-3} + \&c. \ . \ . \ . \ (\mathrm{II})$$

7. *Stirling's Formula*—



This figure includes only the cells horizontally on the right of u_0. Take the upper route and the lower route, and then the mean. Therefore

$$u_n = u_0 + (n)_1 \frac{\Delta u_{-1} + \Delta u_0}{2} + \frac{(n+1)_2 + (n)_2}{2} \Delta^2 u_{-1}$$
$$+ (n+1)_3 \frac{\Delta^3 u_{-2} + \Delta^3 u_{-1}}{2} + \frac{(n+2)_4 + (n+1)_4}{2} \Delta^4 u_{-2} + \&c. \ . \ . \ . \ (\mathrm{III})$$

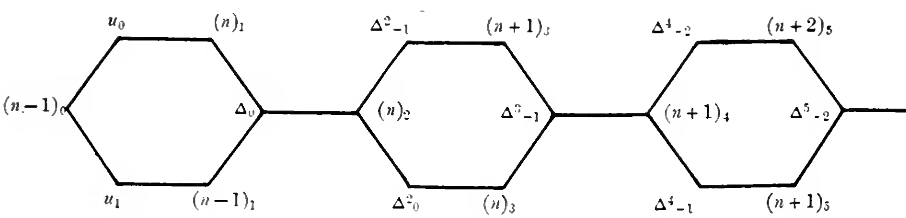
The upper route alone, and the lower route alone, give formulas which may often be conveniently substituted for Stirling's formula, and to even differences these formulas give identically the same results as Stirling's formula.

By giving their values to the coefficients the formula is easily expressed in the customary form

$$u_n = u_0 + n\frac{\Delta u_0 + \Delta u_{-1}}{2} + \frac{n^2}{\underline{2}}\cdot\Delta^2 u_{-1}$$

$$+\frac{n(n^2-1)}{\underline{3}}\cdot\frac{\Delta^3 u_{-1} + \Delta^3 u_{-2}}{2} + \frac{n^2(n^2-1)}{\underline{4}}\Delta^4 u_{-2} + \&c. \quad . \; . \; . \quad \text{(IIIA)}$$

8. *Bessel's Formula*—



In this figure take the upper route and the lower route, and then the mean

$$\therefore \qquad u_n = \frac{u_0 + u_1}{2} + \frac{(n)_1 + (n-1)_1}{2}\Delta u_0 + (n)_2\frac{\Delta^2 u_{-1} + \Delta^2 u_0}{2}$$

$$+\frac{(n+1)_3 + (n)_3}{2}\Delta^3 u_{-1} + (n+1)_4\frac{\Delta^4 u_{-2} + \Delta^4 u_{-1}}{2} + \&c. \quad . \; . \quad \text{(IV)}$$

Put $n = m + \frac{1}{2}$.

$$\therefore \; u_{m+\frac{1}{2}} = \frac{u_0 + u_1}{2} + \frac{\left(m+\frac{1}{2}\right)_1 + \left(m-\frac{1}{2}\right)_1}{2}\Delta u_0$$

$$+\left(m+\frac{1}{2}\right)_2\frac{\Delta^2 u_{-1} + \Delta^2 u_0}{2} + \frac{\left(m+\frac{3}{2}\right)_3 + \left(m+\frac{1}{2}\right)_3}{2}\Delta^3 u_{-1}$$

$$+\left(m+\frac{3}{2}\right)_4\frac{\Delta^4 u_{-2} + \Delta^4 u_{-1}}{2} + \&c. \quad . \; . \; . \; . \; . \; . \; . \quad \text{(V)}$$

The same remark applies as in the case of Stirling's formula.

By giving their values to the coefficients the formula is easily expressed in the customary form

$$
\begin{aligned}
u_{m+\frac{1}{2}} = {} & \frac{u_0+u_1}{2} + m\Delta u_0 \\
& + \frac{m^2-\left(\frac{1}{2}\right)^2}{\underline{|2}} \, \frac{\Delta^2 u_{-1}+\Delta^2 u_0}{2} + \frac{m\left(m^2-\left(\frac{1}{2}\right)^2\right)}{\underline{|3}} \Delta^3 u_{-1} \\
& + \frac{\left(m^2-\left(\frac{1}{2}\right)^2\right)\left(m^2-\left(\frac{3}{2}\right)^2\right)}{\underline{|4}} \, \frac{\Delta^4 u_{-2}+\Delta^4 u_{-1}}{2} + \&c. \qquad \text{(VA)}
\end{aligned}
$$

9. *Everett's Central Difference Formula*—

Returning for a moment to the figure in section 1

$$
\begin{aligned}
AB+CD = AF+ED & = u_0\Delta v_0 + v_1\Delta u_0 \\
& = u_0(v_1-v_0) + v_1(u_1-u_0) = u_1 v_1 - u_0 v_0 \\
& = FC - BE \\
& = \text{the difference of the two inclined diagonals.}
\end{aligned}
$$

Applying this relation to the figure used for Bessel's formula, we can at once write

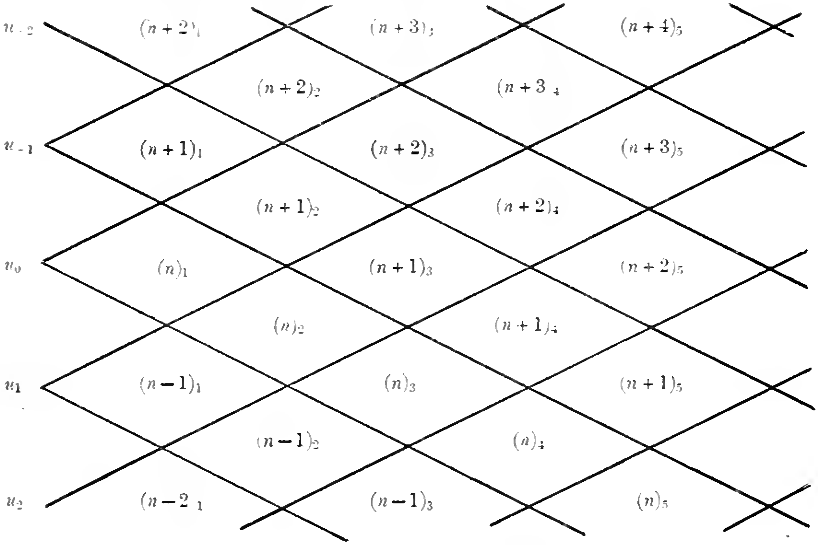
$$
\left.\begin{aligned}
u_n = (n)_1 u_1 + (n+1)_3\Delta^2 u_0 + (n+2)_5\Delta^4 u_{-1} + \&c. \\
-(n-1)_1 u_0 - (n)_3\Delta^2 u_{-1} - (n+1)_5\Delta^4 u_{-2} - \&c.
\end{aligned}\right\} \quad . \quad . \text{(VI)}
$$

Put $n=p$, $1-n=q$; then, giving their values to the coefficients, the formula is easily expressed in the customary form

$$
\left.\begin{aligned}
u_p = p u_1 + \frac{p(p^2-1^2)}{\underline{|3}}\Delta^2 u_0 + \frac{p(p^2-1^2)(p^2-2^2)}{\underline{|5}}\Delta^4 u_{-1} + \ldots \\
+ q u_0 + \frac{q(q^2-1^2)}{\underline{|3}}\Delta^2 u_{-1} + \frac{q(q^2-1^2)(q^2-2^2)}{\underline{|5}}\Delta^4 u_{-2} \ldots
\end{aligned}\right\} \text{(VIA)}
$$

10. The elaborate diagram of section 2 which has been used for purposes of demonstration can be replaced in practice by the simple form given below, in which the differences are left to be supplied mentally. The points to be noticed are, that, at any stage in a formula, the coefficient of the next term is on the right of the difference last employed; and that the coefficient may be used with either the difference above it or the difference below.

Thus, at every stage, there is a choice between two alternatives in proceeding to the next order of differences.



II. *On Premiums payable by Instalments for Whole-Life Assurances.* By S. T. Shovelton, M.A., A.I.A., *Fellow of Merton College, Oxford.*

THE problem dealt with in this paper is that of premiums payable by instalments—m times a year—when those instalments for the current year remaining unpaid at the time of death are deducted from the sum assured. Let it be assumed that the sum assured is payable at the end of that mth part of the year in which the life fails, and let $\Pi_x^{(m)}$ denote the annual premiums payable m times a year when unpaid instalments are to be deducted. Then the sum payable will be

$$1-\frac{m-1}{m}\Pi_x^{(m)},\quad 1-\frac{m-2}{m}\Pi_x^{(m)},\ \ldots\quad 1-\frac{m-r}{m}\Pi_x^{(m)},\ \ldots$$

according as the life fails in the first, second, . . . rth, . . . of the m sections into which the year is divided. The value of this is $A_x^{(m)}\left[1-\Pi_x^{(m)}\right]$ together with the value of an assurance of $\frac{1}{m}\Pi_x^{(m)}$, $\frac{2}{m}\Pi_x^{(m)}$, . . . according as death occurs in the first,

second, . . . section of the year. The value of this assurance is readily seen to be

$$\frac{\Pi_x^{(m)}}{m}\left\{A_x^{(m)}+{}_{\frac{1}{m}}A_x^{(m)}+{}_{\frac{2}{m}}A_x^{(m)}\ldots\right\}-\Pi_x^{(m)}\left\{{}_1A_x^{(m)}+{}_2A_x^{(m)}\ldots\right\}$$

which, by Lubbock's formula, is equal to

$$\Pi_x^{(m)}\left[A_x^{(m)}-\frac{m-1}{2m}A_x^{(m)}+\frac{m^2-1}{12m^2}\Delta A_x^{(m)}-\frac{m^2-1}{24m^2}\Delta^2 A_x^{(m)}\ldots\right]$$

or

$$\Pi_x^{(m)}\left[\frac{m+1}{2m}A_x^{(m)}-\frac{m^2-1}{12m^2}\left(A_{x\overline{1}|}^{\overset{(m)}{1}}-\frac{1}{2}\Delta A_{x\overline{1}|}^{\overset{(m)}{1}}\right)\ldots\right]$$

Since $$\Delta A_x^{(m)}={}_1A_x^{(m)}-A_x^{(m)}=-A_{x\overline{1}|}^{\overset{(m)}{1}}$$

Hence,

$$\Pi_x^{(m)}\mathbf{a}_x^{(m)}=A_x^{(m)}\left[1-\Pi_x^{(m)}\right]+\Pi_x^{(m)}\left[\frac{m+1}{2m}A_x^{(m)}-\ldots\right]$$

Therefore,

$$\frac{1}{\Pi_x^{(m)}}=\frac{\mathbf{a}_x^{(m)}+\frac{m-1}{2m}A_x^{(m)}+\frac{m^2-1}{12m^2}\left(A_{x\overline{1}|}^{\overset{(m)}{1}}-\frac{1}{2}\Delta A_{x\overline{1}|}^{\overset{(m)}{1}}\right)\ldots}{A_x^{(m)}}$$

$$=\frac{1}{{}^{(m)}P_x^{(m)}}+\frac{m-1}{2m}+\frac{m^2-1}{12m^2}\cdot\frac{\left(A_{x\overline{1}|}^{\overset{(m)}{1}}-\frac{1}{2}\Delta A_{x\overline{1}|}^{\overset{(m)}{1}}\right)\ldots}{A_x^{(m)}}$$

where ${}^{(m)}P_x^{(m)}$ denotes the corresponding annual premium when no deductions are made.

At 3 per-cent the ratio of the third term to the second varies from about $\cdot 018\,\frac{m+1}{6m}$ at age 20 to $\cdot 12\,\frac{m+1}{6m}$ at age 75 and, since $\frac{1}{{}^{(m)}P_x^{(m)}}$ is large in comparison with $\frac{m-1}{2m}$, the third term may be neglected. We then have the convenient formula

$$\frac{1}{\Pi_x^{(m)}}=\frac{1}{{}^{(m)}P_x^{(m)}}+\frac{m-1}{2m}\text{ approximately.}$$

This result may also be obtained on the assumption of a uniform distribution of deaths in any year. For, on this assumption, the assurance of $\frac{(m-1)}{m}\Pi_x^{(m)}$, $\frac{(m-2)}{m}\Pi_x^{(m)}$ . . . as above, is equivalent to one of

$$\left[\frac{m-1}{m^2}(1+i)^{\frac{m-1}{m}}+\frac{(m-2)}{m^2}(1+i)^{\frac{m-2}{m}}\ldots\frac{1}{m^2}(1+i)^{\frac{1}{m}}\right]\Pi_x^{(m)}$$

payable at the end of the year.

The value of this is

$$\frac{(1+i)j-\left(1+\frac{j}{m}\right)i}{j^2}\mathrm{A}_x\Pi^{(m)}$$

Writing $\mathrm{A}_x i=\mathrm{A}_x^{(m)}j$ and $(1+i)=\left(1+\frac{j}{m}\right)^m$ and expanding in powers of j the value is seen to be $\frac{m-1}{2m}\Pi_x^{(m)}\mathrm{A}_x^{(m)}$ neglecting the powers of j. Or we may proceed in a slightly different manner. From the self-evident equations

$$1=\left(1+\frac{j}{m}\right)\mathrm{A}_x^{(m)}+ja_x^{(m)}$$

$$1=(1+i)\mathrm{A}_x+ia_x,$$

we have, using $\mathrm{A}_x i=\mathrm{A}_x^{(m)}j$,

$$\frac{(1+i)j-\left(1+\frac{j}{m}\right)i}{j^2}\mathrm{A}_x=a_x^{(m)}-\frac{i}{j}a_x \text{ [\textit{Text-Book}, Ch. IX]}$$

whence $$\Pi_x^{(m)}\mathbf{a}_x^{(m)}=\mathrm{A}_x^{(m)}-\Pi_x^{(m)}\left[a_x^{(m)}-\frac{i}{j}a_x\right]$$

and the same result follows.

2. If the sum assured, less the deductions, is to be paid at the end of the year, we have, in a precisely similar manner, using, for the sake of simplicity, the same symbol, $\Pi_x^{(m)}$,

$$\Pi_x^{(m)}\mathbf{a}_x^{(m)}=\mathrm{A}_x[1-\Pi_x^{(m)}]+\frac{m+1}{2m}\mathrm{A}_x\Pi_x^{(m)}-\frac{m^2-1}{12m^2}\Pi_x^{(m)}\left(\mathrm{A}^1_{x\overline{1}|}-\frac{1}{2}\Delta\mathrm{A}^1_{x\overline{1}|}\right)\ldots$$

Hence, as before,

$$\frac{1}{\Pi_x^{(m)}}=\frac{1}{\mathrm{P}_x^{(m)}}+\frac{m-1}{2m}+\frac{m^2-1}{12m^2}\frac{\left(\mathrm{A}^1_{x\overline{1}|}-\frac{1}{2}\Delta\mathrm{A}^1_{x\overline{1}|}\right)}{\mathrm{A}_x}$$

We thus have

$$\frac{1}{\Pi_x^{(m)}}=\frac{1}{\mathrm{P}_x^{(m)}}+\frac{m-1}{2m}\text{ approximately.}$$

This may be expressed in terms of the annual premium

$$\frac{1}{\Pi_x^{(m)}}=\frac{1}{\mathrm{P}_x^{(m)}}+\frac{m-1}{2m}$$

$$=\frac{1}{\mathrm{P}_x}\frac{\mathbf{a}_x-\frac{m-1}{2m}}{\mathbf{a}_x}+\frac{m-1}{2m}$$

$$=\frac{1}{\mathrm{P}_x}-\frac{m-1}{2m}\cdot\frac{\mathrm{P}_x+d}{\mathrm{P}_x}+\frac{m-1}{2m}$$

$$=\frac{1}{\mathrm{P}_x}\left(1-\frac{m-1}{2m}d\right)$$

The value of Π_x^{m} given in the *Text-Book*, Ch. XVIII. §92 is

$$(1+i)^{\frac{m-1}{2m}}\mathrm{P}_x.$$

Now $$(1+i)^{\frac{m-1}{2m}}=(1-d)^{-\left(\frac{m-1}{2m}\right)}=1+\frac{m-1}{2m}d\ \ldots$$

Hence the two values are practically the same.

If $m=2$ we have

$$\Pi_x^{2}=\frac{\mathrm{P}_x}{1-\frac{d}{4}}$$

which is slightly less than the usual practical approximation

$$\frac{2\mathrm{P}_x}{\left[1+(1+i)^{-\frac{1}{2}}\right]}\quad\text{or}\quad\frac{\mathrm{P}_x}{1-\frac{d}{4}-\frac{d^2}{16}\ldots}$$

In the same way if the sum assured is to be paid at the moment of death we have

$$\frac{1}{\Pi_x^{m}}=\frac{1}{{}^{\infty}\mathrm{P}_x^{m}}+\frac{m-1}{2m}\ \text{approximately.}$$

Thus it is seen that at whatever time the sum assured is payable the difference between the reciprocals of the premiums when the unpaid instalments are deducted and when they are not so deducted is always $\frac{m-1}{2m}$ approximately.

THE INSTITUTE OF ACTUARIES.

SIXTH INTERNATIONAL CONGRESS OF ACTUARIES.

The Council have appointed the following gentlemen as Official Delegates to the Sixth Actuarial Congress, to be held in Vienna from June 7th to 13th, 1909:—Mr. George Francis Hardy, *President*; Mr. Ernest Woods, *Treasurer* (and *pro. tem.* Hon. Correspondent for the Congress in England); Mr. William Peyton Phelps, M.A., *Joint Honorary Secretary.*

The official programme of the Congress was published in the *Journal* for April 1908 (vol. xlii, p. 221).

JOURNAL

OF THE

INSTITUTE OF ACTUARIES.

Some Financial and Statistical Considerations of the Old Age Pension Scheme.

By Vyvyan Marr, F.F.A., F.I.A., *Assistant Secretary, Edinburgh Life Assurance Company, London.*

[Read before the Institute, 22 February 1909.]

There are many references in the pages of the *Journal* to Old Age Pensions, and in view of the Act of last year the subject—from a financial and statistical aspect—may be discussed without transgressing on questions of State policy. I therefore venture to submit the following notes of some of the financial and statistical questions involved, stimulated in so doing by Mr. G. F. Hardy's statement in his Presidential Address, that he believed our most important work lies in the proper application of actuarial principles to the many practical questions which arise from time to time.

Old age pensions ranging from 1*s.* to 5*s.* a week according to the yearly means of the pensioners are granted to British subjects resident in the United Kingdom who have attained the age of seventy years, provided their yearly means do not exceed £31 10*s.*, and provided they are not disqualified on the ground of Poor Law Relief, imprisonment, or the other reasons set forth in Section 3 of the Act.*

The pensions are payable weekly in advance through the Post Office on presentation of an Order which is handed to the pensioner when his claim is allowed. Before, however, a pension

* *See J.I.A.*, xlii, p. 90.

is granted, application has to be made and the circumstances of each case investigated.

Up to 31 December last, 596,038 pensions had been granted —the average amount of each being 4*s.* 10*d.* per week. The distribution of these in the four countries is shown below—

Rate of Pension per week	England excluding Monmouthshire	Wales including Monmouthshire	Scotland	Ireland
	Number	Number	Number	Number
5*s.*	297,332	19,691	60,787	161,578
4*s.*	15,178	864	1,443	3,101
3*s.*	14,830	805	1,488	3,131
2*s.*	7,185	362	656	1,628
1*s.*	4,423	234	395	927
Total	338,948	21,956	64,769	170,365

In addition to these, there will be a considerable number of pensions to be granted in respect of the claims which it was not possible to deal with by 31 December. There will also be a further number in respect of claims still coming in as individuals reach pension age.

The investigation of claims is entrusted to local Pension Committees and Pension Officers. The latter are appointed by the Treasury while the former are appointed by the various county, borough or urban councils. The local Pension Committee may appoint sub-committees with powers. There must be at least seven members on each committee—not necessarily members of the council by which the committee is appointed—and the number must not exceed that of that council. The Local Government Board is constituted the Central Pension Authority. An application for a pension is in the first instance referred for enquiry and report to the Pension Officer. The Pension Officer's report is then considered by the local Pension Committee. If his report is unfavourable and the committee agree that the claim cannot be allowed, they must give the claimant an opportunity of being heard before coming to a final decision. If they decide against the claimant, a notice is sent to the claimant stating on what grounds the application has been rejected and the claimant has the right to appeal to the Central Pension Authority. If the claim is passed, notice is given to the claimant and to the Pension Officer. The Pension Officer has then to provide the

claimant with a book of Pension Orders, and to instruct him in regard to the method of obtaining payment by presenting the Orders at a Post Office. When a fresh book of Pension Orders is required the pensioner has to declare that he is not disqualified from continuing to receive a pension. In order to ensure payment from the date a pensioner becomes entitled to the allowance, claims may be allowed provisionally four months in advance. Pension Officers as servants of the Inland Revenue obtain their remuneration from the Treasury, but members of local Pension Committees do not get paid for their services. They may, however, appoint a clerk to be paid an inclusive fee for his services, the scale of fees ranging from 1*s.* to 2*s.* 6*d.* per quarter per 1,000 of total population for general incidental expenses, and a fee of 5s. for each claim, restricted to 2*s.* 6*d.* for each claim in excess of 20.

The success of the scheme will depend greatly on the tact shown by the executive in carrying out the regulations, and the maintenance of the distinction between an Old Age Pension and Poor Law Relief. The cost of the former is borne entirely by the State. Besides, receipt of an Old Age Pension imposes no loss of franchise. No control is imposed on the manner of expending the allowance, beyond the prohibition of alienation by assignment. This prohibition, however, is ineffective to a certain extent, since the provisions for facilitating payment in cases where the pensioner is unable to attend personally are being taken advantage of by money lenders as a means of obtaining a security for loans. On the other hand the Poor Law Authorities have to supply such information as it is in their power to give, if they are called upon to do so, for the purpose of ascertaining whether any person is disqualified by reason of receipt of Poor Law Relief. The applicant for a pension may also have to appear before the local Pension Committee and answer any questions which may be put to him, and in consequence is subjected to practically the same treatment as if he were applying for Relief. In view of the fact that pensions are not universal, it is difficult to see how this can be avoided. It may be noticed, however, that false statements and representations can be more severely dealt with than they are in cases of applications for Poor Law Relief. The former may be punishable with imprisonment for a term not exceeding six months with hard labour, while persons making false statements for the purpose of parochial relief may be imprisoned for one month with or without hard labour or fined up to £5. There has already been one conviction.

In the Bill as originally introduced the income limit was put at £26 per annum and the amount of pension at £13 per annum, but the principle of a sliding scale was subsequently embodied. It was held that the adoption of the sliding scale offered some encouragement to thrift since it provided for those who are earning between £26 and £31 per annum. Without the sliding scale those persons would not have benefited, whereas, the income of those in recept of £26 per annum would have been increased 50 per-cent. Under the sliding scale the income in both cases is made up to £34 per annum. It will be noticed, however, from the table given above that in 95 per-cent of the number of pensions granted the amount of pension is calculated according to the maximum rate of 5*s*. per week.

If the provisions of the Act are not materially altered, it will be interesting to observe its effect on institutions with which one associates the thrift of the working classes, and I shall now refer to the present position of these institutions. Speaking generally, they may be said to embrace Industrial Assurance Companies, the Post Office Insurance Scheme, and Societies connected with the Friendly Societies Registry Office, namely, Friendly Societies, Building Societies, Co-operative Societies, Trade Unions, Certified Savings Banks, and the Post Office Savings Bank.

Excluding paid up capital and sundry shareholders' balances, the total liabilities appearing in the balance sheets of Industrial Assurance Companies as summarized in the Parliamentary Returns for 1887 amounted to £5,400,141, while the amount stated in the Returns for 1907 is £35,987,373, showing an increase of £30,587,232, or £1,529,362 per annum for a period of 20 years.

The Revenue Accounts for these years are summarized in the following statement:

Income	1887	1907	Outgo	1887	1907
	£	£		£	£
Premiums	3,746,241	12,440,868	Claims under Policies	1,461,832	4,737,281
Consideration for Annuities granted	...	6,746	Surrenders	9,156	129,659
			Annuities	15	5,398
Interest and Dividends (*less* Tax)	169,659	1,126,549	Commission	986,287	3,183,540
			Expenses of management	575,500	2,217,748
Fines, fees, &c.	227	1,504	Dividends and bonuses to shareholders	8,219	610,854
Capital paid up	28,064	168,128			
Transfers from other Accounts	99,349	539,266	Decrease in value of investments, and bad debts, &c.	2,914	8,130
			Increase in funds	999,617	3,390,451
	£ 4,043,540	14,283,061		£ 4,043,540	14,283,061

Premiums and consideration money for annuities received during the past twenty years amount to £154,842,214, while £11,080,007 was received by way of interest and dividends less income tax. On the other hand claims paid amounted to £61,169,402, annuities to £299,322, surrenders to £807,940, commission absorbed the sum of £40,022,435 and expenses of management the sum of £27,337,758.

In the following table is given a summary of the assurances in force as shown in the latest Valuation Returns lodged prior to 1887 and 1907 respectively. The most remarkable feature is the enormous growth of Endowment business, and it is of interest to note that in 1907 the returns only showed seven contracts for deferred annuities, or pensions, amounting in all to the trifling sum of £132.

	1887		1907	
	No. of Policies	Net Amount Assured	No. of Policies	Net Amount Assured
		£		£
Assurances, whole term of life	8,951,210	80,552,116	23,713,681	234,571,308
Endowments	25,192	344,736	2,083,852	19,775,344
Endowment assurances	70,116	1,089,610	629,972	6,276,904
Joint lives	99,326	1,448,025	431,105	6,841,797
Miscellaneous	...	...	8	4,260
	9,145,844	83,434,487	26,858,618	267,469,613
Annuities—				
Immediate	1	15	51	1,761
Deferred	...	...	7	132
	1	15	58	1,893

The Post Office Life Insurance System may be said to date from 1864, when a Bill was introduced authorizing the grant of insurances at certain Post Offices. Prior to 1864 the National Debt Commissioners had power to grant Government Life Assurances for sums not exceeding £100, but this power was not utilized as it had been made a condition of its exercise that a deferred annuity should be effected concurrently with the issue of a policy. The minimum and maximum limits of insurance were respectively fixed at £20 and £100, and of annuities at £4 and £50 per annum. In 1882 the minimum limits were reduced

to £5 in the case of insurances and £1 in the case of annuities, and the maximum limit in the case of annuities raised to £100 per annum, while new tables of rates were introduced and the business linked to the Post Office Savings Bank. In 1896 the rates were again revised. According to the accounts for the year ended 31 December 1907, the premium receipts for the year amounted to £49,610, claims under policies amounted to £11,989, surrenders to £8,742, annuities to £20,543, and management charges to £1,400. The funds invested in 2½ per-cent Consolidated Stock amounted to £893,898, while there was an over-draft of £397.

	No. of Returns	No. of Members	Funds
Friendly Societies:			£
Ordinary Friendly Societies	6,773	3,226,672	18,056,640
Societies having Branches	20,144	2,673,246	23,888,491
Collecting Friendly Societies	45	7,884,307	8,469,767
Other classes	1,999	822,744	2,205,644
	28,961	14,606,969	52,620,542
Building Societies:			
Incorporated Societies	1,904	559,103	55,894,058
Unincorporated Societies	60	57,626	16,364,370
	1,964	616,729	72,258,428
Co-operative Societies	2,782	2,467,806	52,960,807
Trade Unions	645	1,719,031	5,864,342
Workmen's Compensation Schemes	51	103,444	193,794
Friends of Labour Loan Societies	252	33,500	257,643
Total Registered Provident Societies	34,655	19,547,479	184,155,556
	Banks	Depositors	Deposits
Railway Savings Banks	18	64,126	5,865,072
Trustee Savings Banks (including Investments in Stock, and Special Investment Accounts)	224	1,759,228	61,806,617
Post Office Savings Bank (including Investments in Stock)	15,055	10,332,784	174,982,645
Total Certified and Post Office Savings Banks	15,297	12,156,138	242,654,334
Grand Total	49,952	31,703,617	426,809,890

The total number of contracts in force was

2,930 deferred annuities for £61,093 per annum.
13,261 insurances at death for £765,861.

With regard to the societies connected with the Friendly Societies Registry Office, the preceding is a condensed statement of the general summary published in the last returns. The particulars of Friendly Societies refer to the year ending 31 December 1905, and of the other classes for the year 1906.

There is a certain amount of overlapping in the figures given in the totals, because many persons are members of more than one Society, and many may also be depositors in Savings Banks, and at the same time a portion of the funds of Friendly Societies is deposited in Savings Banks.

Dealing in the first place with Ordinary Friendly Societies, Societies having Branches and Collecting Friendly Societies, the last annual returns may be summarized as follows:—

Contributions from members ...	£9,494,246
Other receipts	2,025,623
Total	£11,519,869
Amount paid in sickness claims...	£4,084,301
Death claims	2,063,800
Other benefits	488,040
Miscellaneous payments	508,794
Expenses	2,252,143
Total	£9,397,078

Of the expenses, £1,395,626 is attributable to Collecting Friendly Societies, whose functions are somewhat similar to those of Industrial Insurance Companies. The amount of premiums collected by these Societies amounted to £3,116,235.

There is no doubt that Friendly Societies have suffered an unforeseen strain on their resources through the difficulty of enforcing a distinction between sickness, as insured against by the scales of contribution, and infirmity resulting from old age. It is generally recognized that they are making a gradual improvement in their financial position by amending the rates of contribution and benefit, and by strictly supervising sickness claims and investment of funds. A comparison of a Summary of the latest Valuation Returns of all Friendly Societies with one

relating to twenty-five years ago confirms this, but sheds little light on what the improvement has been. While returns are now received from practically all the Societies on the Register, the same remarks do not apply to the working of the first few years of the Friendly Societies Act of 1875, and the comparison leaves out of account the extent by which the reserves have been increased by change of the basis of valuation. Moreover, in many cases the rates of interest, sickness and mortality assumed in the valuations have little or no connection with the Society's experience. Generally speaking, where quinquennial actuarial valuations have disclosed deficiencies, earnest efforts are made to improve the financial position: in cases where there is an increasing deficiency, it usually is because no attempt has been made to rectify matters.

Practically the only other class of Society performing the functions of insurance are the Trade Unions, whose annual income according to the last return is £2,709,665, as against £671,058 twenty years ago. The expenditure varies from year to year, and in the 1906 returns amounted to £2,283,230. Of this annual expenditure, the average amount devoted to Friendly Society objects, is about 49 per-cent, while about 28 per-cent is devoted to benefits granted during unemployment.

Sir E. W. Brabrook, C.B., late Chief Registrar of Friendly Societies, reviewed the progress of all classes of Societies connected with the Registry in a series of papers read before the Royal Statistical Society on 20 April 1875, 21 April 1885, 23 April 1895, and 18 April 1905. In the last paper he sums up the progress made by these institutions in the following words: "When the comparison is extended over the whole period dealt "with in these four papers we find an apparent increase in the "funds of Friendly Societies in England and Wales of £850,000 "a year over twenty-six years; in the capital of Industrial and "Provident Societies of £1,000,000 a year over thirty-nine years; "in the deposits in Trustee Savings Banks of £350,000 a year "over thirty-eight years; in the Post Office Savings Bank of "£4,000,000 a year over forty years, and a decrease in Loan "Societies of £6,000 a year over twenty-nine years." He infers from these figures that the invested wealth represented by the institutions of which the Friendly Societies' Registry takes cognizance—other than the Post Office Savings Bank—has increased at an average rate of more than £2,000,000 per annum.

If we contrast the figures in the statement given on p. 250 with those in a similar one for 1901 we find the increase to be £59,603,610, including £21,722,967 on account of the Post Office Savings Bank and £9,388,042 on account of Friendly Societies—the latter being in respect of a period of four in place of five years. If the increase in the past five years of the Life Assurance and Annuity Funds of Industrial Insurance Companies be added, namely, £12,300,835, an average increase is shown of over £14,750,000 per annum.

To what extent this annual accumulation of wealth is due to motives of thrift for its own sake as distinguished from motives aroused by the strict administration of the Poor Laws is a question outside the scope of this paper.

It has been mentioned that Friendly Societies have experienced difficulty in distinguishing between sickness, as insured against by the rates of contribution, and infirmity resulting from old age. In this respect the Old Age Pension Scheme should grant a certain amount of relief. It will probably improve the position of some societies, and lead others to adopt either an increased allowance in the event of sickness under the age of seventy years or a small pension between ages sixty-five and seventy in lieu of sickness benefits after the age of seventy. Obviously one of the best methods of extending the sphere of usefulness of Friendly Societies would be to give them facilities for insuring against unemployment and incapacity during the working years of life. In carrying out the work connected with such classes of insurance in an efficient manner so much depends on local or trade conditions and the strict supervision of all claims that many Friendly Societies and Trade Unions already provide the requisite machinery for the purpose.

The proceeds of Industrial Insurance Policies and payments of the nature of "windfalls" are so frequently put to improvident uses that much misery might be avoided were payment made direct to the banks by a system of marked cheques, the moneys being administered by trustees and a fair rate of interest allowed on the amount of such deposits. Any system of granting State aid for auxiliary pensions would probably either interfere with the developments of Insurance Companies and Friendly Societies, or show results similar to those of the Post Office Insurance Scheme when compared with those achieved by private enterprise.

Objections have been urged against the Act on the ground that the Scheme is a non-contributory one. It is difficult to see

in what other manner it would have been feasible to introduce a system of Old Age Pensions without embracing other benefits which would interfere materially with the extension of any of the thrift institutions to which I have referred. Indeed, the payment of an aliment allowance during invalidity and unemployment is almost essential if the Scheme is to be supported to any great extent by the periodical contributions of wage earners. There is a tendency in all Pension Funds connected with railway companies and other large employers of labour to depart from the main object—aliment of superannuated employees—and to introduce the Savings Bank and Insurance elements, and similar extensions would no doubt be demanded in a National Pension Scheme. In addition, we are faced with the questions which always arise at the inception of a Pension Fund; provision has to be made for members who will be placed on the superannuation list at the outset as well as for members who will be classified as contributors. If the latter are called upon to pay contributions in each case equivalent to the value of the benefits granted, the amount to be paid by those who have been in the service for many years is comparatively large, and, as a rule, is more than their means afford, whereas if the charge is spread equally over all the contributors, those still in early years of life have to pay considerably more than they would be asked to do by an insurance company for insuring the same benefits.

There are no means of tracing persons such as exist on the Continent, and in the absence of some ready method of identification, it is unlikely any development of the stamp slip system at present in use in connection with Post Office Savings Bank Accounts would prove a satisfactory means of collecting premiums.

A contributory scheme is unpopular with the working classes. As a rule, female labour is not paid at such rates of wages as would permit of direct levies, and it is questionable whether those wage earners who contribute to Trades Unions and other institutions of an altruistic nature could afford to pay in addition the sum required to provide an adequate pension during old age, incapacity or invalidity.

It might also be held that contributions under a contributory scheme partake of the nature of a poll-tax. If so, and the Fund were kept strictly in trust for the specified objects, it virtually amounts to earmarking a portion of the Revenue for a particular purpose—a proceeding quite opposed to our ideas of taxation.

These points are more or less of an economical character, but there are others which appeal more directly to us as Actuaries.

The Act is restricted in its scope, and unless it were made universal it is difficult to see how equitable rates of contribution could be fixed. Even if rates were calculated, the statistics on which they would have to be based are so unsatisfactory that in a few years' time they would require to be revised: it is easier to alter in each year's Budget the amount required to meet the cost of pensions than to alter fixed contributions. Owing to the difference in the incidence of rates of mortality according to sex, locality and occupation, different scales of contribution should in equity be charged in different circumstances. So many contributory pension funds have been brought to an unsatisfactory financial condition owing to a lack of recognition of the fundamental actuarial principles, that it is questionable whether such a large trust as would be constituted by a universal contributory scheme would be rigorously administered. One realizes the difficulties there would be, on considering the control that such a fund would have over the money market, and looking at the number of alterations which have been made in the permanent charge for reduction of the National Debt since it was fixed by the National Debt (Sinking Fund) Act 1875.

Possibly the question of the statistics on which the yearly estimates of the cost of the scheme will have to be based is the question in which we are most interested. Mr. G. F. Hardy in his Presidential Address referred to the improvements which have been introduced in the Registrar-General's Returns, and to the need of further reform, and also how desirable it is to eliminate from these Returns and the Census figures the systematic errors which at present vitiate the age statements. The necessity is now intensified owing to the Old Age Pensions Act, because it will be vital to be able to predict as closely as possible the annual charge for pensions—if not their capitalized value. One of the greatest difficulties has been encountered in obtaining proof of age. Certain facilities are granted in many cases—such as to members of friendly societies—in obtaining certificates of birth and of death at a minimum of expense and trouble; but the present system of handing to the person who registers a birth a slip notifying that the birth has been registered can hardly be regarded as satisfactory. A parchment document containing the name, date of birth, means of identity and a reference to the full records in the Registrar's office might be of

great use in obviating age questions in future. For instance, subsequent entries might be inserted officially on the occasion of marriage, and production of the document called for at the time of a census enumeration. It might also be useful in facilitating the work of labour exchanges, since its production would be to some extent a certificate of character. Questions of fraud would have to be secured against, but a means of ready access to duplicates retained by the Registrar could surely be devised which would reduce this to a minimum.

When the scheme was introduced in Parliament it was estimated that the number of pensionable persons over 70 years of age would be 572,000. This number was raised to over 600,000 when the sliding scale was adopted. These figures were based on tables prepared by the Local Government Board in July 1907 [Cd. Paper 3618, 1907] which contain estimates of the total population at 30 June 1907 over 65 years of age, 70 years of age, and 75 years of age, and the number of pensionable persons is found by making deductions for those who are disqualified on account of the income limit, pauperism and the other standards. Somewhat similar tables based on the 1881 and 1891 Census are given in the Report of the Departmental Committee on the Aged Deserving Poor [Cd. Paper 67, 1900], but in preparing the 1907 tables the results of the 1901 Census were available and were used.

No estimate was submitted showing the capitalized value of the probable aggregate pensions which would be entered into in this financial year, nor was any distinction drawn between males and females. These are two very important omissions. In the first place a pension once granted is seldom reduced, and, whatever alterations are made in the future, it is unlikely the present pensioners will be the losers so long as they are able to comply with the conditions now in force. In the second place the tests in the case of female applicants for pensions are similar to those for male applicants; since the rates of mortality of female annuitants are, as a rule, much lighter than those of male annuitants and there is no difference in the amount of pension allowed, a greater benefit is virtually given to female pensioners than to male pensioners of the same age.

If therefore, the country can be held to be committed to continue the pensions which have already been granted, the present value of these should be shown as an addition to the National Debt in the Annual Finance Accounts of the United

Kingdom. The same remarks apply to Government Pensions, such as are granted to Civil Servants and others. As a rule, these are merely forms of deferred pay for services rendered, and are virtually a means of returning the accumulations of sums which otherwise would have been demanded by way of salary.

I shall therefore conclude this paper with an attempt to arrive at an estimate of the present value of pensions granted under the Act, taking the end of the present financial year as the date to which the figures refer.

In the absence of detailed statements showing for each age the amount of pensions granted to males and females respectively, it is impossible to estimate with the least degree of accuracy the present value of the pensions granted, and all that can be done is to form a rough idea of the present value as compared with an estimated amount of the pensions payable. I have, therefore, made calculations proceeding somewhat on the lines followed in the tables of the Local Government Board, namely, to estimate the total population at each age from 70 upwards, after making allowance for those who do not comply with the pension tests, and to multiply the number at each age by the present value of an annuity of the average amount of the pensions granted —the annuities being calculated according to the English Life No. 6 Tables at 3 per-cent interest.

If an examination is made of the 1907 Tables of the Local Goverment Board showing the estimated population at 70 years of age and upwards for England and Wales, Scotland and Ireland respectively, it will be found that the figures can be reproduced by assuming that the total population at age 70 and upwards in each of the three countries is increasing in geometric progression, the rates of increase being determined from the enumerated results at the 1891 Census and 1901 Census, and the term being that for $6\frac{1}{4}$ years after the last census, thus representing the population at 30 June 1907. The estimates of the Departmental Committee of 1900, on the other hand, were based on the assumption that the aggregate effect of death, migration and misstatements of age as shown by the ratio of the number living in any age group in the 1891 Census to the number who were grouped as ten years younger in the 1881 Census would be maintained. This method takes into account the rates of migration and death experienced in the last intercensal period, and, on the whole, the figures were confirmed by the results of the 1901 Census. The only objection to it seems to be that it assumes

there are as many age misstatements in the penultimate census as there were in the anti-penultimate, and that these will be repeated in the future. Mr. C. A. Waters, Chief Clerk, General Register Office, in the discussion to Mr. King's paper "On the Construction of Mortality Tables from Census Returns and Records of Deaths", remarked that the officials at Somerset House had good reason to believe that the inaccuracy had decreased very largely in recent years.

In order to obtain the number of pensioners at each age, it was necessary in the first instance to estimate the male and female population in groups, and to make deductions for the number of persons disqualified. Thereafter the number of pensioners at individual ages was obtained by interpolation. In estimating the male and female population in age groups, I adopted, after trial, the method of the Departmental Committee, ascertaining from the census returns of the 1891 and 1901 the probable number of persons who would be alive in 1911, and I adjusted by interpolation the figures to correspond with the present date. I also decided to deal with the total population at certain ages and upwards, in place of taking quinquennial or decennial age groups. For instance, I assumed that the number aged 65 and upwards in 1911 would be equal to the number aged 55 and upwards in 1901, multiplied by the ratio of the number aged 65 and upwards in 1901, to the number aged 55 and upwards in 1891. Three age groups were taken, namely, age 65 and upwards, age 75 and upwards, and age 85 and upwards, so as to eliminate as much as possible the age misstatements. In the 1901 Irish statistics, for instance, the returns for ages 40 to 80 were as follows:—

Ages	Males	Females
40–45	117,142	126,281
45–50	90,049	98,212
50–55	104,425	120,775
55–60	70,887	76,750
60–65	98,305	105,265
65–70	50,028	47,163
70–75	46,273	47,474
75–80	21,021	20,147

It will be observed that in the case of males a greater number were returned at ages 50–55 and 60–65 than at the immediately preceding groups, and the same occurs in the case of females at ages 50–55, 60–65, and 70–75. This is no doubt due to the

tendency to state the age merely in round figures, and estimates based on quinquennial or decennial groups of ages are more affected by this than by the method adopted.

I calculated in this manner for males and females separately for England and Wales, Scotland and Ireland, respectively, the number living in 1911 at and above ages 65, 75, and 85, and from these relative results obtained the numbers for the age groups 65-75, 75-85, 85 and over, in 1911. The figures for 1909 were then deduced from the numbers in these age groups and the corresponding figures of the 1901 Census, by using Mr. A. C. Waters' formula (*J.I.A.*, xlii, p. 263) and are summarized in the following table:—

Estimated Population at age 65 and upwards, April 1909.

Age Groups	England & Wales		Scotland		Ireland		Total	
	Males	Females	Males	Females	Males	Females	Males	Females
65 & upwards	733,959	954,421	95,963	137,636	142,592	142,219	972,514	1,234,276
75 ,,	196,279	282,751	26,054	42,925	44,559	44,993	266,892	370,669
85 ,,	19,860	34,604	2,839	5,821	6,627	7,307	29,326	47,732

If these figures are compared with those of the Local Government Board, it will be observed that the effect of estimating the population in the manner in which I have done has been to show a considerable increase in the first age group. Scarcely any weight can be attached to either of the estimates in view of the time which has elapsed since the last census and the alterations in the age distribution of the population which it is thought has taken place in the interval.

The number of pensions actually granted up to 31 December 1908, in the case of Ireland was 170,365, whereas the total number of persons over 70 years of age in 1907 was 173,359 according to the Board of Trade Estimates. This discrepancy is a strong argument in favour of the necessity of Census Reform, and the advantages attaching to a quinquennial enumeration.

The number of pensioners for each age from 70 upwards was then obtained by interpolation after deductions were made from the figures in the above table for those who are disqualified.

In order to get the deduction for disqualification on account of the income limit the results of the test census in selected Unions made in 1899 were taken. The details are contained in the Departmental Committee's report. The investigation embraced

25,182 cases of persons above the age of 65 and the following is a summary of the results.

Number out of each 100 *of population whose income is* (*a*) *more than* 10*s. per week:* (*b*) 10*s. a week or less.*

Age Groups	Income of more than 10s. per Week		Income 10s. per Week or less	
	England	Scotland	England	Scotland
65 & upwards	37	35	63	65
70 ,,	$31\frac{1}{2}$	30	$68\frac{1}{2}$	70
75 ,,	$27\frac{1}{2}$	27	$72\frac{1}{2}$	73

This table shows that as the age increases the proportion of persons in receipt of small incomes increase, and, were income the only test of disqualification, the proportion, at any age, of persons who would be qualified for pensions to the population living at that age would increase as the age advances. The proportions are affected when we separate males from females, and that this is so, is seen from the following table :

Number of Females in receipt of an income over 10*s. a week, as compared with* 100 *Males in receipt of similar income.*

Age Groups	England	Scotland
65 & upwards	61	66
70 ,,	70	79
75 ,,	78	88

In the case of Ireland estimates were prepared for the Local Government Board in 1891. The Departmental Committee, however, came to the conclusion that the same proportion of people would be disqualified on account of income test as in England. Having regard to the comparatively limited experience, which embraced practically only 2 per-cent of the population above the age of 65, and to the fact that the income limit in the Act is 12*s*. 6*d*. a week in place of 10*s*. a week, I estimate for present purposes the number of persons who are disqualified on account of the income test to be

35 per 100 for ages 65 and upwards,
27 ,, ,, 75 ,,
20 ,, ,, 85 ,,

In the first of these groups there are 65 females to 100 males, in the second 75 females to 100 males, and in the third 80 females to 100 males.*

In estimating the deduction for paupers, allowance has been made for 17 per 100 of the total population aged 65 and upwards,

24	,,	,,	,,	75	,,
30	,,	,,	,,	85	,,

there being 175 females to 100 males in the first age group, 155 females to 100 males in the second, and 150 females to 100 males in the third. These rates were adopted on consideration of the 1907 Tables of the Local Government Board (Appendix, p. 49). It will be observed that the pauper test presses harder as the age advances, and this will counteract to some extent the influence of the income limit test in raising as the age advances the proportion of persons who would be entitled to a pension.

As regards the necessary deductions for aliens, criminals, and lunatics, the Local Government Board assumed the deduction amounted to about

15 per 1,000	for	ages	65	and upwards,
13	,,	,,	75	,,

and I have made use of the same figures, making the deduction for 85 and upwards 11 per 1,000.

Giving effect to these various assumptions, the number of persons who, apart from age, may be eligible for pensions is shown in the table on page 262.

From this estimate it appears that practically 40 per-cent of the male population above age 70 will fulfil the necessary conditions; the slight increase in the proportion in the age group 75 and upwards being partly due to many persons being included in the group who were able to earn a livelihood when they were ten years younger, and are no longer able to do so. In the case of females, the proportion remains practically constant at about 55 per-cent of the total female population of the same age. Taking the combined results, the proportion increases with the age. This is probably due to the fact that at advanced ages of life the proportion of females to males becomes increasingly greater owing to female lives being subject to lighter rates of mortality at these ages.

* All cases in which the weekly income was not stated are included in the above table as cases of income over 10*s*. a week, provided Poor Law Relief had not been granted. This affected the figures relating to females to a greater extent than the figures relating to males.

Table showing estimated number of Pensionable Persons taking into account the various disqualifications other than age.

	Aged 65 and upwards			Aged 75 and upwards			Aged 85 and upwards		
	Males	Females	Total	Males	Females	Total	Males	Females	Total
Estimated Total Population	972,514	1,234,276	2,206,790	266,892	370,669	637,561	29,326	47,732	77,058
Deduct in respect of—									
Persons whose incomes exceed 12*s.* 6*d.* a week	468,107	304,269	772,376	98,366	73,776	172,142	8,112	7,300	15,412
Paupers	136,420	238,734	375,154	60,006	93,008	153,014	9,246	13,871	23,117
Aliens, Criminals, &c.	16,550	16,551	33,101	4,144	4,144	8,288	424	424	848
Total Deductions	621,077	559,554	1,180,631	162,516	170,928	333,444	17,782	21,595	39,377
Estimated number of pensionable persons	351,437	674,722	1,026,159	104,376	199,741	304,117	11,544	26,137	37,681
Proportion thereof to the total population	36·1%	54·7%	46·5%	39·1%	53·9%	47·7%	39·4%	54·8%	48·9%

In order to obtain the number of males and females at individual ages, I adopted a suggestion made by Mr. G. F. Hardy. The ratios were obtained of the pensionable number of males and of females for age groups 65 and upwards, 75 and upwards, and 85 and upwards, to the corresponding values of T_x—the total population at each age and upwards—as shown in the English Life Tables, No. 6, Males and Females (Supplement to the 65th Annual Report of the Registrar-General, Part I, pages xlii and xliv). In the calculations logarithms were used, the logarithms of the ratios being respectively

Age 65 $\bar{1}\cdot30318$ for males $\bar{1}\cdot48583$ for females.
„ 75 $\bar{1}\cdot32317$ „ „ $\bar{1}\cdot45922$ „ „
„ 85 $\bar{1}\cdot32201$ „ „ $\bar{1}\cdot44829$ „ „

The values of the ratios for intervening ages were then obtained by interpolation, assuming second differences to be constant; the results were slightly modified in the case of males for ages 76 to 84 inclusive, and for ages above 85 the ratios were assumed to be constant. These ratios, applied to the successive values T_x in the English Life Tables, gave the total number of pensioners aged 70 and upwards, 71 and upwards, and so on. The numbers at each age were then obtained by subtraction, the number living at age 70 being the difference between the total number living at age 70 and upwards, and the total living at age 71 and upwards. The resulting number of pensioners at each age is summarized in the following table—

Summary of the estimated number of Pensioners.

Ages	Males	Females	Total
70-74	104,820	194,130	298,950
75-79	63,694	116,772	180,466
80-84	29,143	56,831	85,974
85-89	9,416	20,455	29,871
90-94	1,907	4,915	6,822
95-99	209	707	916
100	11	60	71
	209,200	393,870	603,070

The estimated number of female pensioners is thus shown to be almost double the number of male pensioners, and the total

estimate is probably very much on the low side. The number of pensions actually granted up to 31 December last amounted to 596,038, and there were a large number in the course of being investigated. I have no doubt the discrepancy is largely due to what may be regarded as the personal equation in the investigation of applications; the local Pension Committees are not directly responsible for the cost of the pensions, applications for which they consider, and in consequence it is likely that the investigations into the amount of an applicants' income are not so minute as they would have been had the Committees to account to local ratepayers for the amount expended or any part thereof. Taking the average pension at about 4*s.* 10*d.* a week and allowing 3 per-cent for administration expenses, the annual cost of a pension to each of these 603,070 persons will amount to £7,786,000, and the present value thereof, according to the English Life No. 6 Tables at 3 per-cent interest, amounts to £41,807,000.

Many questions require consideration in any attempt to estimate the present value of the liabilities imposed by the scheme, assuming it to be continued in future without material alteration. The above estimates of the number who do not fulfil the necessary qualifications tend to show that applications in the future will not be confined merely to persons who reach the age of 70 years, but several will be received from persons over that age. The fact of there being a pension available when the age of 70 years is attained will probably prove a deterrent to efforts to earn a livelihood which would have been made in the past; and moreover—apart from the probable amendment of the pauper test—the scheme will tend to restrict the number of those above 70 years of age who have to seek Poor Law Relief: any increase in the proportion which the relative number of paupers bears to the population being limited to that which is due to the improved vitality imparted by the care and attention shown to old people in the workhouse. In addition, there is the question of the effect of the personal equation in the investigation of applications to which I have referred, and that of the increase in the numbers who will attain the age of 70 years annually due to the increase in population arising from natural causes and migration.

How far it is possible to make adequate allowance for these influences I hesitate to say; but, of course, so far as the actual mechanical calculations are concerned, Mr. R. P. Hardy's formulas

for determining the values of benefits according to the principle of collective assurance (*J.I.A.*, xxx, p. 78) can be used with suitable modifications. On a 3 per-cent interest basis the capitalized value must, however, be enormous, since an annual allowance for pensions to 25,140 males and 47,042 females on attaining 70 years of age—the respective numbers necessary to sustain the number of pensioners at the figures given above—increases the present value of the liability to a sum approaching two hundred and seventy millions sterling.

NOTE.—The annuity-values used were calculated on the English Life Table No. 6 at 3 per-cent interest. Mr. George King's "Construction A" of the male tables, described in his last paper, was taken for male lives, and the female table was reconstructed on the same basis. I have to thank Dr. Tatham for having supplied me with the necessary figures which, with the resulting values, are given on pp. 355–361 *infra*.

ABSTRACT OF THE DISCUSSION.

MR. O. T. FALK proposed to confine his remarks to the second part of the paper, which dealt with the statistical problem of determining the number of pensioners and the cost of the pensions to be granted to them. Mr. Marr's investigation was of course not the first enquiry into the subject. There was a Special Committee in 1899 which investigated the cost, on the basis of the recommendation of the Chaplin Committee of 1898, and in 1907 the figures were brought up to date by the Local Government Board. Mr. Marr's investigation, however, was probably the first since the passing of the Act last year. The chief impediment was the population difficulty, which he was inclined to think that Mr. Marr had exaggerated. Looking at the estimates of the Registrar-General in 1899, it would be found that he estimated the population for 1901 with great accuracy. To take a single example, he estimated the population aged seventy and upwards in the United Kingdom with an error of less than 1 per-cent. In that connection it was perhaps interesting to note that when the year for which the estimate was required lay outside the two census years for which the figures were given, a direct application of Mr. Waters' formula, with which the members were familiar, was not very satisfactory. He had tested some special cases, and found generally that much better results were obtained by following the method of Mr. Marr, and calculating figures for a third and succeeding census years, and then interpolating by Mr. Waters' formula between the last given census year and the calculated census year.

It was perhaps worth mentioning that one of the reasons why Mr. Marr's figures differed so much from those given by the Local Government Board in 1907 was that the figures in 1907 were not

calculated by the method of the Registrar-General in 1899, and left out of account an abnormally large figure in the age group fifty-five to sixty-five in 1901. Mr. Marr's method took account of that, and he had therefore produced a population figure which was considerably larger than the population figure used by the Local Government Board in 1907. That difference, of course, was not entirely accounted for by the fact that Mr. Marr's estimate was for 1899 and the other figure for 1907. As a matter of interest, he had calculated the population aged seventy and upwards for 1909 in the United Kingdom, and it worked out, in round numbers, at 1,297,000. The figure used in the House of Commons estimates last year was 1,254,000. There was, therefore, only a difference of 43,000, and he thought it would be agreed that, considering that the figures were for different years, and were arrived at, in one case by a good method and in the other by a bad, an error of $3\frac{1}{2}$ per-cent was not very serious. He referred specially to that point because Mr. Marr seemed to suggest that the anomaly of the Irish figures might be partly explained (as suggested in a letter published in the *Times*) by an error in estimating the aged population of Ireland: but, personally, he thought that only a very small part of the error could be accounted for by that. It seemed to be necessary to assume that there had been very considerable misstatements of age for pension purposes, although a good deal of the anomaly and discrepancy might be accounted for by the fact that, in the estimates, the deduction which was made on account of the income test for Ireland was the same as the deduction for the United Kingdom, and, therefore, almost certainly too high. In connection with the census question, he thought that, for the purposes of the pension problem, one required not so much a quinquennial census as greater accuracy in the age returns, and a statement of the ages in individual, instead of in quinquennial, groups. It would be interesting to see whether, in 1911, any appreciable disturbance of the age groups, in Ireland, at any rate, took place, owing to an attempt on the part of the pensioners to reconcile their census ages with the ages stated for pension purposes.

By far the most difficult part of the statistical problem was that which dealt with the estimated deductions on account of the income test and the pauperism test, and there, he thought, Mr. Marr's calculations were most open to criticism. In the first place, he had made the same percentage deduction for the income test and pauperism test in England and Wales, and in the other divisions of the United Kingdom, and that was almost certainly a mistake. In the grand total it was not a very serious mistake, naturally, because the England and Wales figures very largely preponderated in the total: but it was an important error when one came to make the estimate for the smaller parts of the kingdom. To take a single example, the pauperism rate of Scotland was very much lower than the pauperism rate for England and Wales. In the second place, the

corrections made in the 1907 estimates were not quite satisfactory. Taking the deductions in order, the deductions on account of aliens and criminals was an unimportant one, and might be passed over, although it was likely there were errors in it. In the case of the deduction on account of the income test, Mr. Marr seemed to have based his figure on the deductions used by the Local Government Board in 1907, where for ages sixty-five and upwards the deduction was 37 per-cent, but the income limit was then 10*s*. The income limit was now 12*s*. 6*d*., and the author had used a percentage deduction of 35 per-cent. So far as he could see, Mr. Marr must have arrived at that 2 per-cent deduction by considering the figures for pensions which were actually granted up to the end of last year: he had stated in the paper that 95 per-cent of the pensions actually granted were on the full 5*s*. scale, and it therefore seemed a natural step to reduce his figure from 37 per-cent to 35 per-cent. At the same time he himself had investigated the figures of the test census of 1899, which formed a basis for the income test deduction, and he felt confident that that deduction should be reduced to at least as much as 30 per-cent.

In connection with the deduction on account of pauperism, he thought Mr. Marr had erred in the opposite direction, and that criticism applied also to the estimate which was presented to the House of Commons last year. In the Local Government Board estimate of 1907, the pauperism deduction figure was based on a year's count of paupers, and it was well known that that exaggerated the true number of paupers for the year, because many paupers received relief more than once in a year. In 1907, however, the pauperism test was retrospective to a far greater extent than it was now, and there was a very considerable set-off against that exaggeration. That was known and noted in the investigation. Mr. Marr's figure for the pauperism deduction was based on a day's account of pauperism, and it was known that that very largely under-estimated the true number of paupers of the year. Moreover, he had used a percentage for age 70 and upwards, which could not be less than 21 per-cent. Judging from the most recent returns available, those which had been collected for the recent Poor Law Commission Report, that percentage should be at least 24. The pauperism figure was certainly a very hard one to get at, but the publication of the Report of the Poor Law Commission provided a good deal of new data. In the first place there was a more recent age-count of paupers, and in the second place there were more figures with regard to the relation between the true number of paupers of the year and the number as given by a day's count. Data were still lacking, however, for determining the true number of aged paupers from the number given by a day's count.

Since the paper was written, later statistics had been published with regard to the total number of pensions which had been granted. The new figures were in round numbers: England and Wales 371,000, Scotland 66,000, and Ireland 177,000, making a total of 614,000. There were still under consideration at that date in

England and Wales 29,000, in Scotland 5,000, and in Ireland 32,000, or 66,000 in all. Therefore, as far as the total was concerned, Mr. Marr seemed to have got very near to the actual number of pensions which were likely to be granted. At the same time, dividing his total figure into the different sections of the United Kingdom, it would be found that his estimate was not so satisfactory. He had naturally under-estimated the number of pensioners for Ireland, and he did not think any criticism could be made with regard to that; but he had over-estimated the number of pensioners in England and Wales by nearly the same amount, 85,000 to 100,000. In Scotland he had got very near the mark. He thought that comparison justified his criticism of Mr. Marr's deductions. In the case of England and Wales, he had over-estimated the deduction on account of the income test, and very much under-estimated the deduction on account of the pauperism test, and thus had got too large a figure. In the case of Scotland, his two errors balanced one another, because the pauperism rate in the case of Scotland was lower than in England. It would obviously be absurd to give a further estimate now, because it would merely be a question of altering the deduction figures in order to bring the final result nearer the figures published. Another reason for not doing that was, that the author's estimate was admittedly an estimate of the initial cost of the Act, and that was perhaps not the most important figure.

It would be possible to obtain an idea of what the ultimate cost of the Act, in its present form, might be, and, at the same time, an idea of the increased cost if the pauperism test were relaxed, by reducing the deduction on account of income test, and by separating outdoor from indoor pauperism, and deducting only the latter. If that were done, he had calculated, roughly, that the number of pensioners might be increased to 950,000, or even a million. The increase, therefore, would be very great indeed on the present estimate. The figure was calculated on the population of the present year; he had not extended the population to the date on which it might be assumed the conditions would change. At the end of the paper, Mr. Marr had given an estimate of the capitalized cost of the annuities granted in a year. With regard to the method followed he did not wish to say anything, but he doubted if the suggestion that that figure should be included as an item in the National Debt was quite a sound one. If the proportion of aged people in the population remained comparatively constant, he thought the State was more or less justified in working on what might be called the assessment principle. Finally, it should be remembered that an estimate such as Mr. Marr's was none the less important because the figures with regard to which it was made had been already published. It was largely by the comparison of estimates with results that knowledge of data and methods of investigation were improved.

The following communication from Mr. J. W. Thomson (of the Scottish Life Assurance Society) was read by the Secretary:

Mr. Marr's paper is one which, dealing as well as can be

done from the nature of the facts at his disposal, with the financial aspect of an Act bound to be far-reaching in its consequences, will well repay careful consideration. There are one or two minor omissions regarding the working of the Act, which might be amended by him in final form. When a claim is passed and notice given to the claimant and the Pension Officer, the latter has a right of appeal against the decision of the Committee, and, in a case recently under my notice, this is actually being done by him. The fee of 5*s.* for each claim mentioned is a maximum, and I fancy in the large towns will prove the exception rather than the rule. In the country, where claims are fewer and the work, therefore, greater in proportion, the maximum will rule. The Act, I may say—so far as my experience of its working goes—is not being carried out in anything like the inquisitorial manner suggested by Mr. Marr. The applicant appears before the Committee to answer questions when his claim is adjourned, but there is nothing of the Poor Law Relief method adopted. The Local Committees generally are in a position to judge of the circumstances of the claimant, and only in cases of extreme doubt as to the income returned would they proceed to raise any question thereon. The majority of adjourned claims are those where, *prima facie*, the claimant does not appear entitled. It may also be mentioned that the Local Committees are not directly responsible for the investigations, as one would gather from the paper. The Pension Officer is the responsible party. He makes a minute and searching investigation into each individual case, draws up his report, which is considered by the Committee, who largely decide on the strength of it, backed up by their own knowledge of the case. In one way this is a strong safeguard to the proper working of the Act, as the Pension Officer, being himself a Government Official, is removed from all local influence, such as one finds brought to bear in the administration of the Poor Law.

A contributory scheme is, as Mr. Marr states, unpopular with the working classes. This is shown by a resolution adopted at the Trade Union Congress at Bath in 1907, that any pension scheme must be upon a non-contributory and non-discriminatory basis in order to be satisfactory to the working classes. Not only so, but schemes of the kind were examined by the Select Committee of 1885–7, by Lord Aberdare's Royal Commission, and by Lord Rothschild's Commission, and were rudely rejected. Mr. Chamberlain gave it as his opinion that "I have never seen how you could apply compulsion to any but persons who are in regular employment. It is very easy in their case to deduct their contributions from their wages through their employers; but, in the case of persons who are their own employers, or who are in casual employment, a very large part of the population, I have never seen that it would be possible to apply this compulsory provision." Lord Aberdare's Commissioners also reported that, "The accumulation by the State of funds which would have to be kept invested for long periods . . . would be a grave national

danger." This is an aspect referred to in the paper before us.

With regard to remarks on the proof of age, there is undoubtedly great difficulty being experienced in this respect. Where no other evidence is obtainable, the age at marriage is used as a criterion, but even this cannot be held as satisfactory, as the same question crops up in the case of the "gentler sex", as in census returns. There appears to be a chronic tendency to understate the age, even here. In some cases that have come under my notice, the lady was undoubtedly over seventy, so far as appearance and local knowledge of her could lead one to believe, but, unfortunately, the only evidence that could be produced was the marriage certificate, where the age had, in some cases, been understated by from five to ten years. Of course, there was no thought of a pension in those days, and the error had come home to roost. The pension scheme may, however, from this point of view, have a salutary effect on the census returns among the working classes in making a more correct return of age, through a mistaken notion that the pension may be lost in future. It is difficult to see in what way Mr. Marr's suggestion would improve matters. The birth certificate will, of itself, be sufficient in future. The marriage certificate could not, I fancy, help matters much, and how this document could be used as a certificate of character I do not quite see, unless Mr. Marr proposes that employers should endorse the document when an employé leaves, or that the Labour Exchange should do so. And I fancy that this would be going too far in a restrictive direction.

It would be both interesting and useful if Mr. Marr would give us, in an appendix to his paper, a short résumé of the methods adopted in other countries in State pension schemes, and an account of their cost where possible. In Denmark, for instance, the principle is non-contributory, and only the deserving aged poor are pensioned in money or in kind. In 1891 the estimate was 2,000,000 kroner, but by 1905 the actual amount being paid was 7,193,500 kroner. In Germany the principle is compulsory and contributory on persons earning up to £100 a year. In New South Wales, Victoria, and New Zealand the scheme is non-contributory, and the deserving aged poor are pensioned. In Belgium, on the other hand, two schemes are in operation, one contributory and the other non-contributory. The former is an attempt to assist the working classes to save, by an equal contribution from the State for the formation of a fund to secure to them an annuity at sixty-five not exceeding 360 francs.

Turning now, for a moment, to the statistical portion of the paper, and to Mr. Marr's tables, the discrepancy in the Irish figures is certainly very startling. The Board of Trade recently gave the number of persons in Ireland, on 1 January 1908, of seventy years and over, as 184,000, of whom 32,000 were in receipt of poor relief, and, therefore, at once disqualified from pension. The net number who might qualify was therefore 152,000. The claims made to 31 December 1908 were 231,191, 126 per-cent of the total population over seventy. The number of pensions granted to

31 January 1909 was 177,182, or 117 per-cent of the number of seventy and over, less paupers: and this assumes that not a single person of seventy and over in Ireland had an income of £31 per annum.

In concluding, Mr. Marr gives an interesting summary of the estimated number of pensioners at the end of the financial year in April next. This estimate will, however, I fear be found to be much too low. I have not the figures divided into age-groups and males and females, but, in answer to a question in Parliament last week, the Treasury gave the following information:

	Claims made to 31 12 08	Pensions granted to 31 1 09	Claims under consideration at 31 1 09
England and Wales . .	434,070	370,657	29,280
Scotland	76,405	66,123	4,934
Ireland	231,191	177,182	31,554
Total . .	741,306	613,962	65,768

Mr. Marr's estimate at April is 603,070, but it will be seen that this number was exceeded even on 31 January, and not only so but 65,768 claims were still under consideration, and others will have been coming in, between then and April, as the pension age is being attained. It is evident that the number of pensioners in April will probably be nearer 700,000 than 600,000. Taking it as 700,000, and assuming that the distribution in age-groups and sex will be similar, the cost would, roughly, be nearer £9,000,000, and the present value thereof £48,500,000. Even at 675,000, the cost and present value are about £8,700,000 and £46,800,000 respectively. Mr. Marr does not appear to take any account of the saving in the Poor Law Administration by the doing away of further cases of out-door relief. Indoor relief, it is thought, will not be affected to any appreciable extent, but in time the saving in out-relief to the aged would, it is estimated, be about £400,000 per annum for persons aged seventy and upwards, and this, capitalized, might be taken at over £2,000,000.

Mr. A. R. Barrand said there was one paragraph in the paper to which he wished to call special attention. He hardly thought that Mr. Marr was quite serious in the suggestion he made to the effect that, with regard to industrial insurances, in order to obviate the difficulty which sometimes arose from the improvident ways in which the policy money was used, it being often spent either in connection with the funeral or in some form of dissipation, the money should be paid into a bank and invested in the hands of trustees, and, presumably, the income paid out week by week. If the author intended the suggestion to be taken seriously, it would perhaps be necessary to discuss the legal difficulties in the way, *e.g.*,

as to how it was possible to tie the money up, so as to prevent the capital from being used, and to restrict the person entitled to it to the use of the income. When he read that paragraph, however, he looked into the figures of a large industrial institution of which he had some knowledge, and he found that according to the last published statement the average amount of claim was a trifle under £9, and assuming that that was invested at $2\frac{1}{2}$ per-cent, he found that the unfortunate claimants would receive just a penny a week, which could be hardly described as a munificent income! Even this income could only be obtained by leaving out of account all such items as death-bed expenses and funeral expenses, items which were usually supposed to be defrayed out of industrial assurances. In these circumstances, he thought, Mr. Marr's proposal could safely be disregarded. There was just one other suggestion that had occurred to him, namely, whether it was quite safe to take past experience as a guide, in trying to estimate the number of future pensioners. The longevity of life-tenants and annuitants was proverbial, and he thought that by the grant of Old Age Pensions, although the recipients would not exactly be raised to affluence, their lives would be made easier than they were before, and, if easier, it was likely their lives would be prolonged beyond the duration of lives of a similar class in the past. If that were so, it was quite possible that a more favourable view would have to be taken of the mortality, in future estimates, than had hitherto been considered necessary, and this might cause the number of future pensioners to be somewhat larger than had hitherto been estimated.

MR. E. WOODS said that he had been comparing the estimates in the paper with those prepared on previous occasions, namely, the estimates given in the Report of the Departmental Committee, presided over by Sir Edward Hamilton, in 1899, before the Census of 1901, and those prepared by Sir S. B. Provis, for the Local Government Board, about two years ago. The estimates in the paper appeared to be for the year 1909, and he had taken, for the purpose of this comparison, the estimates for the year 1911 given by the Departmental Committee and Sir S. B. Provis. It was very remarkable that, taking the Departmental Committee's estimate of the number of pensionable persons with incomes not exceeding 10*s.* a week, and comparing it with that of Sir S. B. Provis and with the actual figures of the pensions actually granted, it would be found that for England, Wales and Scotland, the number of pensioners on 31 January 1909 was 436,780, and was almost the exact mean between the estimates of the Committee and of Sir S. B. Provis. When, however, the corresponding estimates for Ireland were compared, a very different result was found. The pensioners in Ireland numbered 177,182, a figure which was very largely in excess of the estimates of Sir Edward Hamilton's Committee, and Sir S. B. Provis. The enormous number of pensioners in Ireland, as compared with England, Wales and Scotland, had been put down to misstatements as to age, but Sir Edward

Hamilton's Committee was most careful to emphasize the point that mis-statement of age would not be the only practical difficulty to be dealt with when granting old age pensions, but that there would be unknown quantities which would seriously affect anticipations, and he instanced especially that there were many persons whose incomes were just over the border line of 10*s.* a week, and it would be to their advantage to understate their receipts, and it would be equally to the advantage of others to assign away a portion of their income or income-producing properties, a step to which frequent resort would almost certainly be had in Ireland: and reasons were given why this would be so. He might be pardoned for so frequently mentioning Sir Edward Hamilton's name, because the Report of the Committee of which he was Chairman was a most valuable one, bringing out what appeared to be very accurate results, clearly expressed, and was a good example of modern methods of dealing with more or less unreliable statistics. It was most remarkable how closely the estimate of the annual cost made by Sir Edward Hamilton's Committee, in 1899, approximated to that of the writer of the paper. While, on the one hand, the average pension estimated for by the Committee was higher than Mr. Marr's, there were, on the other hand, a number of pensioners in receipt of incomes exceeding 10*s.* a week who were not allowed for in the Committee's estimate. The Committee estimated the total cost of the pension scheme at £7,942,000 a year, and the author put it at £7,786,000, the amount actually payable on 31 December 1908, being £7,450,785, as calculated from the figures given in Mr. Marr's first table. Anything closer than that he could hardly imagine.

In the later part of the paper the author devoted a great deal of attention to finding out the capital value of these pensions, but he hardly saw the use of that. After all, the particular estimate of £41,807,000 was only the value at the present moment of the present pensions: next year there would be others granted, and the year after so many more, and some ceasing each year. Most certainly, the further grant of pensions would not immediately cease, and the valuation of £41,807,000 was therefore the minimum addition to the National Debt: the real addition being the enormous sum of £270,000,000 mentioned by Mr. Marr, if the present scheme was to be permanent. He did not, however, accept that view of the liability. Personally, he thought that, in order to measure the real cost of the pensions, a good standard to take was the income tax. A penny in the £ yielded about £2,666,867, so that £7,900,000 a year was equal to about 3*d.* in the £. In 1921, if the estimates of Sir Edward Hamilton's Committee were to be trusted, it would amount to as much as 4*d.* in the £, and if the pension age were lowered to sixty-five it would cost $6\frac{1}{2}d.$ in the £. He did not mean to suggest that the cost of old age pensions should be all paid out of the income tax, but merely that the income tax was a sort of standard of measurement of what the scheme would cost. He suggested that the author might amend the table giving

the proportion of the estimated number of pensionable persons to total population, by leaving out the estimates for age eighty-five, and inserting those for age seventy, which was now the pensionable age prescribed by the Act. Reference had been made to the relief of the Poor Rates by the Pension Scheme. According to the Departmental Committee's Report this relief might be expected to amount to £481,000 for the year 1911. That was a small figure as compared with £7,900,000 for pensions, and he very gravely doubted whether the saving would amount to as much as that.

He had had the honour of reading a paper before the New York Congress on the question of Old Age Pensions, and he then submitted that the solution of the problem of the aged poor lay, not so much in any rigid pension scheme, as in a cautious and well-considered amendment both in the Poor Law itself and in its practical administration. Apparently that might now come about, and he should like to repeat what he then said, namely, that, "If as many fear, the ultimate effect of any large State Pension Scheme should be to weaken the support given by the working classes to those great provident institutions which have been slowly and surely built up by their own self-reliant exertions—institutions such as no other European nation can show—a fatal blow would have been dealt to that spirit of independence and of resentment to State leading strings which is so distinguishing a character of the Anglo-Saxon race." Also, "The collection of the necessary funds year by year by means of taxation would doubtless form a far better working basis than the German method of levying contributions and building up enormous reserves."

MR. S. G. WARNER said that the paper contained in its opening section a very lucid and business-like account of the scheme as it at present stood, and it was a good thing to have that recorded in the proceedings of the Institute, as it was valuable for reference, dealing as it did with a subject with which actuaries were so largely concerned. He felt that only a very imperfect opinion could be expressed at present upon the scheme as a whole, because its working was so recent and it was so intimately bound up with the other great question of the administration of the Poor Law. The intimate connection between two subjects had been noted from the first by those responsible for the scheme, and until a few more months had elapsed—sufficiently long to enable the pension scheme to be viewed as a whole, together with the proposed reorganization of the Poor Law, and its financial effect—it was exceedingly difficult to form any definite and final opinion on the subject. The same thing applied with regard to the attempt to make an estimate of the cost of the scheme. It was clear that, from the nature of the material in hand, the making of such an estimate was a matter of the greatest difficulty, and, possibly, there was no conclusion to be arrived at of any permanent value, beyond the fairly obvious one that the scheme was going to be very expensive. Probably the author had made as good an attempt at it as the circumstances permitted, but the difficulties were such that no very great import-

ance could be attached to the accuracy of such an estimate at the present time.

One other question touched upon by Mr. Marr was of the greatest interest in connection with the project, and was keenly debated during its inception: Whether a national pension scheme should be a contributory or a non-contributory one. Probably all actuaries would feel drawn in the first instance towards a contributory scheme: the idea of helping people to help themselves, of making any national effort of the kind an automatic encouragement to thrift, was undoubtedly a very attractive one, and it should not be forgotten that there were large institutions now in the field which had been for many years doing their best to encourage such a tendency amongst the people. When considering the subject in detail, it was impossible to avoid the conviction that there was very great force indeed in the considerations brought forward by the author adverse to a contributory scheme, attractive as the idea at first appeared. One of the objections mentioned in the paper was that a contributory scheme would probably have to exceed the bounds of mere old age pensions: that when the beneficiaries were expected to be contributors they would want other benefits included. Probably, however, those who had a contributory scheme in view would be prepared to admit and face that possibility, and would hold that the larger fund an effective contributory scheme would bring together would, in time, permit of the other benefits being added; in fact, the inclusion of many other benefits than mere old age pensions constituted the ideal towards which many were working. Already there had been reference to other and further reaching benefits, and old age pensions were described as only an instalment of what the public had a right to expect.

The real difficulties, as mentioned by Mr. Marr, appeared to be very great, and one was rather reluctantly driven to the conclusion that, if the system was to be workable, it must be very much upon its present basis. Thrift and self-reliance, with the help of voluntary institutions, had done their utmost during the past century, and, as the result of it all, it was found that there was an amount of poverty in old age so widespread and so acute as to necessitate some scheme of State aid in the way of deferred pensions, advocated, quite independently of party, by statesmen of almost all varieties of political opinion. That fact alone showed that something beyond what aided thrift could do, beyond what voluntary organization could do, seemed to be required. The only fear one had was that, in some way, such admirable organizations as existed of a voluntary kind might be injured or prejudiced, a result which would be very much to be regretted.

Mr. E. A. RUSHER thought that one of the chief values of the paper—if only it would be considered in that way by the public—was to illustrate the extreme caution which it was necessary to exercise when dealing with public statistics. For instance, something had been said of the great number of pensions that had been claimed in Ireland in proportion to other parts of the United Kingdom.

Sir Vesey Knox, in a letter to the *Times* last week, referred to one small cause which might help towards that state of affairs—the famine of sixty or seventy years ago. Taking another instance, Mr. Marr referred to the figures of industrial insurance companies, and gave the annuities paid during the last twenty years by industrial companies as £299,322. From the summary given in the Board of Trade Returns it would appear that during the last twenty years the annuities paid by industrial insurance companies alone amounted to that sum, but over £200,000 of it was from two unfortunate tea pension schemes, which could hardly be considered as of the nature of industrial assurance, although the Board of Trade put them in that category. This showed that the whole subject of dealing with public figures of any kind bristled with difficulties.

Another point in the paper was one to which Mr. Barrand had already referred, where Mr. Marr suggested that the proceeds of industrial assurances should be paid into banks. Seeing that the average claim was £9, and out of that the recipient had to pay the doctor, the bedside expenses, and the funeral expenses, he did not quite see where improvident uses came in. He concluded by stating his full agreement with those who had spoken before, that to add to the National Debt a sum which may ultimately amount to 200 or 300 millions hardly seemed a suitable way of dealing with the subject of old age pensions.

MR. A. W. WATSON, in closing the discussion, said perhaps the most important point was that of the method by which the cost of the Old Age Pension Scheme was going to be borne, and on that there had been a most fair and interesting statement of the position by Mr. Warner. It seemed to him that, if a contributory scheme of old age pensions were to be established, it would certainly have a detrimental effect upon the thrift institutions at present existing in the country; but it was curious that some of the leaders of those institutions were clamouring for a contributory scheme, not seeing, as others had discerned very clearly, that the effect of any contributory scheme would be to divert income from the one type of institution to the Old Age Pension Scheme, and thus to damage the older bodies. With regard to the suggestion that had been frequently made during the last few months that the Old Age Pension Scheme should be linked up with some system of universal disability assurance, he thought any scheme of that kind would undoubtedly tend to damage and ultimately to replace the existing organizations. It was necessary to consider carefully whether the existing bodies, with all their faults, were not a national asset worthy of preservation. It would be extremely easy to set up legislative machinery which would have the effect of ultimately replacing them. With all their efforts and all their attempts at good work there had been failures, and they admitted to-day many shortcomings. Those were the consequences of the inevitable errors into which they fell on their establishment, and not the result of any innate recklessness or indisposition to be advised on

the part of those who had guided them. It would be easy to ground a case against existing institutions, but his personal feeling was that the existing societies and institutions had deserved very well of the community. They were doing a national work, inasmuch as they were providing working men with some opportunity of taking part in the affairs of life and saving them from becoming mere serfs in the daily routine of employment. They were making the working man an independent, self-relying being, and therefore were doing a greater work than could be expressed in pounds, shillings and pence, and shown in an annual balance-sheet. On that account he thought the development of national social organizations should be on such lines as to avoid doing harm to the existing bodies.

The question had been raised by Mr. Marr as to what would be the probable effect of the Old Age Pension Scheme, as it stood, upon the existing friendly societies. So far as their numerical position was concerned, it was difficult to see that the Old Age Pension Act, by itself, would have any particular effect at all, but, no doubt, there had been a gradual weakening of the spirit of self-reliance, and the growth of a spirit of dependence upon the State, of which old age pensions were one manifestation, and that changing spirit would probably have some detrimental effect upon the friendly societies. That was his own personal opinion, but it was only fair to say that there was no subject which was so keenly debated amongst the friendly societies as the probable effect of the Old Age Pension Scheme upon them, and it was quite impossible from the clash of views to gather any real guidance as to what the effect of the scheme would be. As to the financial aspect of the case, there was no doubt that, unless the friendly societies watched the operation of the Old Age Pension Act very closely, the effect of the Act upon them would be most disastrous. At the present time they had an immense difficulty in discriminating between sickness and infirmity in old age, whilst lack of employment and lack of desire to find employment were potent claim-factors in old age sickness. Under the new *régime*, taking particularly the country districts, where the income going into the house of the pensioner would be often 10*s*. a week, and the total income consequently would be as much as had been earned for many years, there would be absolutely no inclination to find employment, and there would be every opportunity to prolong claims for sickness benefits. For this reason, he viewed the operation of the Old Age Pension Act upon the funds of friendly societies with the utmost misgivings. He wished it were easy to persuade the members of friendly societies to act always in their own interests, but, as a rule, they had to be put through a long course of education before they could be persuaded to take action which was obviously directed to their own welfare. He expected to find, when the leaders of the societies endeavoured to persuade the members to deal with the question of sick pay in old age, that, for the first few years, all kinds of sentimental objections would be set up and very little would be achieved. The line of action which

would have to be taken was as follows. The sick pay must be commuted for such annuity benefit as was equivalent to it, and in making that commutation the whole weight of any deficiency existing in the assets at the present time must be thrown upon the period of life after the age of seventy. It would be impossible to make any commutation of sick pay for an equivalent annuity unless the deficiencies were got rid of, because, whatever its defects in other respects, the Friendly Societies Act did provide that no annuity benefit should be granted unless upon a properly certified table; so that no actuary could produce a scheme substituting an annuity for sick pay in the case of any society, unless the extinction of the existing deficiency was an integral part of that scheme. Regarded from that point of view, the Old Age Pension Act ought to be of great value to the Friendly Societies. But, unfortunately, before that value could be realized, the societies had to be induced to see that, unless they took definite action, they would not only derive no benefit from the Act, but they would be seriously damaged by it.

In the communication that had been read from Mr. Thomson, the author's views with regard to Local Pension Committees had been combated. He did not think himself that the difference between Mr. Marr's estimate and the official estimate was due in any degree to laxity on the part of the Pension Committees. The determining factor in the grant of the pension was the Pension Officer, who was a servant of the Crown, and so far as the Local Pension Committees could be said to have taken action on one side or the other, their tendency had been rather in the direction of stringency than of laxity. When all was said, the official estimate consisted of a most elaborate series of deductions drawn from a mere 25,000 cases, and the surprise was, not that the estimates were so far out, but that it had been possible so nearly to approach the facts.

He agreed with the other speakers in not quite seeing the utility of including the value of the pensions in the National Debt. It was known that, with a given amount of pensions per annum, something between five and six years' purchase roughly represented the capitalized value, but why it should be added to the National Debt he was quite at a loss to understand. Nor did he see why there should be any question raised as to the difference between a pension to a man and a pension to a woman, because, other things being equal, the woman was likely to live longer. Those were small points and did not affect the real issues of the scheme. He joined with other speakers in agreeing that the Institute was much indebted to Mr. Marr for bringing forward an essay of distinctly topical interest, and he hoped that the paper and the discussion which had followed it would be of some service to legislators, when they endeavoured to deal with the much more difficult question of how the money to provide the pensions was to be raised.

The PRESIDENT, in proposing that the thanks of the Institute be tendered to Mr. Marr, said he was sure that the subject selected was

an important one, on which the author had submitted a very valuable paper, which had elicited an extremely interesting discussion.

The vote was carried with acclamation.

MR. MARR, in reply, referred to Mr. Falk's remarks about the estimated number of Pensioners, and said it was premature to deal with the actual results, in an attempt to arrive at the number of male and female pensioners respectively, and he therefore thought it as well to follow as closely as he could the figures in the 1907 Board of Trade estimates. In doing so, however, he was confronted with the difficulty of estimating the male and female population after 70 years of age, and therefore decided to resort to the method of interpolation which he adopted. He departed from the methods followed by the Board of Trade in this respect, but followed them more or less in regard to the deductions for the pauper test and the income test. Certainly, in taking the latter at 35 per-cent at age 65, instead of 37 per-cent, he took into account that 95 per-cent of the pensions are at the full scale. With regard to the estimate for pauperism being on the low side, as a matter of fact the number of paupers on 1 January was stated to be 287,000, but if the number of paupers aged 70 and upwards be deducted from the numbers he had estimated at age 65 and upwards, age 75 and upwards, and age 85 and upwards, in the same way as the number of pensioners had been estimated, it would be found that the figure came out about 262,000, thus differing from the actual number by some 2 per-cent of the estimated population. He had not thought it worth while to alter the figure because he was dealing with the net residue, namely, the number of pensioners, and because the recorded number of paupers must be influenced to some extent by the age difficulty. It was to a pauper's interest to make out that he was as old as he could, and, no doubt, among paupers of age 70 and over, there would be proportionately more misstatements than in the average population of ages 70 and over.

It seemed to be the feeling of the meeting that the charge for pensions should be shown as a permanent and increasing part of the national expenditure in place of being capitalized and included in the National Debt. The assets of the State chiefly consisted of the earning power of the people, and the pension scheme virtually made the able-bodied give up a portion of their incomes to those who were not so well off. He, however, adhered to the idea that a pension once granted should be shown in the Consolidated Fund Services Accounts. It was perhaps hardly right to treat the capitalization as an addition to the National Debt as constituted at present, because the scheme was entirely different from the ordinary charges connected with Consols, but he thought this difficulty would be avoided, if a separate fund could be started, wherein the capitalized value of each pension was placed when it was granted, and out of which the payments of pensions were made. Any profit or loss on re-valuation of the fund might go to the National Debt and the amount to be credited to the fund annually, namely, the

capitalized value of pensions granted in the year might be voted annually. By this method, each generation would bear the cost of its own pensioners, and not hand down to future generations the burden of providing for past generations, as they were having to do at present. If, in a series of years, the profit-earning power of the people diminished, it was only right that the pensions of the corresponding pensionable persons should be diminished too; but no doubt such a course was practically impossible.

Exception had been taken to the remarks in regard to the proceeds of industrial insurance policies. Certainly, there was not very much room to go far astray, when a sum of £9 or £10 was involved, but the fact of the whole being spent on funeral expenses, as was frequently the case, showed that there was improvidence in some cases: of course the industrial offices were not responsible, but it would be much better for the persons concerned, if there could be some check on the manner in which the money was to be expended. In striking the average, policies of larger amounts, say up to £50 or so, were no doubt included, and if those could be paid to the widow by a series of instalments spread over a few months it would, he thought, be beneficial. As a rule, the recipients of the proceeds of industrial insurance policies did not seem to know how to handle money. If they received a large sum in cash they did not know what to do with it: but if the money was paid out in weekly or fortnightly instalments, it would be a different matter. He was obliged to Mr. Rusher for pointing out that the figures relating to the operations of industrial insurance companies during the past twenty years, extracted from the summaries published in the Board of Trade returns were affected to some extent by the inclusion of certain special annuity payments. They were also affected by transfers from and to other accounts, in the case of companies which transacted ordinary life assurance business as well as industrial assurance business.

From Mr. Thomson's written communication, evidently some local authorities were not vigorous in their administration of the Act, but no doubt if they had to account to ratepayers they would have recognized their responsibilities in considering the Pension Officer's reports. With regard to the use of the parchment registration certificates as certificates of character, the mere fact of a man producing one would, he thought, be a certain recommendation, because if he did not produce his certificate he might have some good reason for withholding it. It might be wondered if there would be a large percentage of certificates forthcoming at the time of a census enumeration, and he was inclined to think that the proportion would be pretty considerable, as industrial people generally took care of documents, but of course they would require educating in the matter.

He agreed with Mr. Woods in the remarks he made about the 1899 Committee, and he thought it was of great credit to the members of that Committee to have brought out such a clear statement from the figures which they had at their disposal.

On the Annuity Business of British Offices and the Valuation thereof. By H. J. P. OAKLEY, F.I.A., *of the North British and Mercantile Insurance Company, London.*

[Read before the Institute, 29 March 1909.]

INTRODUCTION.

1. THE subject of Annuities, in relation to the transactions of the Life Offices of this country, has seldom received definite consideration at the meetings of our Institute. An examination of the contents of the *Text-Book* and of the Index to the *Journal* furnishes abundant proof of the attention which has been given to the theory, but in regard to the practical working of annuity business, little has of late years been written. The most important contribution in the *Journal*, from the latter point of view, is that by Mr. G. H. Ryan (read before the Institute seventeen years ago) "On a Method for determining the Gain or Loss from Mortality in an Annuity Company." Therein reference was made to another contribution by the same author, read before the Actuarial Society of Edinburgh during the Session 1884-5, and published in the first volume of the Transactions of that Society, "On the Several Mortality Tables Employed in the Valuation of Annuity Contracts." Changes of the utmost importance have taken place during the period which has since elapsed, and it is proposed herein to deal with the cause and effect of such changes, and to gather therefrom some guidance for the future.

2. In the course of his paper before this Institute, Mr. Ryan uttered a warning (which subsequent facts have warranted) regarding Annuity business, and in the discussion it was pointed out, with some pessimism, that the Annuity payments made per annum by British Offices alone had increased from £439,000 to £797,000 in the course of nine years. The fact that these annual payments have since increased to well over £2,000,000 per annum (a figure, which, in comparison with the magnitude of assurance contracts, sinks into insignificance, but which, in itself, is of no inconsiderable dimensions) and that, apart from incidental mention, no general discussion appears since to have taken place, warrants me in bringing the subject before the profession—yet, in view of the elementary nature of this contribution, with some hesitation. It is hoped, however, that it may be the means of

promoting a vigorous discussion—not, I beg, of the paper—but of the subject, and so compensate for the indifferent treatment hitherto accorded to this section of our work. I would hasten to add that the latter remark is not intended, in any sense, as a reflection upon the Institute; on the contrary, the investigation and publication of the Annuity Experience of British Offices 1863–1893 by the Joint Committee of the Institute and Faculty of Actuaries must be considered the most important work yet undertaken in regard to Annuities, and it is in the light of this experience that we review the business of the past decades and look forward to the future.

Business of Past Twenty-five Years.

3. As shewing, in detail, the business transacted during the past quarter of a century, the following table has been framed, compiled from the Board of Trade Returns. The annual premiums paid, for Annuities other than Immediate, are not included in the "Consideration"; but such Annuities, when they become Immediate, are usually included in "Annuities paid."

Table I.

Annuity business of Life Assurance Companies of the United Kingdom.
(Ordinary Companies.)

* Year in which Returns were filed	Consideration	Annuities Paid
	£	£
1883	610,137	573,235
1884	636,510	593,151
1885	644,274	633,091
1886	601,187	658,331
1887	703,993	689,341
1888	812,660	704,989
1889	1,107,787	743,326
1890	1,275,665	797,427
1891	1,184,705	871,070
1892	1,096,870	806,234
1893	1,359,476	1,055,411
1894	1,415,769	1,054,276
1895	1,742,387	1,135,282
1896	2,365,466	1,230,700
1897	2,330,381	1,377,819
1898	1,985,892	1,504,225
1899	2,356,812	1,603,069
1900	2,139,014	1,714,573
1901	1,716,027	1,775,793
1902	1,981,605	1,862,835
1903	2,012,131	1,941,147
1904	1,881,972	2,030,031
1905	1,793,736	2,082,473
1906	2,185,859	2,137,657
1907	2,059,584	2,219,232

* The figures will usually relate to the *previous* year.

4. Mention should, perhaps, be made, in passing, of the outstanding features regarding the course of the business, as revealed by the tabulation of the figures, first amongst which is the very large relative increase in the amount of business transacted, especially when considered in relation to the lower return now obtainable on the purchase money. Secondly, the amount of the consideration has, during the last few years, slightly receded from high-water mark. The latter was reached in the quinquennium preceding the outbreak of the South African War, and coincided with the period of a very low rate of interest. The slight fall in the amount of new transactions was doubtless due to two causes—one, the change in the rates of annuities made by many of the leading Offices because of this low rate of interest obtainable on their funds; the other, the increase in the rate of interest which followed shortly after, due to the slump commencing just before the close of the last century. These two causes almost coincided with one another (the investment of capital with annuity-granting Companies being thus deflected to other securities), and although the better rate of interest still maintains, no office has reverted to the previous terms for annuities, another coincidence occurring in the shape of proof of greater vitality among annuitants being given at the time that offices were themselves beginning to earn more remunerative rates of interest. (The question as to how far the respective changes in the rates of interest and mortality compensate one another is subsequently discussed.) The third point of interest is that in three or four of the recent years, the amount "Annuities paid" has exceeded the "Consideration" received in those particular years.

Annuity Rates, Past and Present.

5. Reference has been made above to the lower return now obtainable on the purchase money. The average rates, published in 1886, of the three British Offices transacting the largest annuity business in that year are shewn in comparison with the average of their present rates. The rates were, and are, quoted by the respective Companies in various ways, but in order to obtain a correct average, they have all been reduced to one common method of purchase and payment—namely, the amount of annuity (quoted for £100 purchase money) payable half yearly, the first payment being due six months after purchase, the annuity ceasing with the half-yearly payment preceding

death (*i.e.*, without proportion) and the Office paying the stamp duty. Allowance has been made for the proportionate payment to date of death, where the rates were so quoted—in the 1886 rates, on the basis of the Government Annuity Tables, 1883, at 3½ per-cent, and in the present rates on the same Experience at 3 per-cent, these being the bases nearest approaching the rates quoted at the respective times. This method of purchase and payment has, for the purposes of comparison, been adhered to in all quotations throughout the paper, the term "rates" being used to indicate the return, or amount of annuity, for £100 purchase money. A reduction in the rate will thus be understood as an unfavourable alteration from the public point of view, acting as an *increase* in the *terms* on which annuities are granted. Incidentally, the question of payment of stamp duty might here be raised. Each Office, no doubt, has its own opinion on the point, but it seems somewhat inconsistent with all other branches of life business to make a separate item of the stamp. The inclusion of the duty in the price presents no difficulty, and it no doubt avoids disappointment on the part of the purchasers when completing. On the other hand, if an Office wish to discourage annuity business, the extra item of the stamp may prove a deterrent.

TABLE II.

Past and Present Average Rates, for £100 purchase money, of three British Offices transacting the largest amount of Annuity Business in 1886.

Age	Males			Females		
	1886 Rates	Present Rates	(3)/(2) per-cent	1886 Rates	Present Rates	(6)/(5) per-cent
(1)	(2)	(3)	(4)	(5)	(6)	(7)
	£ s. d.	£ s. d.		£ s. d.	£ s. d.	
50	7 4 3	6 14 8	93·36	6 15 4	6 2 4	90·39
55	8 1 0	7 12 7	94·77	7 11 1	6 17 10	91·23
60	9 5 4	8 17 10	95·95	8 13 3	7 19 2	91·87
65	10 19 9	10 11 0	96·01	10 5 4	9 9 0	92·04
70	13 7 2	12 16 6	96·01	12 12 1	11 12 4	92·17
75	16 13 1	16 0 8	96·27	15 14 6	14 11 2	92·58
80	20 8 2	20 4 10	99·19	18 17 7	18 10 1	98·01

6. The decrease in the published rates during the period of about twenty years has thus been, roughly, 4 per-cent in the case of male lives and 8 per-cent in the case of female lives. On the

average, the rates of other Offices have doubtless been subject to like reductions. The new business of the three Offices in question, though it does not now bear the same ratio to the total new business that it then did, is very considerably in advance, in amount. Thus, in 1886 they received, as consideration, £265,602 out of the total consideration received by British Offices, £703,993 (=37·7 per-cent), while in 1906 they received £382,923 out of £2,059,584 (=18·6 per-cent). It is to be noted, too, that the 1886 rates were by no means obsolete, but were constructed on what the offices then thought to be safe lines, having been framed by two of the offices in 1884 and by the other in 1885, doubtless having regard to the new Government Experience then published. The rates of all three Companies were changed again in the nineties—one in 1892, the other two in 1895—on this occasion, probably, because of the continual fall in the rate of interest.

Valuation Bases Past and Present.

7. Of equal importance with the change in the terms on which annuities are granted is the change which has taken place

Statement showing classification of the Basis of Valuation of Thirty-five Companies.

Valuation Basis	Number of Companies adopting Basis	
	Group 1 (1875)	Group 2 (1880)
Table of Mortality—		
Carlisle	11	5
Davies' Equitable	6	6
Government Annuitants'	11	14
H^M	1	2
H^F	...	1
English Life No. 3	1	1
17 Offices'	1	1
Swedish	1	1
American	1	1
Special tables	2	3
Total	35	35
Rate of Interest—		
3 per-cent	14	13
$3\frac{1}{2}$ „	6	7
4 „	13	14
$4\frac{1}{2}$ „	2	1
Total	35	35

Note.—One company employs $2\frac{1}{2}$ per-cent interest for annuities on females.

in the basis of valuation. Mr. Ryan, in his paper before the Actuarial Society of Edinburgh, gave, in detail, the total of the Valuation Summary and the basis of valuation (at two consecutive valuations) of the thirty-five Offices then transacting the largest amount of business, and also the statement, here reproduced, showing the classification of the basis of valuation.

8. The bases at present employed are in curious contrast, not one of the above tables of mortality being employed at the last valuation of any British Office (the Government Annuitants' Table mentioned above being, in most cases, the 1860 table). The tables now employed are (1) those based on the Government Experience and adopted by the Government in April 1884, and (2) those based on the British Offices' Annuity Experience, 1863-1893. These tables may, however, be applied in so many ways (because of the element of selection), and the returns are

Statement showing classification of the Basis of Valuation of Immediate Annuities and the total liability under all Annuity Contracts of the Life Assurance Companies of the United Kingdom (Ordinary Companies). Last valuation not later than 1 *March* 1908.

Number of Companies	Basis of Valuation of Immediate Annuities	Liability under all Annuity Contracts
4	Government Annuity Tables, 1883, $3\frac{1}{2}$ per-cent	£2,805,222
1	,, ,, ,, $3\frac{1}{4}$ per-cent	256,171
16	,, ,, ,, 3 per-cent	7,854,000
2	,, ,, ,, $2\frac{1}{2}$ per-cent	448,940
23		£11,364,333
21	British Offices' Annuity Tables . 3 per-cent	£8,800,273
5	,, ,, . $2\frac{1}{2}$ per-cent	1,239,526
26		£10,039,799
49		£21,404,132
16	Small Offices	£105,050
5	"Class" and Benevolent Institutions . .	1,519,079
70	Total liability	£23,028,261

Note.—In the 70 Offices referred to above, some are absorbed Offices making different Returns and at different times, to the Offices which have absorbed them. In such cases the figures have not been combined.

made to the Board of Trade without always defining the exact values employed, that no good purpose would be served by giving the detailed returns, and I have accordingly compiled the following statement on the broad lines as to basis therein given. The latest Valuation Return has been inspected in each case, excepting valuations at dates subsequent to 1 March 1908, and in order to obtain, approximately, the total liability of British Offices under their annuity contracts, the total net liability of each Office has been extracted, although the basis of valuation refers, in many cases, to Immediate Annuities only. (In one case the liability is "gross", the annuity and "life" reassurances being classed together.) The returns are given of forty-nine Offices, each having a liability exceeding £20,000; but the liability of sixteen other Offices, having an individual liability below that amount, and of five "class" and Benevolent Institutions are also added, so as to present the total figures of all "ordinary" British Offices included in the Board of Trade Returns, consistently with the other total Board of Trade figures which have been given.

9. Grouping the 49 Offices according to the rate of interest employed in the valuation, there are:

4	valuing at	$3\frac{1}{2}$	per-cent.
1	"	$3\frac{1}{4}$	"
37	"	3	"
7	"	$2\frac{1}{2}$	"
49			

10. It is not, however, so much in regard to the rate of interest employed as the table of mortality used in the valuations that most comment need be made. It is a significant fact that more than one-half the business of the 49 Offices was valued on the basis of the Government Annuities, although the new tables (constructed on their own collected experience) were available for every Office at its last valuation. The fact is even more significant when it is seen that many of the Offices adopted the O^{M} Table for Assurance Valuations and yet adhered to the mortality table previously employed for Annuities—namely, the Government Annuity Tables, 1883, notwithstanding that the O^{M} Table has far less effect in a valuation (when changing from the H^{M} Table) in respect of assurances—wherein three partly compensating factors are employed, A_x, a_x, and π_x—than have the $O^{[am]}$ and $O^{[af]}$ Tables (when changing from Government Annuity Tables, 1883) on

annuities. The comparison is still more marked if one Office, having a liability under annuity contracts of over £3,000,000 and valuing on the British Offices' Annuity Tables, be excluded from that class, leaving a liability of £7,000,000 valued by the latest experience as against £11,000,000 valued by the Government Tables.

11. In the course of the examination of the Returns one feature which presented itself was the number of times in which the Carlisle Table was referred to as having been employed for the valuation of annuities other than immediate. The Carlisle Table and the numerous monetary tables based thereon, are seldom alluded to in modern contributions, but the above-mentioned circumstance is another tribute to the remarkable energy of Chisholm, Jones and others, in the computation of so many functions and values.

Increase in Liability.

12. An indication of the increase in the liability of British Offices under their annuity contracts is contained in the following statement:

35 Offices transacting largest amount of business.

Valuation about 1875—	
Net Liability under Immediate Annuities	£3,004,998.
Valuation about 1880—	
Net Liability under Immediate Annuities	4,045,257.
Valuation 1901–1908—	
Net Liability under all annuity contracts	20,383,826.

Note—The 35 Offices included in respect of the last above period are not necessarily the same 35 Offices as examined by Mr. Ryan, but are those now transacting the largest amount of business. The minor classes of annuities included in the last item form a very small proportion of the total.

Profits of Past Twenty Years.

13. In view of the facts that, generally speaking, the rate of interest earned by the Companies has fallen during the past twenty years; that the mortality tables and rates of interest now employed in the valuations are much more stringent than those previously adopted; and that the latest investigation deduced from the experience of these Offices themselves reveals greater vitality than do the previous experiences, on which both rates and valuations have been based, the conclusion is forced upon one

that the profit made during the past two decades, if any, can only have been very scanty. With a view to ascertaining how the Companies fared at their succeeding valuations, a search was made for those keeping and publishing separate Annuity Accounts. Twelve such Offices were discovered, and their last four Valuation Returns, as well as their Accounts, have been analyzed, with the results set out in the table below. The basis of the valuation of each Office made just prior to the period under review has been quoted, as affording some indication of the augmentation which has been made to the reserves to bring them into line with modern bases. Although the tables of the new Government Experience published about that time were employed in one or two instances, they were probably "average" (or aggregate) values, no mention of "selection" being made in any of the Returns.

TABLE III.

Table showing, in respect of each of twelve Offices, the combined results of four successive Valuations, and the basis of Valuation prior thereto.

Office	Basis of Valuation made just prior to the period under review: Mortality	Interest	Combined results of four successive Valuations, covering, with slight exceptions, a period of 20 years: Surplus divided (+) or Deficit made good (−)	Depreciation of Investments, or placed to Investment Reserve	Increase (+) or Decrease (−) in sum carried forward
			£	£	£
A	Government Annuitants, 1829 and 1883, Tables	3½%	+ 88,632	36,236	+ 215
B	Government Annuitants Tables, 1860	4 %	− 6,033	...	...
C	"17 Offices' Experience" for old annuities, Government Annuitants, 1860, for new annuities	4 %	− 26,412	...	− 1,938
D	Government Annuitants, 1860	3½%	+ 7,155	...	+ 21,431
E	Government Annuitants, 1883	3 %	− 15,333	...	...
F	Carlisle	3 %	...	...	+ 302
G	"	3 %	...	...	+ 66,141
H	Government Annuitants, 1883	3 %	+ 10,000	...	− 6,539
J	Combination of Carlisle, Government Annuitants, 1825, and Equitable Tables	3½%	...	1,281	...
K	Equitable Table	3 %	+ 21,692	...	+ 4,303
L	Government Annuitants, 1883	3 %	+ 21,807	8,420	+ 7,646
M	Government	3 %	− 28,193	174	...
			+ 149,286	46,111	+ 100,038
			− 75,971	...	− 8,477
	Total net result of twenty years' working		+ 73,315	46,111	+ 91,561

14. If the largest Office be excluded, the net result from the point of view of division of profit, and ignoring the sum carried forward, is a deficit by the other eleven Offices of £15,317 on funds accumulating to the present total of £8,000,000.

15. The grouped figures of each quinquennium are even more instructive, as they reveal the profit or loss in relation to the respective amounts of total funds at each succeeding investigation. The twelve Offices have therefore been classified into three groups, namely—

5, which have made a positive division of profit;
3, which have made no division and suffered no loss;
4, which have made a net loss.

16. It should perhaps be explained that the figures are not those absolutely recorded in the Returns, but have been fixed in this analysis by deduction. For example, it has been necessary in some cases to bring various items spread over two quinquennial periods to one common valuation date; thus, loss on investments may have been taken retrospectively, and an annual division of profit, prospectively.

17. It will be noticed that the three Offices which have made no division at all, but at the same time have suffered no loss, are Offices which have developed their annuity business during the past ten or fifteen years, and entered the field with the knowledge which other Offices had obtained by experience. All three, at their valuations in the first period, and two of them in the second period, gave only combined figures of assurances and annuities, separating the sections of the business subsequently. At those times their liability was exceedingly small, and, to complete the table and keep it uniform, I have assumed the funds at those first two valuations to be equal to the liability.

TABLE IV

Analysis of Results of Twenty Years' working of Twelve British Ordinary Offices which keep separate Annuity Funds and publish Annuity Accounts.

(Extracted, or deduced, from the Returns to the Board of Trade.)

No. of Offices in Group	FIRST PERIOD DATES OF VALUATION RANGING FROM 1888 TO 1892 Brought forward from previous Valuation	Fund	Liability	Surplus (+) or Deficit (−) before allowing for Investments	Depreciation of Investments	Divided (+) or made good (−)	Carried forward	SECOND PERIOD DATES OF VALUATION RANGING FROM 1893 TO 1897. Fund	Liability	Surplus (+) or Deficit (−) before allowing for Investments	Depreciation of Investments	Divided (+) or made good (−)	Carried forward
5	7,756	1,593,779	1,566,618	+27,161	...	+21,485	5,676	3,439,956	3,399,420	+40,536	6,123	+22,138	11,975
3	...	11,243	11,243	...	...	...	...	76,041	76,041	...	...	...	...
4	1,938	1,987,224	2,003,684	−16,460	...	−16,617	157	2,579,986	2,548,703	+31,283	174	+30,443	666
12	9,694	3,592,246	3,581,545	+10,701	...	+ 4,868	5,833	6,095,983	6,024,164	+71,819	6,597	+52,581	12,641

No. of Offices in Group	THIRD PERIOD DATES OF VALUATION RANGING FROM 1898 TO 1902 Fund	Liability	Surplus (+) or Deficit (−) before allowing for Investments	Depreciation of Investments	Divided (+) or made good (−)	Carried forward	FOURTH PERIOD DATES OF VALUATION RANGING FROM 1903 TO 1907 Fund	Liability	Surplus (+) or Deficit (−) before allowing for Investments	Depreciation of Investments	Divided (+) or made good (−)	Carried forward
5	4,445,623	4,354,379	+91,244	4,806	+50,187	36,251	5,288,615	5,164,900	+123,715	33,427	+55,476	31,812
3	881,165	867,565	+13,600	...	...	13,600	1,802,391	1,734,670	+ 67,721	1,281	...	66,443
4	3,517,919	3,541,191	−23,272	...	−14,909	21,637	3,971,619	4,016,507	− 44,888	...	−44,888	...
12	8,844,707	8,763,135	+81,572	4,806	+ 5,278	71,488	11,062,628	10,916,077	+116,551	34,708	+10,588	101,255

18. The total net result of each valuation period may be summarized thus—

TABLE V.

	Valuations made during the periods			
	1888–1892	1893–1897	1898–1902	1903–1907
	£	£	£	£
Sum divided	4,868	52,581	5,278	10,588
Carried forward . . .	5,833	12,641	71,488	101,255
	10,701	65,222	76,766	111,843
Less brought forward . . (ignoring interest)	9,694	5,833	12,641	71,488
Total Net Surplus . .	1,007	59.389	64,125	40,355
On Funds accumulating to .	3,592.246	6,095,983	8,844,707	11,062,628

19. In the last three valuations, the ratio of Total Net Surplus to the funds, forms a decreasing progression, the surplus on the last occasion, if looked upon in the light of interest margin, resulting from a realized rate in excess of valuation rate of less than two shillings per-cent per annum. As there is no question as to the realized rate being actually well in excess of the valuation rate (such margin being nearer £1 per-cent per annum than two shillings), it follows that the difference between the rates has practically compensated for the losses due to greater longevity, depreciation of investments, the strengthening of reserves in respect of existing business, and, in some cases, to the loss on transaction of new contracts.

Valuations of Model Office.

20. In order to show the relative cost of strengthening the reserves, and the ratios subsisting between various bases of valuation, I have had recourse to the Model Office, framed by Mr. Ryan, in his paper (already referred to) before the Actuarial Society of Edinburgh. A word of explanation as to the construction of this Model Office might here be given. Mr. Ryan brought together from the valuation returns of the ten British Offices whose annual payments on account of annuities were, at that time, in excess of £10,000, the annuities payable (on single lives) at each age from 50 upwards. In order to obtain a smooth

TABLE VI.

Particulars of Mr. G. H. Ryan's Model Office.

Age last Birthday	Annuites in course of payment Ungraduated	Annuities (Male and Female) Graduated	Annuities (Females) Graduated	Annuities (Males) Graduated	Age last Birthday
	£	£	£	£	
50	1,208	2,239	1,444	795	50
1	2,212	2,438	1,572	866	1
2	1,715	2,795	1,803	992	2
3	3,687	3,246	2,094	1,152	3
4	3,569	3,710	2,393	1,317	4
55	5,604	4,134	2,667	1,467	55
6	5,766	4,544	2,931	1,613	6
7	3,106	4,984	3,215	1,769	7
8	4,883	5,495	3,544	1,951	8
9	4,667	6,116	3,945	2,171	9
60	6,899	6,872	4,433	2,439	60
1	7,617	7,704	4,969	2,735	1
2	9,005	8,500	5,483	3,017	2
3	11,294	9,185	5,925	3,260	3
4	9,880	9,726	6,274	3,452	4
65	9,777	10,078	6,501	3,577	65
6	10,836	10,204	6,582	3,622	6
7	12,226	10,215	6,589	3,626	7
8	8,793	10,150	6,547	3,603	8
9	9,833	10,006	6,454	3,552	9
70	10,124	9,821	6,335	3,486	70
1	9,838	9,684	6,246	3,438	1
2	8,220	9,516	6,138	3,378	2
3	10,959	9,276	5,983	3,293	3
4	9,679	9,008	5,810	3,198	4
75	7,520	8,667	5,590	3,077	75
6	9,310	8,204	5,292	2,912	6
7	8,105	7,653	4,936	2,717	7
8	7,949	7,100	4,580	2,520	8
9	4,324	6,488	4,185	2,303	9
80	7,343	5,897	3,804	2,093	80
1	3,877	5,341	3,445	1,896	1
2	5,468	4,831	3,116	1,715	2
3	4,194	4,298	2,772	1,526	3
4	4,384	3,800	2,451	1,349	4
85	2,675	3,280	2,116	1,164	85
6	3,386	2,767	1,785	982	6
7	1,955	2,268	1,463	805	7
8	1,330	1,829	1,180	649	8
9	1,066	1,439	928	511	9
90	780	1,129	728	401	90
1	802	878	566	312	1
2	1,078	677	437	240	2
3	224	513	331	182	3
4	235	378	244	134	4
95	198	263	170	93	95
6	83	181	117	64	6
7	...	124	80	44	7
8	62	94	61	33	8
Total	257,745	257,745	166,254	91,491	..

series, he graduated the figures and then split them into annuities on female and male lives in the proportion which the sexes were found to bear to one another in the experience of Government Annuitants investigated by Mr. A. J. Finlaison, namely: 19,859 females to 10,929 males. The resulting Model Office is shown on the previous page.

21. Such Model Office was valued by several mortality tables (the most modern table employed at that time being the Government Annuitants, 1883, "average" (or aggregate) values), and at three rates of interest, namely—3 per-cent, $3\frac{1}{2}$ per-cent and 4 per-cent, the valuation factor employed being $(\frac{1}{2}+a_x)$, the addition of ·5 having been made as roughly representing the adjustments for half-yearly and other instalments, and for proportion to day of death.

22. These valuations I have supplemented with others on modern bases, though in the tables of reserves I have not included many of the bases given by Mr. Ryan, but have only selected those which were most popular at that time, namely—Carlisle, Davies' Equitable, and Government Annuitants, 1860, adding, of course, Mr. Ryan's standard basis, Government Annuitants, 1883, "average" (or aggregate) values. Further, I have not employed the rate of 4 per-cent which is now quite obsolete, but only the two lower rates, 3 and $3\frac{1}{2}$ per-cent. The factor used, is, for purposes of comparison, the same as that employed by Mr. Ryan, namely $(\frac{1}{2}+a_x)$ or $(\frac{1}{2}+a_{[x]})$ as the case may be. The supplementary bases which I have included are—

Government Annuitants, 1883—Select-values.
" " Ultimate-values.
British Offices' Annuity Tables, 1893—Select-values.
" " Ultimate-values.

I have also made an arbitrary assumption in order to approximate to the reserve obtained by valuing the business of less than five years' duration as all "select", and the remainder as "non-select." This assumption is based on the facts that, roughly, one-third of the contracts of an Office transacting a large and almost stationary business are of less than five years' duration, and that practically no annuities are purchased at ages more advanced than 80. It will be found that in the Model Office, 40 per-cent of the annuities from age 50 to 79 inclusive equals 33·8 per-cent of the total annuities (*i.e.*, practically one-third), and the business has therefore been split into "select" and "non-select"

Table VII.

Reserves of Model Office (Mr. Ryan's construction) according to various Bases.

I.—*Interest 3 per-cent.*

Age Period	Carlisle Table	Davies' Equitable	Government Annuities, 1860 (Males and Females)	Government Annuities, 1883 (Average Values) (Males and Females)	Government Annuities, 1883 (Ultimate Values) (Males and Females)	Government Annuities, 1883 (Select and Non-Select) (Males and Females)	Government Annuities, 1883 (Select Values) (Males and Females)	O[am] & O[af] (Ultimate Values) (Males and Females)	O[am] & O[af] (Select and Non Select) (Males and Females)	O[am] & O[af] (Select Values) (Males and Females)	Age Period
50–59	505,707	510,441	M 178,313	M 179,860	M 176,407	M. 177,144	M 178,250	M 179,583	M 181,108	M 183,394	50–59
			F 355,371	F 357,929	F 350,669	F 353,195	F. 356,984	F 357,941	F 361,281	F 366,290	
			533,684	537,789	527,076	530,339	535,234	537,524	542,389	549,684	
60–69	874,929	912,762	M 307,172	M 308,566	M 301,563	M. 305,673	M 311,837	M. 308,038	M 312,397	M. 318,935	60–69
			F 615,157	F. 617,434	F 607,072	F 614,561	F 625,795	F 631,141	F. 639,158	F 651,182	
			922,329	926,000	908,635	920,234	937,632	939,179	951,555	970,117	
70–79	540,352	575,918	M 191,212	M 188,003	M 183,828	M 190,169	M 199,680	M 190,266	M 195,160	M 202,500	70–79
			F 379,993	F 371,598	F 367,706	F 378,484	F 394,650	F 388,094	F 396,868	F 410,030	
			571,205	559,601	551,534	568,653	594,330	578,360	592,028	612,530	
80–89	147,765	139,382	M 49,966	M 47,712	M 46,933	M 46,933	M 49,451	M 48,455	M 48,455	M 54,207	80–89
			F 97,033	F 91,946	F 91,450	F 91,450	F 96,470	F 97,578	F 97,578	F 108,151	
			146,999	139,658	138,383	138,383	145,921	146,033	146,033	162,358	
90–98	13,031	7,984	M 3,197	M 3,173	M 3,184	M 3,184	M 3,184	M. 3,255	M 3,255	M 3,918	90–98
			F 6,116	F 5,937	F 6,039	F 6,039	F 6,039	F 6,492	F 6,492	F 7,737	
			9,313	9,110	9,223	9,223	9,223	9,747	9,747	11,655	
Total	2,081,784	2,146,487	M 729,860	M 727,314	M. 711,915	M 723,103	M 742,402	M 729,597	M 740,375	M 762,954	
			F 1,453,670	F 1,444,844	F 1,422,936	F 1,443,729	F 1,479,938	F 1,481,246	F 1,501,377	F 1,543,390	
			2,183,530	2,172,158	2,134,851	2,166,832	2,222,340	2,210,843	2,241,752	2,306,344	
Ratio to Government, 1883 (Average values)	95.9	98.8	100.6	100	98.3	99.8	102.3	101.8	103.2	106.2	
Ratio to Government, 1883 (Average values) Females only			100.6	100	98.5	99.9	102.4	102.5	103.9	106.8	

II.—*Interest 3½ per cent.*

Age Period	Carlisle Table	Davies' Equitable	Government Annuities, 1860 (Males and Females)	Government Annuities, 1883 (Average Values) (Males and Females)	Government Annuities, 1883 (Ultimate Values) (Males and Females)	Government Annuities, 1883 (Select and Non Select) (Males and Females)	Government Annuities, 1883 (Select Values) (Males and Females)	O[am] & O[af] (Ultimate Values) (Males and Females)	O[am] & O[af] (Select and Non Select) (Males and Females)	O[am] & O[af] (Select Values) (Males and Females)	Age Period
50–59	483,292	487,455	M 170,510	M 171,138	M 168,704	M 169,405	M 170,458	M 171,623	M 173,073	M 175,247	50–59
			F. 338,525	F 340,831	F 334,266	F. 336,660	F 340,250	F 340,780	F 343,941	F 348,683	
			509,035	511,969	502,970	506,065	510,708	512,403	517,014	523,930	
60–69	845,279	880,544	M 296,785	M. 298,262	M 291,665	M 295,619	M 301,551	M 297,763	M 301,891	M 308,173	60–69
			F 591,913	F 595,378	F 585,588	F 592,787	F 603,586	F 607,939	F 615,767	F 627,420	
			888,698	893,640	877,253	888,406	905,137	905,702	917,656	935,593	
70–79	527,185	561,717	M 186,597	M 183,608	M 179,603	M 185,764	M 195,005	M 185,766	M 190,524	M 197,659	70–79
			F 370,060	F 362,382	F 358,650	F 369,106	F 384,790	F 378,066	F 386,582	F 399,355	
			556,657	545,990	538,253	554,870	579,795	563,832	577,106	597,014	
80–89	145,302	137,291	M 49,183	M. 46,996	M. 46,236	M 46,236	M 48,694	M 47,721	M 47,721	M 53,359	80–89
			F 95,435	F 90,503	F 90,016	F 90,016	F 94,915	F 95,961	F 95,961	F 106,307	
			144,618	137,499	136,252	136,252	143,609	143,682	143,682	159,666	
90–98	12,854	7,925	M 3,171	M 3,147	M 3,156	M 3,156	M 3,156	M 3,227	M 3,227	M 3,880	90–98
			F 6,065	F 5,883	F 5,985	F 5,985	F 5,985	F 6,430	F 6,430	F 7,656	
			9,236	9,030	9,141	9,141	9,141	9,657	9,657	11,536	
Total	2,013,902	2,074,932	M 706,246	M 703,151	M 689,364	M 700,180	M 718,864	M 706,040	M 716,436	M 738,318	
			F 1,401,998	F 1,394,977	F 1,374,505	F 1,394,554	F 1,429,526	F 1,429,236	F 1,448,681	F 1,489,421	
			2,108,244	2,098,128	2,063,869	2,094,734	2,148,390	2,135,276	2,165,117	2,227,739	
Ratio to Government, 1883 (Average values)	96.0	98.9	100.4	100	98.4	99.8	102.4	101.8	103.2	106.2	
Ratio to Government, 1883 (Average values) Females only			100.5	100	98.5	100.0	102.5	102.5	103.9	106.8	

accordingly, 40 per-cent of the annuities at ages 50 to 79 both inclusive being regarded as "select", and the remaining 60 per-cent of the annuities at these ages, with the total annuities at age 80 and upwards, as "non-select." The reserves, on such partly select and partly non-select basis, in the case both of Government Annuitants, 1883, and the British Offices' Annuity Tables, 1893, have accordingly been added to the Table. (*See* Table VII.)

23. Below the total reserve in each case is given the ratio to Mr. Ryan's standard basis. In considering these ratios as a guide to the basis of valuation to be employed for modern business, it should be borne in mind that the ratio subsisting between female and male lives, as revealed at the last investigation, differs very considerably from the ratio between the two sexes as given in the Government Experience. The proportion of females to males in the Government Annuity Experience was 1,817 females to every 1,000 males. The number of lives under observation in respect of "new" annuities in the British Offices' Experience was 18,928 females and 7,252 males, the proportion being 2,610 females to every 1,000 males, the increase in the ratio in comparison with the Government Experience being about 44 per-cent. This is a very striking development of the business, especially in view of the greater longevity of female than of male annuitants. In order that this point should not be overlooked in considering the results of the Model Office, I have also shown the ratios subsisting between the reserves on the various bases (in which the lives are separated according to sex) on the female section of the business only. It will be observed that the ratios to the standard reserves (females only) of the reserves under other bases (females only) are greater than the ratios subsisting between the same respective reserves for the combined sections.

24. Additional tables of ratios are given to show the relation subsisting between the reserves by the various tables and by two modern bases of valuation. The two standard bases which have been adopted for this purpose are—

(*a*) British Offices' Annuity Tables—40 per-cent of annuities at ages 50–79 inclusive, select; remainder, non-select. Interest 3 per-cent.

(*b*) British Offices' Annuity Tables—All select. Interest 3 per-cent.

In the first table are shown the figures resulting from the use of Mr. Ryan's complete Model Office, while in the other the

TABLE VIII.

Reserves of Model Office (Mr. Ryan's construction) according to various bases of valuation, with ratio to modern standards.

(*a*) *British Offices' Annuity Tables—40 per-cent of annuities at ages 50–79 inclusive, select: remainder, non-select. Interest 3 per-cent.*

(*b*) *British Offices' Annuity Tables—All select. Interest 3 per-cent.*

Basis of Valuation		Reserve by Model Office	Ratio to Standard (*a*)	Ratio to Standard (*b*)
		£		
Carlisle	3 %	2,081,784	92·9	90·3
,, ...	3½%	2,013,902	89·8	87·3
,, ...	4 %	1,949,937	87·0	84·5
Davies' Equitable	3 %	2,146,487	95·8	93·1
,, ,, ...	3½%	2,074,932	92·6	90·0
,, ,, ...	4 %	2,007,886	89·6	87·1
Government Annuities, 1860 . .	3 %	2,183,530	97·4	94·7
,, ,, ,, . .	3½%	2,108,244	94·0	91·4
,, ,, ,, . .	4 %	2,040,279	91·0	88·5
Government Annuities, 1883—Average values.	3 %	2,172,158	96·9	94·2
Government Annuities, 1883—Average values.	3½%	2,098,128	93·6	91·0
Government Annuities, 1883—Average values.	4 %	2,030,660	90·6	88·1
Government Annuities, 1883—Ultimate values.	3 %	2,134,851	95·2	92·6
Government Annuities, 1883—Ultimate values.	3½%	2,063,869	92·1	89·5
Government Annuities, 1883—Partly select, partly non-select.	3 %	2,166,832	96·7	94·0
Government Annuities, 1883—Partly select, partly non-select.	3½%	2,094,734	93·4	90·8
Government Annuities, 1883—Select values.	3 %	2,222,340	99·1	96·4
Government Annuities, 1883—Select values.	3½%	2,148,390	95·8	93·2
British Offices' Annuity Tables—Ultimate values.	3 %	2,210,843	98·6	95·9
British Offices' Annuity Tables—Ultimate values.	3½%	2,135,276	95·3	92·6
British Offices' Annuity Tables—Partly select, partly non-select.	3 %	2,241,751	**100**	97·2
British Offices' Annuity Tables—Partly select, partly non-select.	3½%	2,165,117	96·6	93·9
British Offices' Annuity Tables—Select values.	3 %	2,306,344	102·9	**100**
British Offices' Annuity Tables—Select values.	3½%	2,227,739	99·4	96·6

figures are in respect of the Female section only. The greater difference from the standard bases shown in the latter, than in the combined sections, will be noted.

Table IX.

Reserves of Model Office (Mr. Ryan's construction), Female Section only. *According to various bases of valuation, with ratio to modern standards—*

(*a*) *British Offices' Annuity Tables—40 per-cent of annuities at ages 50–79 inclusive, select; remainder, non-select. Interest 3 per-cent.*

(*b*) *British Offices' Annuity Tables—All select. Interest 3 per-cent.*

Basis of Valuation		Reserve by Model Office (Female Section only)	Ratio to Standard (*a*)	Ratio to Standard (*b*)
		£		
Government Annuities, 1860 . . .	3 %	1,453,670	96·8	94·2
,, ,, ,, . . .	3½%	1,401,998	93·4	90·8
,, ,, ,, . . .	4 %	1,356,263	90·3	87·9
Government Annuities, 1883—Average values.	3 %	1,444,844	96·2	93·6
Government Annuities, 1883—Average values.	3½%	1,394,977	92·9	90·4
Government Annuities, 1883—Average values.	4 %	1,348,774	89·8	87·4
Government Annuities, 1883—Ultimate values.	3 %	1,422,936	94·8	92·2
Government Annuities, 1883—Ultimate values.	3½%	1,374,505	91·5	89·1
Government Annuities, 1883—Partly select and partly non-select.	3 %	1,443,729	96·2	93·5
Government Annuities, 1883—Partly select and partly non-select.	3½%	1,394,554	92·9	90·4
Government Annuities, 1883—Select values.	3 %	1,479,938	98·6	95·9
Government Annuities, 1883—Select values.	3½%	1,429,526	95·2	92·6
British Offices' Annuity Tables—Ultimate values.	3 %	1,481,246	98·7	96·0
British Offices' Annuity Tables—Ultimate values.	3½%	1,429,236	95·2	92·6
British Offices' Annuity Tables—Partly select and partly non-select.	3 %	1,501,377	100	97·3
British Offices' Annuity Tables—Partly select and partly non-select.	3½%	1,448,681	96·5	93·9
British Offices' Annuity Tables—Select values.	3 %	1,543,390	102·8	100
British Offices' Annuity Tables—Select values.	3½%	1,489,421	99·2	96·5

25. The results of the various valuations are very interesting, but reference need only here be made to one or two particular instances. The reserves of the Carlisle Table, at 4 per-cent (a basis employed by some of the offices detailed in Mr. Ryan's paper) are only 84·5 per-cent of those required by the British Offices' Annuity-Tables 1893 (all select) at 3 per-cent, or 87 per-cent of the partly select and partly non-select values, at the same rate; while, to make a comparison between more modern bases, the reserves by the Government 1883 Tables, partly select and partly non-select, at 3 per-cent interest is but 94 per-cent of the reserves by the British Offices' Annuity Tables, all select, at the same rate of interest. In regard to the bases employed 20–30 years ago, there can be little wonder that Offices have shown no profit when endeavouring to augment their reserves, say from Carlisle, 4 per-cent, to the British Offices' Tables at 3 per-cent; while as regards modern valuations, there is considerable room for greater stringency in the reserves, looking to the large proportion of business still valued by the Government Tables.

Valuation of an Actual Experience.

26. Although the results of the valuations which I have made on the arbitrary assumptions regarding the proportion of "select" business may prove useful to some extent as a guide, I have thought it would be interesting to see the results of valuations, on various bases, of an actual experience which has come under my notice, and in which the select lives are separately classified. The valuations have all been made at 3 per-cent, using throughout the factor for annuities payable half-yearly, without proportion to day of death, and ignoring, for the present purpose, the usual necessary adjustments. The bases as regards mortality are—

1. Government Annuities, 1883.—Less than 5 years' duration —Select values (as at entry). More than 5 years' duration—Non-select values.
2. Government Annuities, 1883.—All select (as at entry).
3. British Offices' Annuity Experience, 1893.—Less than 5 years' duration—allowing for period elapsed since entry, but ignoring fractions of a year. More than 5 years' duration—Non-select values.
4. British Offices' Annuity Experience, 1893.—Less than 5 years' duration—Select values (as at entry). More than 5 years' duration—Non-select values.
5. British Offices' Annuity Experience, 1893.—All select (as at entry).

TABLE X.

Valuation of actual experience of Immediate Annuities, classified according to age and date of entry. Valued as payable half-yearly, without proportion to day of death. No other Adjustments.

Section of Experience	Amount of Annuities	Government Annuities, 1883, 3 per-cent: Old—Non-select New—Select	Government Annuities, 1883, 3 per-cent: All select	British Offices Annuity Tables, 3 per-cent: Old—Non-select New—Allowing for selection and period elapsed since entry	British Offices Annuity Tables, 3 per-cent: Old—Non-Select New—Select	British Offices Annuity Tables, 3 per-cent: All select
Males—"Old" (duration more than 5 years) . .	43,994 18 5	343,840·6	361,195·1	353,511·3	353,511·3	369,206·9
Males—"New" (duration less than 5 years) . .	17,352 12 5	161,332·7	161,332·7	161,763·6	165,516·4	165,516·4
Total Males.	61,347 10 10	505,173·3	522,527·8	515,274·9	519,027·7	534,723·3
Females—"Old" (duration more than 5 years) .	82,009 16 11	721,082·5	754,077·4	747,303·9	747,303·9	777,428·3
Females—"New" (duration less than 5 years) .	33,562 11 11	334,089·1	334,089·1	337,211·0	344,741·8	344,741·8
Total Females	115,572 8 10	1,055,171·6	1,088,166·5	1,084,514·9	1,092,045·7	1,122,170·1
Grand Total	176,919 19 8	1,560,314·9	1,610,694·3	1,599,789·8	1,611,073·4	1,656,893·4
Ratio to British Offices' Tables— Duration more than 5 years, non-select Duration less than 5 years, select	...	96·85	99·98	99·30	**100**	102·84
Ratio to British Offices' Tables—All select . .	...	94·17	97·21	96·55	97·23	**100**

27. From the results it will be seen that if an Office has been using the Government Annuity, 1883 (all select), values as a precautionary measure because of greater vitality among the annuitants than indicated by the Government Experience, the change can be made to British Offices' Annuity Tables, valuing the last five years' business only as select and the remainder as non-select, without suffering any loss on such change. In Mr. Ryan's Model Office the ratio between Government Annuities, 1883 (all select), and British Offices' Annuity Tables (partly select and partly non-select) is 99·1, but, of course, in any change from select to non-select, results will vary according to the proportion of select business and the age incidence thereof.

28. If an office has been valuing by the Government tables, using non-select values for business of more than five years' duration and select values for the remainder, the British Offices' Tables, allowing for period elapsed since entry, will prove a useful basis as a stepping stone to pass to more stringent reserves at a subsequent valuation.

29. The point might here be mentioned that in using select values (as at entry) for all annuities of less than five years' duration, and non-select for those of longer duration, the reserves may fluctuate somewhat if there is a wide difference between the new business of the last year and the corresponding year of the previous quinquennium, owing partly to the magnitude of $(a_{[x]}-a_{x+1})$ as compared with $(a_{[x-4]+4}-a_{x+1})$. This feature is more noticeable in the results of annual valuations.

Basis of Present Rates.

30. But whatever the basis of valuation may be, the future course of annuity business will not be satisfactory unless the terms on which existing contracts are being entered into are fixed in accordance with the experience which has been gained in the past, and a reasonable forecast of the future. From the examination of the rates of many Companies it would appear that they are based (with various degrees of loading) on the Government Experience, with 3 per-cent interest. Whatever the feeling may be as to the compensation for the greater vitality according to the British Offices' Experience, by the somewhat more remunerative rate of interest now obtainable than was the case before the close of the last century, such set-off of one factor against another is not scientific. The following table is therefore given showing the ratios subsisting between the British Offices' Select Annuities at

$3\frac{1}{4}$ per-cent, and the Government, 1883, Select Annuities at 3 per-cent, also between the former at $3\frac{1}{2}$ per-cent and the Government 3 per-cent.

TABLE XI.

Tables of Ratios.

Age $[x]$	Males		Females	
	$\frac{a_{[x]}\ O^{[am]}\ 3\frac{1}{4}\%}{a_{[x]}\ \text{Govt. } 3\%}$	$\frac{a_{[x]}\ O^{[am]}\ 3\frac{1}{2}\%}{a_{[x]}\ \text{Govt. } 3\%}$	$\frac{a_{[x]}\ O^{[af]}\ 3\frac{1}{4}\%}{a_{[x]}\ \text{Govt. } 3\%}$	$\frac{a_{[x]}\ O^{[af]}\ 3\frac{1}{2}\%}{a_{[x]}\ \text{Govt. } 3\%}$
40	1·0399	1·0067	·96986	·93730
45	1·0286	·99862	·97483	·94469
50	1·0148	·98826	·98682	·95914
55	1·0041	·98075	1·0000	·97479
60	1·0053	·98480	1·0140	·99152
65	1·0064	·98876	1·0227	1·0032
70	1·0041	·98919	1·0338	1·0171
75	1·0028	·99035	1·0250	1·0113
80	·98637	·97649	1·0126	1·0016

31. It will be observed from these ratios that, in the case of male lives, the difference of $\frac{1}{4}$ per-cent in the rate does compensate very nearly for the difference in mortality (particularly at the purchasing ages—say, ages sixty to seventy-five), though throughout the table the $O^{[am]}$ $3\frac{1}{4}$ per-cent annuity-value is greater than the Government 3 per-cent value, producing a smaller rate of annuity per-cent.

32. In the case of female lives the difference in the rate of interest of $\frac{1}{4}$ per-cent fully compensates for the greater vitality of lives entering before age fifty-five, but from that age onwards (and it is at the later ages that most of the business is transacted) the $O^{[af]}$ $3\frac{1}{4}$ per-cent values considerably exceed the Government 3 per-cent values, while from age sixty-five onwards the $O^{[af]}$ $3\frac{1}{2}$ per-cent values exceed the Government 3 per-cent.

33. The arbitrary nature of the set-off of excess interest against lighter mortality and the inequitable compensation resulting therefrom, is perhaps still more manifest when it is considered that, whereas a difference of one-half per-cent in the rate of interest (keeping to the same mortality basis) results in a difference nearly *constant* at all ages—ranging between the narrow limits of 7*s*. 6*d*. and 9*s*.—in the rate of annuity for £100 purchase-money, a change in the mortality basis from the Government to the British Offices' Experience (keeping the rate of interest constant) results in a *varying* difference in the rate of

annuity, ranging, in the case of male lives, from over 8*s.* per-cent at age 40 to zero at age 80, and in the case of female lives, conversely from almost zero at age 40 to 12*s.* per-cent at the older ages. These differences were tested at three rates of interest and were found, at any particular age, to be almost identical. The facts just stated may therefore be accepted as a general guide to the effect on annuity rates of any proposed change in the basis.

Suggested Basis for New Rates.

34. With the object of obtaining the views of the profession on this question, and particularly as to the desirability of a change in the rates generally current, I have constructed tables of rates on what are submitted as the maximum terms on which annuities should be granted, if any remuneration is to be obtained and future loss avoided. The rates are of annuities payable half-yearly, without proportion to day of death (for £100 purchase money, the office paying the stamp duty on the bond).

35. As to the basis of mortality, no question will doubtless be raised—British Offices' Annuity Experience, select at entry.

36. As regards the rate of interest, it is submitted that as this will prove the only source of profit and that the excess over the rate assumed will be liable to have set against it any loss arising from greater longevity, depreciation of investments, or undue expenses, the margin in the rate of interest should be at least $\frac{1}{2}$ per-cent. A moderate estimate of the rate of interest obtainable by life offices on the whole of their funds may, after deduction of tax, be placed at $3\frac{3}{4}$ per-cent, resulting in the employment of $3\frac{1}{4}$ per-cent as the maximum rate.

Expense Ratios.

37. The loading should, as far as can be gauged, be in accordance with fact. I have assumed initial expenses of £2 per-cent on the purchase-money (being £1 per-cent commission, 10*s.* per-cent stamp duty, and 10*s.* per-cent for general initial expenses), and as charges in connection with the payment of instalments, $1\frac{1}{2}$ per-cent on the amount of the annuities. In connection with these suggested loadings (though the matter is also of interest by itself), I have taken out the expense ratios of the twelve offices publishing separate annuity accounts for

the year 1906, with the results shown in the table below. The following assumptions have been made:

That stamp duty is included in the expenses, whether obtained from the purchaser as a separate item or not.

That, with one or two exceptions, the initial expenses (excluding commission, but including stamp duty) are £1 per-cent.

That annual premiums for deferred and other annuities are subject to initial and renewal commission of 5 per-cent, and to 5 per-cent for general expenses.

TABLE XII.

Expense Ratios of Twelve Offices publishing separate Annuity Accounts.

Office	Initial Expenses expressed as a percentage of the Consideration	Annuity Payment Charges per-cent
A	1·77 per-cent, ·77 per-cent being commission	1·27 per-cent
B	2·26 ,, 1·26 ,, ,, ,,	1·99 ,,
C	2·92 ,, 1·92 ,, ,, ,,	3·40 ,,
D	2·00 ,, 1·00 ,, ,, ,,	Nil
E	1·89 ,, ·89 ,, ,, ,,	2·06 ,,
F	·53 ,, ·25 ,, ,, ,,	Nil
G	2·00 ,, 1·00 ,, ,, ,,	4·58 ,,
H	3·25 ,, 2·25 ,, ,, ,,	·39 ,, alternative views
	2·00 ,, 1·00 ,, ,, ,,	1·40 ,,
J	1·85 ,, ·85 ,, ,, ,,	2·01 ,,
K	1·76 ,, ·76 ,, ,, ,,	1·76 ,,
L	1·53 ,, ·53 ,, ,, ,,	2·10 ,,
M	2·00 ,, 1·00 ,, ,, ,,	1·00 ,,
On the combined figures for the twelve offices	1·997 ,, ·997 ,, ,, ,,	1·745 ,,

38. In fixing the loading at £2 per-cent on the price and $1\frac{1}{2}$ per-cent on the annuity I have been guided, besides general considerations, by the figures of the largest office and by the desire to produce the maximum terms on which annuities could with safety be granted. (In the case of many of the companies the figures have, it is obvious, been fixed quite arbitrarily, and in every case there is, no doubt, some difficulty in apportioning the share of expenses of the annuity section of an office.)

39. The following tables have accordingly been prepared on the formula—

$$\frac{98}{1{\cdot}015\,(a_{[x]}+{\cdot}25)}$$

(taking the nearest even penny below the resulting decimal) and

Table XIII.

Table of rates of Annuities, payable half-yearly, without proportion to day of death (for £100 purchase-money), the company paying the stamp duty.

Age	Males: Suggested maximum rate	Males: Average of Six Offices (to nearest penny)	Males: Highest rate of any of the Six Offices	Females: Suggested maximum rate	Females: Average of Six Offices (to nearest penny)	Females: Highest rate of any of the Six Offices	Age
	£ *s.* *d.*	£ *s.* *d.*	£ *s.* *d.*	£ *s.* *d.*	£ *s.* *d.*	£ *s.* *d.*	
40	5 11 8	5 15 3	6 0 10	5 7 10	5 5 3	5 7 4	40
1	5 13 6	5 17 0	6 2 5	5 9 4	5 6 6	5 8 10	1
2	5 15 6	5 18 9	6 4 1	5 11 0	5 8 1	5 10 4	2
3	5 17 6	6 0 7	6 5 11	5 12 6	5 9 8	5 12 0	3
4	5 19 8	6 2 7	6 7 9	5 14 2	5 11 5	5 13 8	4
5	6 1 10	6 4 8	6 9 9	5 15 10	5 13 3	5 15 5	5
6	6 4 4	6 6 10	6 11 9	5 17 8	5 15 3	5 17 3	3
7	6 6 10	6 9 1	6 13 11	5 19 8	5 17 3	5 19 2	7
8	6 9 6	6 11 7	6 16 3	6 1 8	5 19 5	6 1 2	8
9	6 12 4	6 14 1	6 18 8	6 3 10	6 1 9	6 3 4	9
50	6 15 4	6 16 6	7 1 3	6 6 0	6 4 3	6 5 6	50
1	6 18 4	6 19 6	7 4 1	6 8 4	6 6 10	6 7 11	1
2	7 1 10	7 2 8	7 7 0	6 10 10	6 9 6	6 10 5	2
3	7 5 4	7 6 0	7 10 2	6 13 6	6 12 4	6 13 1	3
4	7 9 0	7 9 7	7 13 7	6 16 4	6 15 4	6 15 11	4
5	7 13 0	7 13 6	7 17 3	6 19 4	6 18 7	6 19 0	5
6	7 17 4	7 17 8	8 1 3	7 2 6	7 2 1	7 2 6	6
7	8 1 10	8 2 1	8 5 6	7 6 0	7 5 10	7 6 6	7
8	8 6 6	8 6 11	8 10 3	7 9 8	7 9 10	7 10 10	8
9	8 11 8	8 12 2	8 15 5	7 13 10	7 14 2	7 15 6	9
60	8 17 0	8 17 9	9 0 11	7 18 2	7 18 9	8 0 4	60
1	9 2 8	9 3 7	9 6 8	8 2 10	8 3 9	8 5 6	1
2	9 8 10	9 9 8	9 12 7	8 8 0	8 9 1	8 11 0	2
3	9 15 4	9 16 2	9 19 0	8 13 6	8 14 9	8 16 10	3
4	10 2 2	10 3 2	10 5 10	8 19 6	9 0 11	9 3 2	4
5	10 9 8	10 10 8	10 13 3	9 5 10	9 7 8	9 10 2	5
6	10 17 6	10 18 6	11 0 11	9 12 10	9 15 0	9 17 8	6
7	11 5 10	11 6 10	11 9 0	10 0 4	10 2 11	10 6 0	7
8	11 14 10	11 15 8	11 17 8	10 8 6	10 11 5	10 14 8	8
9	12 4 6	12 5 3	12 7 0	10 17 2	11 0 7	11 4 2	9
70	12 14 8	12 15 7	12 17 2	11 6 8	11 10 4	11 14 2	70
1	13 5 8	13 6 10	13 8 0	11 16 8	12 0 1	12 4 4	1
2	13 17 4	13 18 9	13 19 8	12 7 6	12 11 0	12 15 2	2
3	14 10 0	14 11 5	14 12 4	12 19 2	13 2 7	13 6 8	3
4	15 3 4	15 4 9	15 6 0	13 11 6	13 15 2	13 18 10	4
5	15 17 10	15 18 11	16 0 8	14 4 10	14 8 6	14 12 0	5
6	16 13 4	16 13 8	16 16 6	14 19 2	15 3 8	15 6 0	6
7	17 9 10	17 9 7	17 13 4	15 14 8	15 18 10	16 1 0	7
8	18 7 8	18 6 6	18 11 6	16 11 2	16 15 0	16 17 0	8
9	19 6 8	19 4 9	19 11 0	17 8 10	17 12 5	17 14 2	9
80	20 7 2	20 4 3	20 12 0	18 7 10	18 11 1	18 12 6	80

Note.—Some of the Offices do not publish rates for ages above 70 or 75.

I have included for comparison, the average (to the nearest penny) of the rates of the six British offices transacting the largest amount of annuity business in 1906, which publish their rates, payable half yearly, without proportion to death, and themselves pay the bond stamp. I have also added a column giving the highest rate quoted by any of these six offices, at each age.

Comparison of Rates.

40. It will be observed that the average rates are, in the case of males, higher throughout the tables than the suggested rates; while in the case of females, the suggested rates are higher than the average at the younger ages, but lower at the older (and annuity granting) ages, crossing one another at or about age 57. These results are practically in keeping with the ratios subsisting between the British Offices' Annuity Values at $3\frac{1}{4}$ per-cent and the Government Annuities, 1883, at 3 per-cent, shown in Table XI. If the six largest Offices be selected simply according to the amount of business transacted, irrespective of the method of purchase and payment on the basis of which their rates are published, the differences between the latter and the suggested rates would be more marked.

41. The comparison is also made, at quinquennial ages, between the suggested rates, the highest rate at the particular age of *any* British Office (whatever the published terms of purchase and payment), the annuities granted through the medium of the Post Office and Trustee Savings Banks, and the annuities granted by the Commissioners for the Reduction of the National Debt (taking two prices of Consols—(*a*) price during January last, say not exceeding £83. 13s. 8d.,* (*b*) at par). In all cases the terms, however quoted, have been reduced to the standard method of purchase and payment adopted throughout this paper.

* The lowest price reached during January was $82\frac{1}{16}$.

TABLE XIV.

Table of comparative Rates of Annuities, reduced to a common method of purchase and payment—namely, Annuity payable half-yearly, without proportion to day of death, for £100 purchase-money, the stamp duty being paid by Grantors, or avoided.

Age	Males: Suggested maximum rate	Males: Highest of any British Office	Males: Post Office	Males: Commissioners of National Debt—Consols between £82. 19s. 9d. & £83. 13s. 8d.	Males: Commissioners of National Debt—Consols above £99. 10s. 1d.	Females: Suggested maximum rate	Females: Highest of any British Office	Females: Post Office	Females: Commissioners of National Debt—Consols between £82. 19s. 9d. & £83. 13s. 8d.	Females: Commissioners of National Debt—Consols above £99. 10s. 1d.	Age
	£ s. d.	£ s. d.	£ s. d.	£ s. d.	£ s. d.	£ s. d.	£ s. d.	£ s. d.	£ s. d.	£ s. d.	
40	5 11 8	6 0 10	5 12 8	6 0 3	5 12 8	5 7 10	5 10 0	5 1 2	5 8 6	5 1 2	40
45	6 1 10	6 9 9	6 2 4	6 9 10	6 2 4	5 15 10	5 17 6	5 9 11	5 17 2	5 9 11	45
50	6 15 4	7 2 11	6 11 9	7 2 2	6 14 9	6 6 0	6 8 5	6 1 8	6 8 10	6 1 8	50
55	7 13 0	8 0 7	7 11 10	7 19 3	7 11 10	6 19 4	7 4 6	6 17 2	7 4 4	6 17 2	55
60	8 17 0	9 5 1	8 16 11	9 4 4	8 16 11	7 18 2	8 6 2	7 18 11	8 6 1	7 18 11	60
65	10 9 8	10 16 1	10 11 1	10 18 6	10 11 1	9 5 10	9 14 2	9 9 8	9 16 10	9 9 8	65
70	12 14 8	13 3 5	12 17 4	13 4 11	12 17 4	11 6 8	12 0 2	11 15 0	12 2 5	11 15 0	70
75	15 17 10	16 9 6	16 2 4	16 10 1	16 2 4	14 4 10	15 1 8	14 14 8	15 2 3	14 14 8	75
80	20 7 2	...	20 8 5	20 16 5	20 8 5	18 7 10	...	18 17 9	19 5 7	18 17 9	80

42. The difference between the suggested rates and the highest rate of any British Office at each particular age is, of course, much more marked than in the previous table.

National Debt Annuities.

43. In comparing the rates suggested, with those of the annuities granted by the Commissioners for the Reduction of the National Debt, it may here be well to give some information regarding the terms on which the latter are granted. The rates of these annuities are dependent upon the price of Consols at the time of purchase, the rates quoted being those which £100 $2\frac{1}{2}$ per-cent Consolidated Stock will purchase at various prices. The annuities are payable quarterly on the Government quarter-days, 5 January, April, &c., and a final payment is made at death, of one quarter of the annuity. Although the rates of annuity quoted fall with the price of Consols, the actual return on the purchase-money increases with the falling prices of Consols, the latter simply ruling the rate of interest employed in the computation of the rate of annuity. When the price of Consols lies between £82. 19*s*. 9*d*. and £83. 13*s*. 8*d*., exclusive of accrued dividend, the annuity is based on 3 per-cent interest, being the yield on Consols at £83. 6*s*. 8*d*., while, when the price is above £99. 10*s*. 1*d*., exclusive of accrued dividend, the annuity is calculated at $2\frac{1}{2}$ per-cent, being the yield on Consols if bought at par. At intermediate prices, the rate of interest employed ranges between these two rates, 3 and $2\frac{1}{2}$ per-cent. For the rest, the mortality is Government Annuitants 1883 (select at entry), and the loading, nil. The purchase may be made either by the transfer of Consols or by payment of money, but in the latter case, a charge of 2*s*. 6*d*. is made for every complete £100 of the Stock which the money so paid would purchase. The charge is so insignificant that in the comparison no allowance therefor has been made.

44. It will be observed that at the prices of Consols which have been (with intervals) ruling for some time past, the annuities granted are considerably in excess of the suggested rates and also in excess of the rates of many of the leading offices. The question naturally arises as to how far the present experience of the Government confirms their experience published in 1883, or whether it conforms rather to the British Offices' Annuity Experience, 1893. The class of annuitant contracting with the Government, cannot, it is thought, differ widely from the class transacting business with the British Life Offices, in which case

it is probable that the mortality experienced nearly coincides. From this it would follow that the Government is making a loss on annuity business at current rates, though whether this is the case or not is doubtless receiving due consideration. The fact remains that, at the present time, the terms of the Commissioners (having behind them the premier security of the world) on the one hand place out of competition Offices quoting rates on the terms which have been suggested as the maximum, and on the other hand prevent those Offices which (whether from desire or as an effort to maintain connections) endeavour to take a share of annuity business, from reducing their terms to a more remunerative basis. The result of the next investigation into the Government experience will, no doubt, be received with very considerable interest.

POST OFFICE ANNUITIES.

45. The rates of annuities granted through the medium of the Post Office and Trustee Savings Banks (between the limits, on any one life, of £1 to £100 per annum) are based on the Government Annuity 1883 Tables (select at entry) at $2\frac{1}{2}$ per-cent without loading. They are payable half-yearly on two of the Government quarter-days, a payment at death being made of one-fourth part of the annuity. When brought into line with the common standard of payment, as shown in the table, the rates coincide with those of the National Debt Commissioners when Consols are at par.

GOVERNMENT ANNUITY BUSINESS.

46. In order to compare the amount of annuities granted by the Government, as well as the rates thereof, I have extracted the following figures from the account relating to "National Debt Annuities" for the year ended 5 January 1908—

TABLE XV.

Extract from Annual Return of National Debt Annuities, for year ended 5 January 1908.

	Transferred and Paid to Commissioners		Annuities Granted
	Total Stock	Money Paid including Commission	Immediate
	£ s. d.	£ s. d.	£ s. d.
For Life Annuities, per 10 Geo. 4, c. 24, and 51 and 52 Vict., c. 15 ... (Being annuities purchased on the terms published by the Commissioners.)	139,620 2 8	463,994 6 3	45,818 6 8
For Life Annuities, per 16 and 17 Vict., c. 45; 27 and 28 Vict., c. 43; 45 and 46 Vict., c. 51; and 50 and 51 Vict., c. 40 ... ... (Being annuities purchased through the medium of Post Office and Trustee Savings Banks.)	...	494,102 19 9	39,933 15 6
Total for Life (Immediate) Annuities	£139,620 2 8	£958,097 6 0	£85,752 2 2

47. The total amount of life annuities paid during the financial year ended 31 March 1908 reached the sum of £1,434,679. 6*s.* 10*d.* The estimated capital liability on that date in respect of existing annuities for life and terms of years (which amounted to £1,468,465. 14*s.* 6*d.*, the annuities for terms of years included therein being only about £18,000) was £14,265,649. From these figures it will be noticed that the existing annuities granted by the Government are roughly equal to about two-thirds of the total annuity business of the British Offices.

48. In considering the relative rates of, and the competition between, the British Offices and the National Debt Commissioners, it must be recognized that while, at the present time, the terms of the latter may really be more favourable, they are not shown in so simple a form, but necessitate a calculation which many proposers do not understand; that the resulting price may vary from day to day, and that the annuities are on a slightly more expensive basis (being payable quarterly with a quarter's payment after death) than those quoted by the Offices. The rates

of British Offices, however, are tabulated that "he who runs may read"; any reasonable request, such as the desire for payment on particular due dates, is met, and there is lastly the payment of commission, which is no doubt a very influential factor. It must in any case be regarded as an indication of the confidence reposed in the leading Insurance Institutions of this country that in the face of, broadly speaking, equal terms, they are able to show a larger volume of business than that transacted on the unsurpassed security of the British Government.

Loss on Post Office Deferred Annuities.

49. Before leaving the subject of Government annuity contracts, and reverting to the question as to whether profit or loss has resulted from the grant of annuities on the statutory basis, it is interesting to review the Return of "Government Insurances and Annuities" for the year ended 31 December 1907 (being an account of the receipts and payments, &c., in respect of Deferred Annuities and Payments on Death) in the light of the evidence taken by the Departmental Committee "on the Encouragement of the Life Insurance System of the Post Office" at the beginning of last year. The receipts during the year in respect of Deferred Annuities amounted to £26,191. 14*s.* 9*d.*, the new contracts entered into were 169 for £3,593. 9*s.*, and the total contracts current at the end of the year were 2,930 for £61,092. 12*s.* 10*d.* The receipts in respect of Deferred Annuities and Payments on Deaths are (contrary to the practice in the case of Immediate Annuities) invested to form a fund, the investments in respect of each section of this (apparently common) fund being £487,761. 0*s.* 9*d.* against Deferred Annuities, and £406,136. 11*s.* 4*d.* against Payments on Death, the average rates of interest realized being £2. 11*s.* per-cent and £2. 11*s.* 8*d.* per-cent respectively. The evidence taken by the Committee showed that there had been a steady deficiency in the fund during the past series of years on Deferred Annuities amounting to £96,000, and a steady increase in the surplus of the life insurance fund during a similar period of £77,000, leaving a net deficit on the common fund of £19,000. The reasons given for the deficit were—

(1) That the premiums could not be invested at a sufficiently high rate of interest, and

(2) That the annuitants, it was believed, lived longer than was expected.

It would almost seem from the facts that the rates of interest earned on the two sections of the fund were practically the same, and that a surplus really accrued in respect of the life insurance section, that the second reason, namely, greater vitality of the annuitants, was the more potent factor in causing the deficit.

Suggested Rates in relation to Valuation Bases.

50. Referring once more to the suggested rates for British Offices, it may be argued that the basis still errs on the side of liberality, and that the rate of interest employed should, at least in the case of younger lives, not exceed 3 per-cent, first because of the greater length of time for which the annuities on younger lives may run, and secondly because of the desire, now very general, to value at 3 per-cent. The arguments may, however, be employed that, in the first case, the purchase money is invested at once, at current prices (unlike the annual premiums for assurances) and need be liquidated very slowly, if at all; and in the second case, that a margin of $\frac{1}{2}$ per-cent in the interest is sufficient for safety and to yield a small profit, there being no periodical bonus to maintain. But while it is submitted that rates may at present be based on $3\frac{1}{4}$ per-cent interest (except 3 per-cent be used, as a matter of caution, at the younger ages) a reduction to 3 per-cent as a valuation rate is a sound and precautionary measure, and having cut the loss by such a drop at the outset, a larger fund is built up on which to earn greater excess interest. This principle may, in the opinion of the writer, be carried too far, a valuation by the British Offices' Annuity Tables, select values, at $2\frac{1}{2}$ per-cent (which has been adopted by a few Offices) being quite inconsistent with the grant of annuities on current terms.

Model Office showing proportion of Select Annuities.

51. In order to consider the result of bringing new business, effected at the suggested rates, into the valuation at 3 per-cent (among other bases) and at the same time to frame a Model Office in which the element of selection shall be quite definite and which may serve as a guide to possible future valuations, the number living according to the British Offices' Experience (Females), graduated, commencing with select lives at age 60, has been selected as constituting an Office, replenished at the end of every

year by entrants at age 60, so that on 31 December of any year, the total of the column—

$$
\begin{aligned}
l_{[60]} &= 61{,}721 \\
l_{[60]+1} &= 61{,}143 \\
l_{[60]+2} &= 60{,}273 \\
l_{[60]+3} &= 59{,}127 \\
l_{[60]+4} &= 57{,}751 \\
l_{65} &= 56{,}217 \\
l_{66} &= 54{,}587 \\
\vdots & \quad\quad \vdots \\
l_{104} &= \quad 2
\end{aligned}
$$

may be looked upon as the amount of annuities or number of lives. The result of this (admittedly arbitrary) formation is to produce an Office conducting a stationary business, which may be followed from quinquennium to quinquennium; though as the amount of new business is shewn, no difficulty would be experienced in comparing the results with an Office granting a larger proportion of new annuities.

52. In fixing upon females and entry-age 60 only, I have been guided by the following considerations. First, that the incidence of the entry-age in the case of the "new" annuities in the British Offices' Experience should be borne in mind. The following table (of interest quite apart from the question of the moment) shows the number of entrants in the quinquennial age-groups—

TABLE XVI.

Table of Entrants in respect of "New" Annuities in the British Offices' Annuity Experience, 1863-1893.

Age-group	Males	Females	Age-group
1–39	196	536	1–39
40–44	167	504	40–44
45–49	324	931	45–49
50–54	636	1,862	50–54
55–59	885	2,904	55–59
60–64	1,415	4,215	60–64
65–69	1,527	3,858	65–69
70–74	1,177	2,522	70–74
75–79	629	1,119	75–79
80–84	235	355	80–84
85–89	56	104	85–89
90–96	5	17	90–96
Total	7,252	*18,927	

* NOTE.—There would appear to have been a slight slip in calculating the number of "new" annuities (females) in the unadjusted data, the number thereof being given in the volume as 18,928.

The actual years of age in which the highest numbers entered were—In the case of male lives, age 65, the number of entrants being 321; in the case of female lives, age 62, the number of entrants being 886. (In Mr. Ryan's Model Office the existing business at each year of age reaches the maximum at age 67.) Secondly, it was thought desirable to give some effect to the business transacted below the most popular entry-age, and, as in using the column l_{x} to l_{104} a difference of a year or two at this part of the table makes a very appreciable difference in the total, 60 was fixed upon as the common entry-age. If an age earlier than 60 had been adopted (the business in force at that age being larger than that at any subsequent age) a change in the rate of interest would probably have had rather more effect on the reserves than would be the case in practice. Thirdly, it was thought that the figures would be unduly complicated by the adoption of more than one entry-age, and as female lives form the bulk of modern annuities and contain the greater element of danger, the one section of entrants has been selected to meet the particular purpose.

53. It is assumed, for simplicity, that the annuities are payable annually on 31 December, without proportion to day of death. The consideration being fixed according to the suggested basis of rates, modified only for yearly payment; the expenses being exactly met; the mortality, as assumed from the nature of the formation of the office, exactly coinciding with the female experience (graduated) of the British Offices; and the net rate of interest earned on the total funds being $3\frac{3}{4}$ per-cent, it follows that the profit, if the valuation were made on the basis of the purchase price, would be exactly one-half per-cent on the fund. The reserves on various bases are given, and it is proposed to trace the course of future profit at succeeding valuations, the business meanwhile running in accordance with above assumptions. The figures have been cut down to one-tenth, so as to form an Office having reserves of about £900,000 instead of nearly nine millions.

54. The respective reserves under the various bases are all, of course, greater than the purchase basis, but attention will doubtless be called principally to the fact that the reserves on the $2\frac{1}{2}$ per-cent "all select" basis exceed those based on the purchase terms by nearly 10 per-cent (a difference similar to that existing between Government Annuitants, 1860, Table at 4 per-cent and British Offices' Annuities, partly select and partly

Reserves of Model Office represented by the Column $l_{[60]}$ to l_{104} (graduated) of the British Offices' Annuity Experience (Females), valued by the British Offices' Annuity Tables.

(The figures have been reduced throughout to one-tenth.)

Age groups	Amount	Interest, 3½ per-cent			Interest, 3 per-cent			Interest, 2½ per-cent			Age groups
		Values according to period elapsed since entry	First five years, select values; thereafter, non-select values	Select values throughout	Values according to period elapsed since entry	First five years, select values; thereafter, non-select values	Select values throughout	Values according to period elapsed since entry	First five years, select values; thereafter, non-select values	Select values throughout	
[60]	6,172·1	73,792·4	73,792·4	73,792·4	75,486·6	75,486·6	75,486·6	79,070·1	79,070·1	79,070·1	[60]
[60] + 1	6,114·3	70,076·0	70,938·1	70,938·1	71,635·1	72,521·7	72,521·7	74,930·7	75,860·1	75,860·1	[60] + 1
[60] + 2	6,027·3	66,330·4	67,758·9	67,758·9	67,758·9	69,229·6	69,229·6	70,778·6	72,315·5	72,315·5	[60] + 2
[60] + 3	5,912·7	62,574·1	64,312·4	64,312·4	63,880·8	65,660·5	65,660·5	66,636·1	68,498·6	68,498·6	[60] + 3
[60] + 4	5,775·1	58,830·9	60,679·0	60,679·0	60,020·6	61,909·1	61,909·1	62,527·0	64,502·1	64,502·1	[60] + 4
60– 64	30,001·5	331,603·8	337,480·8	337,480·8	338,782·0	344,807·5	344,807·5	353,942·5	360,246·4	360,246·4	60– 64
65– 69	26,401·8	239,483·2	239·483·2	248,413·4	243,888·1	243,888·1	253,003·2	253,117·7	253,117·7	262,608·5	65– 69
70– 74	21,477·1	155,073·2	155,073·2	163,087·5	157,448·7	157,448·7	165,600·3	162,399·8	162,399·8	170,826·5	70– 74
75– 79	15,497·7	85,729·6	85,729·6	92,162·5	86,810·3	86,810·3	93,326·5	89,035·0	89,035·0	95,741·5	75– 79
80– 84	9,228·3	37,710·6	37,710·6	41,847·7	38,091·4	38,091·4	42,276·2	38,875·6	38,875·6	43,157·3	80– 84
85– 89	4,094·4	11,900·3	11,900·3	13,800·7	11,996·6	11,996·6	13,914·3	12,191·8	12,191·8	14,146·3	85– 89
90– 94	1,164·9	2,310·7	2,310·7	2,843·2	2,325·6	2,325·6	2,861·8	2,355·6	2,355·6	2,900·3	90– 94
95–104	179·8	226·8	226·8	301·5	227·8	227·8	303·3	230·0	230·0	306·3	95–104
Total	108,045·5	864,038·2	869,915·2	899,937·3	879,570·5	885,596·0	916,093·1	912,148·0	918,451·9	949,933·1	Total
Ratio to Purchase basis	...	**100**	100·7	104·2	101·8	102·5	106·0	105·6	106·3	109·9	Ratio to Purchase basis

NOTE.—The effect of multiplying the l_x column by annuity-values tabulated to only three places of decimals is to produce results which will not absolutely coincide with those based, for theoretical accuracy, on more exact values. At age 60, therefore, which is the purchase, as well as the most weighty valuation, age, annuity-values running into four places of decimals have been used, and the results of the assumed business and valuations, &c., are given to the nearest integral £1 per annum.

non-select at 3 per-cent—a change not unlikely during the past twenty to thirty years).

55. From the first line in the table it will be seen that the cost of transferring one year's new annuities, just effected, from the purchase basis ($3\frac{1}{4}$ per-cent) to 3 per-cent is, allowing for $1\frac{1}{2}$ per-cent loading reserve, £1,720, or to $2\frac{1}{2}$ per-cent, £5,357. The consideration for annuities purchased in each year (in respect of which the above loss by transference would have to be met) is £76,428, and the annuities paid in each year by this Model Office are £101,873.

Future Valuations on Assumed Bases.

56. Without showing in detail the exact profit or loss which may be made by passing from all the various bases in turn, let it be assumed that a valuation has just been made as at 31 December 1908, on the same basis as the purchase money ($3\frac{1}{4}$ per-cent, allowing for period elapsed since entry) and that the subsequent valuations are made—

As at 31 Dec. 1913 on 3 per-cent $O^{[af]}$ allowing for period elapsed since entry.
,, 1918 ,, $2\frac{1}{2}$,, ,, ,,
,, 1923 ,, $2\frac{1}{2}$,, $O^{[af]}$ select throughout.
,, 1928 ,, $2\frac{1}{2}$,, ,, ,,

The reserve at 31 December 1908 would be...	£864,038·2
plus $1\frac{1}{2}$ per-cent reserved for future expenses	12,960·6
	£876,998·8

and it is assumed that the new quinquennium is started with a fund of that amount.

57. The annuity account for 1909 will read as follows (taking the figures to the nearest pound)—

	£		£
Amount of Annuity Fund at beginning of year	876,999	Annuities paid ...	101,873
Consideration for annuities granted	76,428	Commission . .	764
Interest (less tax)	32,887	Expenses ...	2,293
		Amount of Annuity Fund at end of year	881,384
	£986,314		£986,314

the fund increasing during the year by £4,385, being the excess interest of $\frac{1}{2}$ per-cent. At the end of five years the fund will

have increased to £900,631 due to the excess interest of $\frac{1}{2}$ per-cent, which will have amounted to £23,632 in the five years.

58. The following table gives the results of each succeeding valuation (as well as the surplus which would have been shown on each occasion if the basis of valuation last employed had been adopted a second time)—

TABLE XVIII.

Table of Assumed Future Valuations of a Model Office.
(See paragraph 51 for construction.)

Date of Valuation	Accumulation of Fund from previous liability	If Previous Basis again Adopted		New Basis actually Adopted		
		Liability	Surplus	Mortality and Interest	Liability	Surplus (+) or Deficit (−)
	£	£	£		£	£
31 Dec. 1913	900,631	876,999	23,632	British Offices' Annuity Tables allowing for period since entry 3%.	892,764	+ 7,867
31 Dec. 1918	919,581	892,764	26,817	British Offices' Annuity Tables allowing for period since entry $2\frac{1}{2}$%.	925,830	− 6,249
31 Dec. 1923	959,330	925,830	33,500	British Offices' Annuity Tables —All select— $2\frac{1}{2}$%.	964,182	− 4,852
31 Dec. 1928	1,005,437	964,182	41,255			

59. In the quinquennium 1914–18 the excess interest of $\frac{3}{4}$ per-cent on the 3 per-cent reserves would amount to £36,087, but this would be reduced, even if the former basis be adhered to, by a loss of £9,270 due to transference of the last five years' business from purchase basis to the stronger reserve.

60. In the next quinquennium 1919–23 the excess interest of $1\frac{1}{4}$ per-cent on the $2\frac{1}{2}$ per-cent reserves would amount to £62,371, reduced by loss on transference of basis of new business to the extent of £28,871.

61. In the quinquennium 1923–1928 the excess interest on the $2\frac{1}{2}$ per-cent "select" reserves would amount to £64,952,

augmented by heavier mortality than assumed, £5,174, but reduced by loss on transference of basis of new business, of £28,871, the net surplus being £41,255.

62. From the figures under the bases assumed to be successively adopted, it will be noticed that the result of the fifteen years' working 1909–1923, is a net loss. Due, however, to the stringent basis and the building up during the previous period of unduly large reserves (in comparison with the purchase basis), the considerable profit for the ensuing quinquennium 1924–1928 is shewn. But even this ultimate profit would not result unless the assumptions as to rate of interest and mortality be maintained; if they be not maintained and the gradual passing to 2½ per-cent "select" reserves be justified by subsequent events, the profit during the ensuing twenty years may be no greater (unless the purchase basis be increased from that assumed) than has been shown for the past two decades; if, on the other hand, the current rates of interest and the mortality according to the latest experience continue, there will, in the writer's opinion, be no justification for such stringent valuation as assumed in the latter part of the above table. If a reasonable basis might be suggested, as being in keeping with current conditions and the purchase basis which has been mentioned, it would be the British Offices' Tables, at 3 per-cent, using "select at entry" values for business of less than five years' duration, and non-select thereafter, a method which is of extreme simplicity in working and allows a small margin for lighter mortality. Such a valuation would result in a quinquennial surplus (when once the fund has been established on such basis) of about £28,000, none too large, it is true, on a liability of £900,000 (being about 3 per-cent for the whole quinquennium), looking to other demands which might be made upon it and possible adverse experience.

63. It is to be remembered that in the working of this Model Office, the purchase money is based on terms more stringent than those now generally current, and that the assumed new business is not large in proportion to the existing business of the Office. Actual experience may therefore prove even less fortunate than that portrayed. The figures, in any case, would, on the one hand, seem to confirm the suggested table of annuity rates (in so far as they can be tested from the one entry-age chosen) as the maximum which should be granted, and on the other, to show that the combination of current rates of annuities with reserves of

$2\frac{1}{2}$ per-cent on the $O^{[am]}$ and $O^{[af]}$ tables is illogical. Of course, where only a small amount of annuity business is transacted, and then only for the sake of keeping connections, this stringent basis may be advisable owing to the improbability of so small an experience coinciding with the theoretical basis, in which case the loss due to building up a stringent reserve to safeguard against adverse experience might almost be looked upon in the nature of expenses chargeable to the Life Department. Indeed, this will be the result where no separate annuity fund is kept. In such cases (as for example, where the liability under annuities reaches little more than 1 per-cent of the total funds) the most stringent valuation would have no appreciable effect on the total surplus.

MISCELLANEOUS NOTES.

64. It has been thought that the opportunity might be taken while the subject of annuities is under consideration, of raising the question of the method of valuation, quite apart from the bases employed, and also of discussing the minor classes of annuities. The following notes have therefore been made, not because they contain anything either of novelty or merit, but in order that no section of the business may be overlooked, and in the hope that some hints for students may be gleaned from them, or from their discussion.

CLASSIFICATION OF ANNUITIES.

65. It is, of course, important in view of the effect of selection on annuity-values, to classify the business both according to age and date of purchase (unless "select at entry" values are universally applied), and a specimen sheet is shown of a continual classification register, in which effect thereto may be given. Each Office will no doubt form the valuation factors to cover the usual conditions under which their annuities are granted; the following sheet has been framed for use where the majority of the annuities are payable half-yearly, without proportion to day of death. Columns are accordingly given in which the necessary adjustments can be made for annuities granted otherwise. At the valuation date, the proportion of annuity accrued is mentally calculated in each case, and the annuity is valued as payable half-yearly from the valuation date, the factor being $(a_x + \cdot 25)$ and the portion of accrued annuity added later. The factor for an annuity payable quarterly being $(a_x + \cdot 375)$, or if payable

Specimen Classification Sheet.

Annuity Classification. Immediate Annuities on Single Lives classified according to year of Birth.

1842^{1} *Year of Birth.*—Females. Females.—*Year of Birth* 1842^{1}.

Year of Entry	Number of Annuity Bond	If with Proportion	Annuity: Amount	Annuity: Payable	Annuity: Last due date in year	Proportion of Annuity payable at Death	Proportion of Annuity accrued to 31st Dec.	Adjustments for Annuities not payable Half-Yearly: Yearly (−·25)	Adjustments for Annuities not payable Half-Yearly: Quarterly (+·125)	Written-Off: Date	Written-Off: Cause	Remarks
1892	23,496	...	40 0 0	H	30 Sept.	... ...	10 ...	... ...	...	...	...	...
1900	29,781	P	60 0 0	Q	1 Oct.	7·5	15 ...	... ...	7·5	...	...	...
1903	31,257	...	25 0 0	H	31 July	... ...	10·4	... ...	... ...	...	...	...

The figure 1 or 2 above the year of birth is used to indicate the first or second half of the year.

annually simply a_x, the adjustments to the half-yearly annuity factors are $+\cdot125$ and $-\cdot25$ respectively. In entering these adjustments, therefore, it is only necessary to enter one-eighth of the annuity in the quarterly instalment column or one-quarter in the yearly column. For the proportion to death, in the case of proportionate annuities, one-half, quarter or eighth of the annuity is entered according to whether the annuity is payable yearly, half-yearly or quarterly, the total at each age being multiplied later by A_x.

66. The annuities may be classified according to half-years of birth, those born from 1 January to 30 June being classified in the first half of any year, and those born from 1 July to 31 December, in the latter half. The assumption then is that those born in the first half of the year attain the integral age on 31 March (on the average) and those in the latter half of the year, on 30 September, so that in no case is there a difference of more than three months. If the valuation be made at 31 December, the factors $a_{x+\frac{3}{4}}+\cdot25$ and $a_{x+\frac{1}{4}}+\cdot25$ could be used, though probably $a_{x+\frac{1}{2}}+\cdot25$ and $a_x+\cdot25$ will usually be preferred, the understatement of ages by one quarter of a year being on the safe side. The periods of half-years could be run from 1 October to 31 March, and from 1 April to 30 September respectively, in which case the lives would, on the average, attain the exact integral or half age on the valuation date (if that be 31 December). It will, however, be simpler in practice to divide the year into the two usual halves. In some offices it may be considered sufficient to use integral ages only, classifying the lives according to years of birth, 1 July to 30 June, so that, on the average, the integral age is attained on 31 December (the valuation date).

VALUATION SCHEDULE.

67. The annuities being entered, and adjustments made, and effect having been given to "written off" and alterations, the various sections, according to age and year of entry, may be cast and entered in the valuation schedules, a specimen of which is given below. Such schedules may usefully be framed with a second set of columns for factors and values on a second basis, in case it may be decided to make two trial valuations. Where the business is split into "select" and "non-select", separate schedules should be used for "old" (non-select) annuities and "new" (select) annuities.

SPECIMEN VALUATION SCHEDULE.

Annuity Valuation, 190...... *Valuation of Immediate Annuities granted prior to* 31 *December* 190......

FEMALES. FEMALES.

Year of Birth	Valuation Age x	No. of Bonds	Amount of Annuities	Proportions to-day of death	Valuation Basis..............				Proportions accrued to 31 Dec.	Adjustments for	
					Factors		Values			Yearly (−)	Quarterly (+)
					$a_x + {\cdot}25$	Λ_x	Annuities	Proportions to death			

The thin inner-column lines are inserted instead of decimal points.

68. The use of cards will be found a convenience, particularly during the period in which the annuities are classified amongst the "select" portion of the business, while if the number of annuities at each age is small, a Classification Sheet is hardly necessary, the particulars being entered at once into the Valuation Schedule from the cards. Where more than one annuity at the same age and duration exists, the items may be mentally cast.

Annuities on more than One Life.

69. Immediate Annuities during joint lives, and also during joint lives and to the survivor, are equally subject with annuities on single lives to previous observations, except that individual valuation is usually necessary. The law of uniform seniority does not apply to female lives in the British Offices' Annuity Tables, and it invariably happens that in this class of annuity one female life, at least, is involved. The exhaustive treatise by Mr. T. G. Ackland in his contribution to the Faculty of Actuaries,* and the valuable notes by Mr. Lidstone, at the end of the volume of Annuity Tables, render it unnecessary, however, to add anything regarding Joint Life annuity-values, except to point out that values are only given as "at date of purchase" and "after five years from purchase", so that where the period elapsed since entry is less than five years, the value as "at date of purchase" is doubtless usually employed.

Deferred Annuities.

70. Deferred Annuities present many problems, chief amongst which is the basis, necessarily fixed at the outset, on which the immediate annuity, to be entered on many years hence, shall be calculated. If the annuity be granted without return of premium in event of death or withdrawal before the annuity age, the basis for the immediate annuity may be taken on the assumption that the life will then be equivalent to a non-select one on the most favourable mortality experience, *i.e.*, O^{am} or O^{af} as the case may be. The rate of interest in the calculation of the immediate annuity should not exceed 3 per-cent, and it is questionable whether in long periods of deferment it should not be even lower, though the premium payable during such period may be calculated at a higher rate, the value of the annuity to be entered upon being considered in the light of a pure Endowment.

* Notes on the British Offices' Life Annuity Tables (1893), by T. G. Ackland. *Transactions* of the Faculty of Actuaries, vol. iii, p. 285.

71. No doubt the majority of Deferred Annuities are granted with return of premiums in event of death or withdrawal, and in this case there is no necessity to consider mortality during the period of deferment, but simply to accumulate the premiums less a charge for expenses. The basis of the ultimate annuity should depend to a certain extent on the nature of the return which will be made if the purchaser elects to withdraw. The nature of this return varies considerably in different Offices; in some cases a percentage for expenses is deducted before accumulating, and in other cases no such deduction is made; while the rate of interest employed in the accumulation, to provide the return, varies from a low simple interest rate to a return with 3 per-cent compound interest. Where a full accumulated return is to be made, the basis of the ultimate annuity should certainly be for a "select" life, because indifferent lives, quite apart from bad lives, will exercise the option of taking the accumulated cash. As regards the return to be made during the period of deferment, it seems desirable that a better return should not be made than would be obtained by the surrender of a Sinking Fund Policy. The tendency to grant Deferred Annuities with option of withdrawing at any time, the whole of the premiums with compound interest, seems more of the character of Savings Bank business, and appears to be out of keeping with the general transactions of Life Offices. Where such returns are made, it is necessary for the Office to look simply to the excess of interest to cover expenses and yield a profit. Such expenses will usually reach $7\frac{1}{2}$ per-cent, in view of the practice to allow renewal commission of 5 per-cent on the annual premiums for Deferred Annuities. The expenses on the ultimate annuity need only be sufficient to cover payment charges, say $1\frac{1}{2}$ per-cent.

72. The option of selecting an annuity, as one of the privileges under the many schemes of "Option Policies" now being devised, should be looked upon in the light of a Deferred Annuity, with return, at least in regard to fixing the basis of the annuity which may be ultimately selected.

73. In the valuation of Deferred Annuities, with return of premiums, it is only necessary to accumulate the premiums, with or without a deduction for expenses. The rate of interest employed for such accumulation should certainly not be less than that on which the premiums were based. Such rate will usually exceed the valuation rate employed in respect of other annuities.

Contingent Annuities.

74. Contingent Annuities are seldom the subject of competition, and they form so small a section of annuity business that no great reliance can be placed on the experience of the Offices in regard to them. Mr. S. F. M. Cumming, F.F.A., however, collected the experience of 27 British Offices and submitted the results in a paper before the Fifth International Congress of Actuaries.* The total entrants (including annuitant and counter lives) only numbered 1,614, the majority of the annuitant lives being females, and of the counter (or assured) lives, males. The total deaths were 208 amongst annuitant lives and 140 amongst assured lives. The resulting mortality in both classes was exceptionally good, though in comparison with standard tables the mortality in the annuitant section was relatively better than that in the assured section. Mr. Cumming calculated the pure premiums on seven different mortality bases (with interest at 3 per-cent in each case) for both single and annual premiums for £10 annuity, taking three different combinations of ages, and these have doubtless proved very useful, at least as a guide to avoid the lowest basis according to the nearest combination of ages given in his paper.

Application of Tables.

75. If it be desired to employ the British Offices' Annuity Tables for this class of business, it must be borne in mind that there are no commutation columns for joint lives, no tables of M_x and R_x even for one life (for benefits "with return") and no factors below age 20. These tables, however, were framed from the experience relating to immediate annuities, and it cannot be expected that they will prove applicable to every class of annuity contract. From the Returns made to the Board of Trade it would appear that the valuations of Contingent Annuities were based either on the Carlisle or the British Offices' Annuity Tables. In view of Mr. Cumming's investigation and of the fact that the mortality amongst both classes was so good, it may be thought sufficient to adhere to the British Offices' Annuity Tables for both lives in the valuation of the simpler contingent annuities, rating up the assured life, if thought desirable, to an age equivalent to that on another mortality table. One of the most troublesome features in connection with the employment of

* Note.—Notes on a Mortality Experience in connection with reversionary annuity business in Great Britain.

suitable tables for the valuation of this section of business occurs when the annual premiums are limited in number. If the British Offices' Annuity Tables be employed, the joint life terminable annuity may be calculated by the formula (interpolating for the joint life annuity values if the difference in the ages is other than a multiple of 5)—

$$a_{xy} - a_{x+n:y+n}\frac{D_{x+n}}{D_x}\frac{l_{y+n}}{l_y}$$

employing select functions or otherwise as necessary. It will often be possible, if there are sufficient annuities of a class to warrant it, to devise a rule of thumb, which, while simplifying and expediting the work, will not involve a serious error. Thus if the valuation factor employed for future premiums be—

$$\frac{a_{x:y:\overline{n}|} + \mathbf{a}_{x+1:y+1:\overline{n}|}}{2}$$

(based on the assumptions that the premiums are due six months after valuation date and that the lives are born, on the average, in the middle of the year) the calculation of one of the joint life term annuities may be avoided by the following rule—

> Calculate $\mathbf{a}_{x+1:y+1:\overline{n}|}$ and deduct a constant of ·05, plus ·01 × n. Thus, for an annuity of 18 years' term deduct ·23. The deduction for 30 years' term (·35) should be fixed as the maximum.

76. A similar rule for Joint Life Whole-Life annuities (where the mean is required for valuing future premiums on the same assumptions as above) is—

> Calculate $\mathbf{a}_{x+1:y+1}$ and deduct ·35 (being ·5 for half a year's premium accrued less ·15 for the mean of the difference between two annuity-values at successive ages). Or, if $a_{x:y}$ has already been calculated, use such value + ·35.

On two separate occasions when the values of twenty or thirty of such annuities were required, the employment of the last-mentioned rule only made an average difference of ·001 and ·004 respectively from the correct means, the individual difference seldom exceeding ·03. The saving of time occupied in making several interpolations is warranted in special cases of this kind, by the small error involved.

77. In such cases as Contingent Temporary Annuities on the lives of children, the annuity to commence on death of parent or

other adult, and to cease on the child attaining age, it must be assumed that the child is a healthy one, with a probability almost approaching certainty of reaching maturity. There being, however, no select tables at young ages, while other tables show too heavy mortality in such a case, the simplest plan is to use an annuity-certain. Thus, an annuity to a girl aged five, after the death of a male aged 35, but ceasing in any case on the girl attaining age 25, may reasonably be valued—

$$a_{\overline{20}} - a_{35\cdot\overline{20}}$$

or in another case, where an annuity was purchased on a child aged one, deferred till age nine, and then to run for 11 years only, the deferred annuity-certain value sufficiently meets the case. These arbitrary values slightly overstate the true, but the differences are small.

78. In most office valuations there are doubtless found one or two contracts of an unusual nature, the consideration for which was probably fixed on an arbitrary basis, looking to the special circumstances of the case, and to the fact that being isolated from any other business of the same class there would be no opportunity of obtaining anything approaching an average experience. In such cases the valuation might reasonably be made on the purchase basis, modified only as to any difference in the rate of interest, lapse of time, and expenses involved.

Limitation of Risk.

79. A question of considerable importance is the limitation of risk under annuity contracts. In regard to assurances, Dr. Sprague dealt with the matter very thoroughly in his paper* read before the Institute 43 years ago. It was his opinion that risks of various kinds might be fairly classed together, but I am not sure that he would have classed a large annuity with assurances. The two sections of business should, I think, be looked upon separately in regard to the limit retained under respective contracts. One important point is, that under an annuity it is impossible to make a heavy loss suddenly, but this would not be a sufficient argument to warrant an office retaining a single annuity of an exceptionally large amount. Whatever may be fixed as the limit, it should be such that there are several annuities at or about the same figure. The utmost loss which

* On the limitations of risks. *J.I.A.*, xiii, p. 20.

can be made under any one contract, from the prospective point of view, is

$$(\text{Amount of annuity}) \times (a_{\overline{104-x}|} - a_{[x]})$$

and if the opinion be held that such diverse risks as immediate annuities and assurances can be classed together, the utmost loss may be considered in relation to the office rule for the limit under assurances. If the latter be fixed at £10,000, the limit of the annuity to be retained (represented by L) would be

$$\mathrm{L} = \frac{10{,}000}{a_{\overline{104-x}|} - a_{[x]}}$$

This rule would break down at the very old ages and in this connection it may be said that the greater the possible loss in comparison with the consideration, the lower should be the limit retained. The latter remark applies to contingent annuities, though they might, perhaps, be more fairly classed with assurances. If x be the annuitant life, the utmost possible loss at the outset would be $a_{\overline{104-x}|} - \pi$, assuming the assured life to die at once. But as every year lived by the assured life would decrease the amount of the greatest possible loss, the utmost loss might be looked upon at the outset as represented by $a_{[x]}$ and if the limit on assurances be £10,000, the limit of contingent annuity would be $\frac{10{,}000}{a_{[x]}}$. It would be well, however, to consider the respective premiums for a Whole-Life assurance of £10,000 and for the amount of contingent annuity represented by $\frac{10{,}000}{a_{[x]}}$, and to fix the limit of the annuity in proportion thereto. In regard to deferred annuities, it is doubtful whether the question of reinsurance has ever arisen, the contracts of this class invariably being for comparatively small amounts. Should, however, a large deferred annuity be effected, the rule as to the limit should follow that fixed by the Office in the case of immediate annuities, except that as there is the risk, besides that of the ultimate annuity, of further possible loss due to extensive changes in the basis of annuities taking place during the period of deferment, it would be well to take a more moderate view of the limit to be retained under a deferred, than under an immediate, annuity. But, whatever the class of annuity, it is questionable whether any universal rule can be laid down. Each case should be considered on its merits, looking, at the same time, to the existing business of the Office.

SUMMARY.

80. The arguments which, at the outset, it was the object of the paper to enforce, may be briefly stated thus—

That, whether from desire, or as one of the necessities for the maintenance of connections, a large annuity business is being transacted by British Life Offices.

That the profit on this business has been almost a negligible quantity, first, because the experience has proved less favourable than the bases originally adopted, and secondly, because of the endeavour to build up reserves in keeping with the real experience.

That notwithstanding such endeavours, there is still room, in respect of about one-half of the business, for further strengthening of reserves, as regards mortality, up to the standard of the British Offices' Annuity Tables.

That although the effort has been and is being made to bring the reserves into line with such (standard) basis, the terms on which (generally speaking) current contracts are being entered into are inconsistent with the standard reserve, and that the maximum terms for new contracts should not exceed the rates resulting from the $O^{[am]}$ and $O^{[af]}$ tables at $3\frac{1}{4}$ per-cent interest (or even 3 per-cent for younger lives, graduating up to $3\frac{1}{4}$ per-cent) with suitable loading.

That the terms on which annuities are at present granted by the State through the medium of the Commissioners for the Reduction of the National Debt, are such as to restrict competition—at least in those cases in which a true comparison is made—to those Offices which adhere to unremunerative rates (from which it would seem to follow that the State annuities are likewise unremunerative).

That if a universal reduction be made in current rates, taking those suggested as the maximum, and the latest experience be reproduced, there is the probability, in view of present strength of reserves, of more remunerative trading being experienced, than has been the lot of the Offices hitherto.

That simplicity in the valuation may be combined with precautionary measures against possible adverse experience without entirely effacing profit (as in the use, for example, of "select at entry" values for all business having duration 0-5 years, instead of values allowing for period elapsed since entry), the effort being made to secure the maximum of simplicity with minimum of error, any departure from the true value to be on the side of safety.

Conclusion.

81. In conclusion, I wish to express my thanks to the many Companies of which it has been necessary to make enquiries, for the courtesy with which the desired information has been given; to the Company with which I am connected for the privilege of making use of many facts and figures; to Mr. G. H. Ryan for the permission, so freely granted, to make use of his Model Office; and finally, to my colleague, Mr. C. W. Winstanley, A.I.A., for his careful and painstaking labours in checking the various statistics and calculations contained in, or involved by, this paper.

Abstract of the Discussion.

Mr. WALTER T. MAY remarked that he did not think there was any doubt that the practical subject of the grant of annuities had not received from the members in their corporate capacity the consideration to which it was entitled. Actuaries had, of course, long been aware that companies granting annuities had not received anything like a reasonable profit on those transactions, in exchange for the risks they ran and the guarantees they gave, but he thought most actuaries would have their eyes more widely opened by a study of the paper. With the conclusions arrived at by the author he agreed in the main, but wished to emphasize the immense difficulties that of late years had confronted the grantors of annuities, and to point out how much had been done to cope with them. In the first place, the rate of interest fell very rapidly in the first half of the period under review, and although during the last ten years there had been some improvement in that direction, Mr. Oakley had shown that annuity considerations received had tended to fall during that time, and were now considerably exceeded by the payments made, so that on the annuity account there could not have been a very large sum to invest at the higher rate of interest which might have been obtained in the last few years. He had taken thirteen large annuity

offices, with funds exceeding half-a-million, and he found that in the last twenty years they had, on the average, reduced their valuation rate of interest by ·4 per-cent, or over a period of twenty-five years a reduction of ·7 per-cent had been made, and he estimated from the Model Office, as revised by Mr. Oakley, that the effect of such a reduction over the latter period would be to increase reserves by about 5 per-cent.

In the second place, and of far more serious import than the fluctuations of the interest, which was a factor that rose as well as fell, had been the gradual discovery of the tremendous effects of selection exercised against the offices by annuitants, and by the extraordinary longevity, in particular, of female annuitant lives. He found that, out of thirteen offices, with annuity reserves aggregating $12\frac{1}{2}$ millions, seven, with reserves amounting to $7\frac{1}{2}$ millions, had already passed to a valuation by the British Offices' Tables, thereby strengthening their reserves, if previously made on the equivalent select and non-select basis of the Government Annuity Tables, by about $3\frac{1}{2}$ per-cent, or £260,000. If a fairly progressive business was being done, it would appear from a case that had come under his notice, that to pass from a select valuation on the Government Annuity Tables, allowing for the years elapsed since entry, to a similar valuation on the British Offices' Experience, might mean an increase of 4 per-cent in the reserves. He had only quoted those two instances of what had been done, to prepare for the future, and to show what enormous changes had been rendered necessary by the publication of the British Offices' Annuity Experience, while the changes in the reserves for assurances on passing to the British Offices' Experience had been comparatively insignificant. When it was considered that, with remunerative rates, constructed on a true table of mortality and properly loaded for expenses, with all things running smoothly and the money market not subject to violent fluctuations, the offices could only expect to earn $\frac{1}{2}$ per-cent on their funds as profit, the increase of $3\frac{1}{2}$ per-cent in the reserves to pass from one table to another was very significant, and he was not surprised that more offices did not at once make the plunge.

Actuaries had a scientifically constructed instrument in Finlaison's Table of 1883, based on the class of lives with which the subject dealt, separated according to years of insurance elapsed since entry, and offices had been strengthening their annuity reserves for years. Was a further sacrifice necessary? Unfortunately, there could be no doubt that the British Offices' Annuity Experience did not, for reasons he would give later, minimize the mortality of annuity nominees, and a question of public interest then arose as to whether the Postmaster-General and the Commissioners for the Reduction of National Debt were justified in considering that the Government nominees nowadays were so different a class from those who effected annuities with the offices that they still experienced the rates of mortality ascertained by Mr. Finlaison. He did not think that was so, and he was of opinion that grantors of annuities generally had three primal reasons for pinning their faith to the

British Offices' Tables. In the first place, those tables were of later date, and had been more scientifically constructed, with the view, especially, of finding the true values of $a_{[x]}$. In that connection, he would refer students to the words of the President in the Memorandum on the graduation of that experience, in the volume of Principles and Methods, page 125. In the second place the results were corroborated to a remarkable degree by the figures produced by Mr. King, in his investigation into the mortality of the nominees of the General Annuity Trust, in vol. xxxiii of the *Journal*, page 262, where the expectations of life ascertained by Mr. King were in so close accord with those of the British Offices' Experience, five years and more elapsed since entry, that taking an average of the expectations under both tables from 55 to 75 inclusive the average expectation under Mr. King's table was less than 1 per-cent in excess of that under the other. Mr. King's investigation was comparable in point of date with the Offices' Tables, but related to nominees under reversionary annuities, so that no selection in the ordinary sense of the term could be exercised. Thirdly, the terms to female nominees were increased by the Government in 1883, and the older experience of 1829 showed much lighter female mortality than that of 1883. Thus at age 60, $e_{[x]}$ under the 1883 table equalled 16·3, whereas under the aggregate table of 1829 e_x equalled 17·3.

It was remarkable to note that, under the table of 1829, the expectations of life at all ages were in excess of the select expectations of the British Offices for female lives. He thought, therefore, it would be agreed that the British Offices' Annuity Tables were the only safe ones to use, but, as the author had stated, no permanent improvement in the earning power of life funds could set in, however conservatively the valuations were made, until the terms granted were materially reduced, and therefore the all-important question of rates came up for consideration. He thought a 3 per-cent basis should be aimed at, suitably loaded to meet actual expenditure, which, apart from commission and stamp, should not be very heavy, and he preferred to use that basis, scantily loaded, to using $3\frac{1}{4}$ per-cent, if the valuations were to be conducted on a 3 per-cent basis, as he saw no reason with annuity business why the rates should differ, there being no bonuses to provide for, and as annuity contracts were in force on the average for so short a time, the reserves should not be strained by each new contract. As offices were dependent entirely on the excess of interest earned over that assumed, and could not expect, for some years to come, any profit from mortality, he did not think it would be safe to adopt, say, $3\frac{1}{4}$ per-cent throughout rates and valuations. He was referring to an average office earning a little over $3\frac{3}{4}$ per-cent, and doing, as the majority of companies must be at the present time, stationary or slightly decreasing annuity business, and not therefore likely to have many investments to make on annuity account. In the exceptional case of an office with small annuity funds, courageous enough to think of

developing a large annuity business in the near future, he did not say that better terms might not be granted, if there was a reasonable assurance of being able to earn over 4 per-cent.

Having thus spoken as to the sacrifices that had been made in the past, and what it seemed necessary to do in the way of reducing rates to make those sacrifices fruitful, he wished to say something on the severe competition the offices had to contend with from the Government, especially at the present time, with Consols at a price paying nearly 3 per-cent. On page 208 of the National Debt Return, referring to the grant of annuities under the Act of 1808, it was stated that "the economical success of the life annuity plan as a means for the redemption of the National Debt primarily depended upon the correctness of two assumptions, namely, the probability of a rise in the price of stocks during the continuance of the annuities which might be granted, and the sufficiency of the estimate of the duration of the lives of the nominees upon which the annuities depended." The first proviso of economical success had, he supposed, an equal chance of happening, though the experience of the last ten years had been adverse, and therefore some slight loss had arisen in that direction. With regard to the second and more important proviso, as to the sufficiency of the estimate of the duration of the lives, it was not, of course, possible for them to judge. It was known that the greatest care and actuarial skill were used in the construction of the tables of 1883, but, for the reasons he had given, he did not think the lives dealt with in that experience could have been a similar class to those who effected annuities to-day, and he saw no reason to suppose that the rates of mortality, ascertained by the most accurate processes to be in force amongst the nominees of the British offices, should not be similar to those in force among the Government nominees, now that speculative annuities were not granted by the Government.

Taking, however, an extreme case, if it should so happen that the mortality to which Government nominees were now subject coincided with that of the British nominees, he should like to point out that the values of $a_{[x]}$ for male lives under the Government 1883 Tables were about equivalent, from ages 55 to 75, to $a_{[x]}$ under the British Offices' Tables, at a rate of interest $\frac{1}{3}$ per-cent in excess, while for female lives the rates under the two tables were about equivalent, at a rate of interest $\frac{1}{2}$ per-cent in excess, at the more popular ages for effecting annuities. Therefore, if the Government nominees were found to be now subject to the rate of mortality shown in the British Offices' Tables the Government would have to be in a position to grant Savings Bank Annuities on a basis approaching 3 per-cent, with an allowance for expenses, in order to fulfil sub-section 4 of section 5 of the Government Annuities Act 1882, which enacted: "The Tables shall be framed in such manner that the fund formed by the receipt of sums in respect of deferred annuities and of insurances and the amounts paid for immediate annuities shall respectively be adequate (after payment of

expenses) to meet all claims without causing any loss to the Exchequer." The same remarks applied to the National Debt Annuities, and as Mr. Oakley had shown, when Consols were at about 83, the basis was 3 per-cent. and in their case if the nominees were subject to mortality according to the British Offices' experience the equivalent basis under that table would be about $3\frac{1}{2}$ per-cent. which was therefore the rate, on the assumption referred to, at which the Government would be reducing debt.

He wished also to refer to one or two isolated points in the paper which had particularly attracted his attention. It would be seen from Table I that the business transacted by the offices rose very rapidly from 1883 to 1897, especially during the last five or six years, when the value of money was rapidly cheapening, and that the greatest amount of consideration was received in the year in which Consols reached their maximum: also, that, since the commencement of the South African War, the amount of consideration had remained fairly stationary, with a tendency to decline. Similar progression was shown by the Savings Bank Annuities of the Government, though the increase was not quite so rapid. The consideration money rose from £269,986 in the year ending 5 January 1888, to £868,490 in the year ending 5 January 1897, falling to an average of about £600,000 in the last few years. The consideration money shown in Table XV of the paper for the year ending January 1908, £494,102, was exceptionally small. The consideration money received under the Act of George IV, commonly called National Debt Annuities, did not exhibit any regular progression, the dearness of money off-setting the good terms granted when Consols were low, and the cheapness of money making up for the poorer terms granted when Consols were above par.

In considering the very instructive, but disappointing, results of Tables III, IV, and V, it has to be remembered that in the first two quinquennium the annuity funds contributed a share in the appreciation of securities which would not be shown in the profits, and, in other ways, the annuity fund might have been of advantage to the office. For instance, annuities might have been set up on better terms than those granted to the public in connection with reversionary life interests, &c., but those details in no way obscured the hard fact that unduly unfavourable mortality had been nearly sufficient to eat up the profit from interest. Turning to Table XIII, in which Mr. Oakley compared his suggested maximum rates with the average of six offices which quoted in the usual way, he had found, to compare with those, the average rates for females of the thirteen offices he had already mentioned with funds exceeding $12\frac{1}{2}$ millions, and he found in all cases his average rates were higher than the average of the author's six offices, and crossed his suggested maximum rates at age 55. As only about one-fifth of the new entrants effected annuities below age 55, it would be seen how great would be the change necessary to pass to scientifically-constructed rates, loaded to provide for actual expenses, and with a very small

interest margin for contingencies and profits, such as Mr. Oakley had suggested.

In paragraph 48 the author congratulated the offices on being able to show a larger volume of business than the Government on "broadly speaking, equal terms." He should like to substitute "much less favourable terms." Taking the average rates shown by the thirteen offices, after adjustment he found that they were only better than the Post Office rates up to age 61; but from Table XVI it would be seen that 66 per-cent of the transactions, called new annuities, were effected on lives aged 60 and upwards at purchase. Therefore the offices' rates did not really compare very favourably with those of the Post Office, and much less so with those granted by the National Debt Commissioners, with Consols at 84. It was interesting to note that in the Government Annuity Experience of 1883 the percentage of new entrants aged 60 and upwards was only 52 per-cent, as against the 66 per-cent above mentioned. It was also noteworthy that the proportion of females to males was very appreciably greater, as compared with the Government Experience, females forming 64 per-cent of the Government Experience, and 72 per-cent of the British Offices' New Annuity Experience.

For the work done by Mr. Oakley in bringing Mr. Ryan's Model Office up to date actuaries would be most grateful. He rather wished he had retained the valuation by the Government Table of 1829, as the very high reserves it brought out were worthy of notice, and it was a table based on annuitant lives. He found from the original valuations of the Model Office in Mr. Ryan's paper before the Actuarial Society, that the reserves by that table actually exceeded those by the British Offices' Tables, Select and Non-Select, by $1\frac{1}{2}$ per-cent, and only fell short of the all-select reserves by about $1\frac{1}{4}$ per-cent.

Mr. A. Hewat said that there were two remarks which had occurred to him. The first was with regard to a company that kept one fund, a combined assurance and annuity fund. The one was very useful in counteracting the other, because it might happen in one year that the mortality among both the annuitants and the assured lives was very light. He remembered, at the time the influenza was raging very badly, being told that a number of people were dying off, and replying that although the assured were dying off so also were the annuitants, and therefore it was suiting the office very well! When, however, separate funds were kept it was not quite so easy to deal with the matter. Another thing was that in the annuity business the office had not to consider bonuses in the valuation. He thought it was a good thing for an office doing life business to have an annuity business in connection with it. There were other reasons, of course, for having annuity business along with insurance business, but from the point of view of the paper he thought it should be kept in mind that the one helped the other in more ways than one, more especially as, if the funds were kept separately, perhaps the shareholders obtained the profit, and at the end of the period had a large addition to their

dividend; but when one was running into the other he thought it enabled a safer and surer business to be done from the mortality point of view.

MR. H. J. RIETSCHEL said that in paragraph 65 the author said: "It is, of course, important, in view of the effect of selection on annuity-values, to classify the business both according to age and date of purchase (unless 'select at entry' values are universally applied)", and to do that separately for males and females. That was certainly necessary if absolute accuracy was desired, but an examination of the $O^{[M]}$ annuity-values would show that, if the whole business was valued in the first place by the Ultimate Table, it was possible without much labour to make a sufficiently close approximation to the extra liability incurred on account of selection for the business of less than five years' duration. All that was required was the total amount of the annuity business still in force that had been effected during the last five years, analyzed according to calendar years of issue. No breaking up into years of birth or according to sex was required for that purpose. In order to obtain the addition to the "Ultimate" liability on account of selection, an average age at entry was assumed, and the total annuity business in force for each year of entry was multipled by the difference between the select annuity-values for the appropriate duration and the ultimate annuity-values. As an error of five years in the assumed age at entry made very little difference in the amount of the additional liability, it was obvious that no great liability to error existed if a mistake was made in the age.

In order to test that, he had assumed that in the Model Office, Table VI, 40 per-cent of the business from fifty to seventy-nine was less than five years in force, and that one-fifth of that might be ascribed to each of the last five years. He had assumed three average ages at entry, sixty, sixty-five and seventy, and he had in the first place divided the business as to sex, and then, alternatively, treated it as if all were female business. The additional liability on account of selection brought out on those assumptions varied from £10,800 to £12,800, or on those widely different assumptions the greatest possible difference in the liability was £2,000, and in practice it would be very much less than that. The total annuity liability produced by the Model Office was £2,000,000, so that the greatest possible error represented only £1 in £1,000, which he was sure was quite accurate enough for an annuity valuation, especially as the future rate of mortality was so doubtful a factor.

If the valuation was made on a net 3 per-cent basis, making no reserve for future expenses, and trusting to the surplus interest to provide those expenses, he ventured to suggest that the office annuity rates should be based on the 3 per-cent select annuity values loaded merely for initial expenses, reliance being placed on the surplus interest to provide the future expenses. No immediate strain would then be thrown on the office in respect of its new business. The adoption of a $3\frac{1}{4}$ per-cent interest basis for ascertaining the office purchase price seemed to obscure the connection

which should subsist between the office reserves and the terms upon which the company could grant annuities. With regard to the paragraph in the paper on limitation of risk, in view of the fact that annuity business and ordinary life business were usually treated as one, for the purpose of ascertaining the surplus belonging to the participating policyholders, he thought the two sections should be regarded as one, in regard to the limit retained. Then, with respect to the limit to be retained, the actuary in fixing it should have regard to the resources of the company. Thus, an office which had in the past produced quinquennially a surplus over and above that required to declare its bonus, could obviously incur the chance of a greater deviation in its expected claims than a society which could only with difficulty maintain its bonus. The limit should be fixed not so much upon a consideration of the number of risks of the same class and amount which the office might expect to obtain, as upon a consideration of the percentage of fluctuation from the expected mortality which it could afford to bear, without endangering its rate of bonus. That view of looking to the surplus, in considering the limit to be retained, was another reason for regarding the annuity and life assurance business as one for that purpose. He noticed that Mr. Young in his book on Insurance also looked to the profit in order to fix the limit which an office could retain for any class of risk.

Mr. E. A. Rusher said that with reference to a valuation by the Government 1883 All-Select Table being practically equivalent to one by the British Offices' partly Select and partly Non-Select, he understood that throughout the paper Mr. Oakley had dealt with that valuation by simply dividing up his annuities into two groups, one in which the lives had been on the books five years and upwards, and the other where they had been less than five years, the latter being valued by the annuities as at the moment of entry. It seemed to him that, in doing that, the author had somewhat over-valued his liabilities. Annuities were so easy to classify and could be valued so readily that there seemed no reason why they should not be divided up accurately into years 0, 1, 2, 3, 4, 5 and upwards: but as Mr. Rietschel had already said there was a method of arriving practically at the same result, even without going to the trouble of dividing them up quite so minutely.

He had made a trial of the method on a large annuity business which came before him, and he found that by valuing by the Government Tables, using, throughout, annuities as at the moment of entry, he obtained practically the same result as though he valued on the British Offices' Annuities, making allowance for each year of selection up to the fifth. For females, the liability by the two methods agreed exactly, whilst for the males the liability by the Government Tables, all select, came out about 1 per-cent in excess of the British Offices'. It occurred to him that that might enable any company which had a large amount of annuity business on its books to pass from the Government Annuity basis to the British Offices' basis, if it thought fit, by a series of steps, by valuing

back gradually at each period the lives as one year nearer the period of selection, ultimately getting back to a valuation, all select, which, as stated, would correspond to one by the British' Offices Tables, and so forming a series of stepping stones to get over the difficulty of having a large addition to the liability at any one period of time.

Another point was with regard to Mr. Oakley's reference to the valuation of joint and survivor annuities. The author dismissed the subject rather briefly, by implying that it was necessary to make an individual valuation. In the case of joint and survivor annuities, those who had to deal with any large figures knew that they had of late been increasingly popular, and therefore it behoved them to look into the matter to see if it were not possible to shorten the valuation. He found that it was possible, by a simple method, to get a very close approximation to the correct liability. Taking either the Government 1883, or the British Offices', Table, by deducting two-and-a-half years off the age of female lives the female lives could be treated as male. Further, although it was not strictly accurate in theory, equivalent equal ages could be taken as though Makeham's law held. When treated in this way it would be found that the valuation obtained would be very close indeed to an individual valuation. The resultant equivalent equal ages were practically the same under both tables, Government and British Offices'. It seemed to him that by this means a great deal of labour was saved in valuing joint and survivor annuities.

Mr E. H. BROWN said that Mr. Rietschel had already referred to a short method of valuation by Select Tables, but this method would probably be recognized as an adaptation of Mr. Ackland's method, published in vol. xl of the *Journal.* He thought it might perhaps interest the Institute to hear that he had tested that method as applied to the valuation of annuities. He found that by constructing a table of weights with a ratio of 1·04 and applying it as explained by Mr. Ackland, remarkably accurate results could be obtained. He had tested it by valuing a large number of annuities; in the case of male lives, the error was positive and amounted to ·008 per-cent; in the female lives, it was negative, amounting to ·014 per-cent, and combining male and female lives, the error was only ·005 per-cent. Making use of the Model Office, given by Mr. Oakley, he distributed the new business of 40 per-cent over durations 0 to 4 in the proportion of 9, 8·5, 8, 7·5 and 7 per-cent respectively, and, employing Mr. Ackland's method of valuation, he found that the ratio of the Select and Non-Select British Offices' Annuities to the Government Annuities 1883, given in Table VII, should be 102·4 instead of 103·2, and similarly the ratio for female lives would be reduced from 103·9 to 103·1; and the males, which he regretted the author had not included in that table, would be reduced from 101·8 to 100·9.

Mr. R. R. TILT said that he did not, however, altogether agree with Mr. Oakley in his observation that the annuity figures sank into insignificance as compared with the figures of insurance business. He

would like to take as the text of his present comments "Selection exercised against the office, according to the variation in the rates of interest available from good investments." He believed that this selection not only affected the profit or loss from interest receipts, but it involved a considerable risk of loss of capital. He thought that the figures of Table I showed clearly that, when stocks were high and the return consequently small, there was a greater flow of business to the office than when stocks were cheap. People who could eke out a small income, representing 4 or $4\frac{1}{2}$ per-cent on their investments, found it necessary to sink the whole or part of their capital in annuities, when the return on their investments was only 3 per-cent. It was well known that there was a rapid rise in stocks between 1894 and 1897, after which prices were stationary for a year or two, and then rapidly fell. In 1895 and 1896 the largest aggregate amounts were received, and they fell away as prices dropped back. In the quinquennium 1895 to 1900 the companies received about 11 millions, whilst in the next quinquennium they received 9 millions, that being a period when a larger return could be obtained from the Stock Exchange.

He did not think that Mr. Oakley's figures fully showed the effect, because there were more competitors in the field for annuity business in the latter part of the period than at the commencement. Presumably, with more offices seeking business, more annuities would be granted (other things being equal). Taking the figures of the six offices that paid more than £50,000 each in annuities in 1896, he found out that they received in that year more than £1,000,000 consideration money; ten years later (in 1906) they received about two-thirds of that sum. This was not because they had ceased to compete. In four cases out of the six the present rates were above the author's suggested rates, and above the average office rates. One of the other two offices received a larger amount in 1906 than in 1896. He thought that it might be taken that, if there had not been increased competition for business, the drop in the amounts would have been more than was shown in the table. Allowance must also be made for increase in the population and in the funds of the nation. With regard to the Government Annuities, some figures given by Mr. Oakley showed that the amount now paid annually by the Government was about 17 times the amount of the annuities they granted last year, he inferred from this that there had been a considerable drop in Government business, because the average life of annuities was certainly not 17 years. He thought he had shown that annuity business increased when stocks were high. It would be well to consider how the companies invested the money thus paid to them.

In the last session of the Institute an interesting paper was read by Mr. Newman on the "Investments of Insurance Offices", (*J.I.A.* vol. xlii, p. 294) which contained a table giving an analysis of the assets of British life offices, year by year. In 1895 the assets were 226 millions, of which 134 millions were represented by loans, agents' balances and cash; that is to say, investments which involved

covenants to pay the capital value in cash at some time or another. The balance (92 millions) was invested in stocks and shares, land, houses and reversions—securities which (except some redeemable debentures) did not represent covenants to pay the capital in cash. Money invested in these securities represented rights to income only, and the future cash values of these rights was not known. In 1900 (five years later) the total assets had increased by 52 millions, but loans, agents' balances and cash had diminished by two millions, so that stocks, land, &c., had increased by 54 millions. Now between 1895 and 1900, there was a very large increase in the annuity funds, and it was a fair inference that a large portion of the annuity considerations was invested in the purchase of rights to income—a commodity which fluctuated in value from day to day. What was the nature of annuity business? Stripped of its trappings, it was simply a contract to return the cash by instalments over a short period of years with interest on the out-standing balance. He had shown that during the quinquennium 1895-1900 the money was invested in a way which did not involve a return of the cash. If it were permissible to count on a continuously level flow of annuity business into an office, it might be argued that over a long period any loss on investments would probably equal the profit, and that therefore the business might be transacted, and the cash invested in stocks. Nevertheless, there was a risk that the depreciation would exceed any increase in value, and wherever there was a risk in assurance business, a premium should be charged.

The important point, however, to which he desired to call attention was the fact that the office had an excess of money to invest just when stocks were high, and investments representing fixed amounts of cash were probably exhausted. That seemed to him a grave objection to annuity business at fixed rates, and he did not see how it was to be easily overcome. The Government provided for alterations in the rate of interest by the automatic way in which the annuity price was made to depend on the price of Consols, but the Government system did not appear to compensate for the risk of loss of capital. They might turn cash into Consols at 112 and subsequently return the cash by turning Consols into cash at 85.

The remedy appeared to be to alter the terms periodically. If this could not be done, it seemed to him that Mr. Oakley's proposed rates were not sufficient, taking into account the risk of loss of capital. The author gave, in Tables III, IV and V, a summary of consecutive valuations of a group of twelve offices, which published separate annuity accounts. The depreciation shown in these valuations, £45,000 or so in twenty years, on funds which had grown rapidly to £11,000,000, seemed ridiculously small, and he was afraid that he could place no reliance on the figures. The fact was that, although these companies kept separate annuity revenue accounts, they did not keep separate assets for the annuity funds. He had looked through several Blue-Books, and had found one office only that had a separate investment of assets against its

annuity liabilities. The other companies mixed their annuity and life assurance assets, and he believed that in this way the true depreciation of the annuity fund was concealed. It seemed to him that the effect might be illustrated as follows:—An office, when Consols were at 100, valued its holding at 90. The same day that it paid a claim on its life fund of £1,000 it received an annuity consideration of £1,000: thereby it was enabled to retain its Consols, and, in effect, the annuity consideration of £1,000 was represented as having been invested in Consols at 90. An arrangement of this kind was not sound finance. The annuity consideration was in reality invested at 100.

He thought that if the companies' business had been confined to annuities—which meant that the annuity considerations would have been the only moneys available for investment—that the depreciation shown, considering the large amounts invested between 1895 and 1900, would have been much greater. Whilst making these criticisms he was well aware that the modern tendency was to combine various businesses under one roof: and managerial policy might require the inclusion of annuity tables in the office prospectus.

The President, in proposing a vote of thanks to Mr. Oakley for his admirable paper, said it was not often that the first essay read by a contributor gave rise to so interesting a discussion. The author had dealt very thoroughly with the subject, and put a good deal of work into his communication, both in connection with the numerous tables and the very valuable statistics he had given. The conclusions also were very important, one of the most significant being perhaps what might be termed the main conclusion, namely, that for the last twenty years the offices granting annuity contracts had practically been carrying on that business at cost price, or, at all events, whatever profit they had made had been profit arising out of the difference between the rates of interest earned and those assumed, and the whole of that profit had been absorbed in bringing up their reserves to a proper figure. Looking at the table in paragraph 7, a table which he thought was taken from a paper by Mr. Ryan, and observing the basis of valuation adopted by offices in 1880, it would be seen on the whole how inadequate they were, as compared with the present ideas of what proper annuity reserves should be, and yet that very date, 1880, practically represented about the average date of the British Offices' Annuity Experience. When it was remembered also that while those reserves were being made the offices were actually experiencing the mortality as shown by the British Offices' Tables, it revealed how inadequate the reserves were and also showed that, practically speaking, the Government Annuity Tables of 1883 were really obsolete the moment they were brought out.

Mr. Oakley, and one or two members in the discussion, had referred to the question of Government annuity business, and that was connected with another very important conclusion to be drawn from the paper, a conclusion which he supposed had been in the minds of everyone in a vague sort of way for many years past—the

impossibility that the Government could be really carrying on its annuity business at a profit, considering the experience that had been brought to light in recent years as to the real mortality of annuitants. No reason could be seen why there should be the extraordinary distinction between Government annuitants and annuitants who patronized the life assurance offices, neither was there any reason why the Government should persist, in the light of more recent experience, in carrying on their business at what must be manifestly a loss. It could only be hoped that fresh ventilation of the subject might bring the matter once more forcibly to the notice of those in authority, and that before very long they would look into it and discontinue what must be more or less a drain on the national resources.

There were one or two interesting points raised in the discussion. Mr. Rietschel had referred to the question of limitation of risk in respect of annuity business, and students would find a very valuable paper by Dr. Bremiker in vol. xvi of the *Journal* which dealt with that and some kindred questions. Although that paper had been written a great number of years ago, he thought it would still repay the study of anyone interested in the subject. Another point was raised by Mr. Tilt, a more practical question, namely, the selection against the office which took place when the rate of interest was extremely low and the price of securities high. It seemed to him that Mr. Tilt had shown that it was extremely probable that such selection by annuitants had an adverse effect in the long run on the profits of annuity transactions, and that was another reason why annuity business should not be done on the very small margin of loading which was usually thought sufficient for that class of business. In the case of insurance, even non-profit insurance, there were numerous sources of profit to fall back upon, if the assumptions as to interest, mortality, and so on, were erroneous; whereas, in the annuity business, if the assumption as to mortality were erroneous, there was only the interest profit to fall back upon. Quite a large margin of interest was required to make up for the difference in the mortality. Therefore, he thought, as insurance premiums were loaded comparatively heavily to secure against loss and to provide the company with a profit, the offices should not be content with loading their annuity premiums with a mere nominal loading to cover the commission, and rest there. There was a further important consideration, that, as far as might be known, the British Offices' Annuity Experience might already be somewhat out of date. It was fifteen years since the Annuity Experience closed in 1893 and actuaries were quite in the dark as to what the result of a more modern experience might be. That seemed to him to be another reason for providing against the contingencies of still further falls in the rate of mortality, and securing against loss on that class of business by putting on a fairly substantial loading.

The vote of thanks was carried unanimously.

Mr. H. J. P. Oakley, in reply, said that he had been very interested in the remarks of Mr. Rietschel and Mr. Brown, in

regard to a short method of giving effect to selection. That point crossed his own mind also when preparing the paper, but he had in view the fact, which the President had just mentioned, that it was possible that the British Offices' Tables might even at present be slightly out of date, and he thought it was preferable to value the annuities by assuming those who had entered during the last five years as "select at entry." With regard to Mr. Rusher's remarks as to the arbitrary assumption between select and non-select portions of the business, that arbitrary assumption was made only in respect of Mr. Ryan's Model Office, in which no figures were given showing how the annuitants entered. In paragraphs 27 and 28, which related to an actual experience, the valuations were made in accordance with fact, taking the annuitants as they entered, and allowing for the period elapsed. Reference had been made to the suggested rates not being yet sufficiently stringent to yield a profit. He put those forward as maximum rates, suggesting that no higher rates should be given to the public than those shown. He was almost in accord with the speakers in thinking that a rate of interest of 3 per-cent should not be exceeded in the calculation. Mr. Tilt had referred to the investigation into the accounts of twelve companies, and in that connection he was very glad the offices had not adopted Mr. Hewat's view of forming only one fund, because it would not have enabled him to see how the offices had been working. He could hardly agree with Mr. Tilt that no separate assets had been kept. As there had been no bonuses to give or profits to divide, the offices had not written down their investments, and that was the reason why no depreciation had been shown. They had, doubtless, been hoping for better times. Perhaps, when another five years had passed from the dates when the last respective accounts had been analyzed, it might be seen that the offices had written off a certain amount for depreciation.

LEGAL NOTES.

By ARTHUR RHYS BARRAND, F.I.A., *Barrister-at-Law.*

Interest of a husband in the life of his wife.

THE case of *Griffiths* v. *Fleming* [1909] 1 K.B. 805, recently decided by the Court of Appeal, is concerned with the interest of a husband in the life of his wife, and is one of the most important cases dealing with the subject of insurable interest decided in recent years. The facts of the case were given in these Notes when the proceedings in the High Court were reported (*J.I.A.*, xlii, 479), but it will be convenient to recapitulate them briefly. The plaintiff and his wife effected a joint life

assurance for £500 with the United Kingdom Temperance and General Provident Institution, of which the defendants are the trustees, the policy bearing date 8 October 1907. The policy, as usual, was payable to the survivor. Each of the assured signed an ordinary proposal, the only variation being that in answer to the question as to the amount of the assurance, each indicated the table under which the assurance was being effected, and stated that it was to be for £500, jointly with the other assured. Shortly after the granting of the policy the female assured committed suicide. The assurance company contested the claim, *inter alia*, on the ground that the male assured had no insurable interest in the life of his wife, and, the other pleas failing, this became the sole issue. The case came before Pickford, J., who held that, as the wife had rendered certain services to her husband, and as, by reason of her death, he had been compelled to hire someone to perform those services, he had such an insurable interest in her life as would support the policy. He therefore gave judgment for the plaintiff. The defendants appealed against this decision, but the Court of Appeal upheld it, though on different grounds. In delivering a considered judgment, Vaughan Williams, L.J., said: "I think it was "unnecessary to go into evidence to show a pecuniary interest "in the husband, as was done before the learned judge at the "trial. I agree with his ultimate decision, but on the ground "that the husband is to be presumed to have an interest in the "wife's life in such a sense that it is unnecessary to give "affirmative evidence as to the existence of an interest." Kennedy, L.J., did not deliver a separate judgment, but read the judgment of Farwell, L.J., with which he expressed his entire concurrence. In the course of that judgment, Farwell, L.J., said: "I have come to the conclusion that the decision of "Pickford, J., can be supported on a broader ground, and I "desire to rest my judgment on it, namely, that a husband has, "as such, an insurable interest in his wife's life. The contrary "appears to be stated in some of the text-books, but the "proposition is affirmed in Bullen and Leake, 2nd edition, p. 161 "(6th edition, p. 205). The learned authors say: 'The interest "'in this statute means, in general, pecuniary interest . . . But "'a wife may insure her husband's life, and the husband his "'wife's.'" After pointing out that the Gambling Act applied in its terms to an assurance affected by a man on his own life, he went on to say: "Every man is presumed to have an interest in

"his own life . . . But this must be on the ground that an "insurance by a man on his own life is not within the mischief "of the Act . . . Lord Kenyon went a step further, and held that "a wife, as such, has an insurable interest in her husband's life, "and he refused to allow evidence to be given by her that her "late husband was entitled to a life interest of large amount . . . "In my opinion Lord Kenyon excluded the evidence on the same "grounds on which evidence of insurable interest in the insurer "for his own benefit would be excluded, namely, that the case "was not within the mischief of the Act. If this be so, it "follows, in my opinion, that the same principle must be applied "to the insurance by the husband of the wife's life; a husband "is no more likely to indulge in 'mischievous gaming' on his "wife's life than a wife on her husband's. It is not a question "of property at all; it is that for this purpose husband and "wife stand on the same footing, and that the ruling of "Lord Kenyon a century ago in favour of the wife's claim "ought now to be applied in favour of the husband's."

Joint-life policy may be construed as two settlement assurances.

It may also be pointed out that Farwell and Kennedy, L.JJ., were prepared to hold that a policy in this form could be regarded "as an assurance by "the wife on her own life expressed to be for the benefit of her "husband contingently on his surviving her, and by the husband "on his own life for the benefit of his wife contingently on her "surviving him", within the meaning of section 11 of the Married Women's Property Act, 1882.

Banker's pass-book as evidence of state of account.

The case of *Holland* v. *Manchester and Liverpool District Banking Company, Limited* [1909] 25 T.L.R., 386, came before Lord Alverstone, C.J., at the Manchester Assizes in February last, and deals with an interesting point respecting the duty of a bank to honour its customers' cheques. The plaintiff, who had an account with the defendants, after examining his pass-book and finding a balance of £70. 17*s*. 9*d*. entered there in his favour, drew a cheque for £67. 11*s*. in favour of a firm to whom he owed a trade debt. The latter presented the cheque, which was dishonoured without any communication being made to the plaintiff. As a matter of fact, the real amount of the plaintiff's balance at the bank at the time was £60. 15*s*. 9*d*., but one of the defendants' clerks had entered to his credit a sum of £10. 2*s*. twice over, with the

result that from the pass-book he appeared to be in credit to the amount of £70. 17*s.* 9*d.* The plaintiff having suffered damage by reason of his cheque being dishonoured, brought an action against the bank, and judgment was given in his favour for £100. Lord Alverstone, C.J., in the course of his judgment, said that "The effect of a pass-book entry did not seem to have "been clearly decided in the Courts, but he considered that, "whilst the bank in this case were entitled to have any wrong "entry ultimately corrected, until the correction was made the "customer had the right to act upon the bank's statements "in the pass-book, and to receive them as statements by the "bank that there was so much money to his credit. The "pass-book in all such cases, although subject to adjustment, "was *primâ facie* evidence against the bank of the amount "standing to the credit of a customer, upon which that customer, "in the absence of negligence or fraud on his part, was entitled "to rely."

Right to surplus funds of Friendly Society.

The case of *Braithwaite* v. *Attorney General* [1909] 1 Ch. 510, was an action brought by the trustees of a society called The Benefit Society for Girls educated at the School of Industry, Kendal, and duly registered under the Friendly Societies Act, 1793, to ascertain what should be done with the funds of the society, it having practically ceased to exist. The society consisted of honorary members and benefited members. The funds consisted of contributions by both classes of members, but only benefited members could receive any benefit. In addition to certain sick benefits, annuities were granted at age 56, of amounts increasing at certain intervals with the age of the annuitant. The society was carried on in accordance with the rules down to the year 1845, when the School of Industry ceased to exist. After that date no new benefited members were admitted, and the society was carried on for the benefit of the members enrolled before the abolition of the school. Subscriptions continued to be received from both classes of members, and at the date of the action the funds amounted to £2,205. 8*s.* 6*d.*, apportioned as to £1,901. 1*s.* 2*d.* to the honorary members' fund, and as to £304. 7*s.* 4*d.* to the benefited members' fund. There were only two benefited members of the society surviving, aged respectively 85 and 68, both being entitled to annuities. Swinfen Eady, J., held that the

fund contributed by the honorary members was not the subject of a charitable trust, and therefore declined to accept the contention of the Attorney General that it ought now to be applied *cy près*. He also held that "the donors, having parted once for all with all interest in the sums so given", there was no resulting trust in their favour, and they had no claim to any portion of the fund. With regard to the contention that the two remaining benefited members of the society were entitled to the whole funds, he said: "Their only right is under their contract of membership, pursuant "to which they are entitled to certain annuities out of the "benefited members' funds, but not to the surplus of those "funds after satisfying their annuities. . . . . The entire "beneficial interest has been exhausted in respect of each "deceased benefited member, and when the annuities to the two "surviving members cease to be payable, upon their respective "deaths, they too will have exhausted all their beneficial interest "in the funds. All possible claimants to the fund having now "been disposed of, I decide that the surplus belongs to the "Crown as *bona vacantia*."

Three revenue cases of considerable importance have been decided recently, and although so far as the actual facts of each case are concerned, they have no connection with assurance matters, it has been deemed advisable to notice them here, as the principles laid down in them are far-reaching in their consequences, and are applicable in many other circumstances than those in which they were actually applied in these cases.

Increase of authorized capital. Liability to duty although not issued.

The first of these is the case of the *Attorney-General* v. *The Anglo-Argentine Tramways Company Limited* [1909] 1 K.B., 677. Here the defendants, who were an English company, were registered in 1887 with a capital of £800,035. In July 1907 the company passed a resolution approving a provisional agreement for the acquisition of the undertaking of another company, and authorizing the increase of the capital by an amount not exceeding £5,000,000. In August 1907 the directors, acting on this resolution, increased the capital by £200,000, and paid duty on this amount; and in July 1908 the capital was further increased by £2,800,000, and duty was paid on this amount also. The Inland Revenue authorities contended, however, that they were entitled to duty on the whole of the £5,000,000 authorized in July 1907, whether

such capital was actually issued or not, and they accordingly claimed the sum of £12,500 as a debt due to the Crown. Judgment was given in favour of this contention, and in delivering it, Channell, J., said: "The question seems to me to turn "on the meaning of the words 'increase of registered capital' "(Stamp Act 1891, section 112). What is that which is to be "increased in order to attract the duty? It seems to me to be "clear that it is the maximum limit of the capital which the "company are authorized to ask from the public, that is to say, "the nominal capital. The Act contemplates that just as the "amount of the original nominal capital is registered, and is, "therefore, open to the inspection of the public, so also any "increase of that registered capital will itself be registered and "open to public inspection, so that anyone can, by looking at "the register, see at once what is the maximum amount of "capital which the company are authorized to raise. Therefore, "it seems to me, that 'registered capital' means, not the capital "which has been actually issued, but the authorized capital, that "is, the maximum amount of capital which the company are "authorized to raise. When that capital is increased, the "company have to pay *ad valorem* duty on the increase."

Stamping of agreement securing minimum quarterly payments over a term of years.

The second case is that of the *County of Durham Electrical Power Distribution Company* v. *Commissioners of Inland Revenue* [1909] 1 K.B., 737. This was concerned with a question as to the proper stamping of an agreement entered into by the plaintiffs with a customer, under which the latter was, for a term of seven years, to take from the plaintiffs all the electric current required, and to pay for the same by means of a fixed charge of £57. 10*s*. per quarter, and in addition, a sum of 1*d*. per Board of Trade unit, together with a charge of 10*s*. per quarter as rental for the meter. There was also a proviso under which, in certain circumstances, the quarterly payments might be reduced, but in no case was the fixed charge to be less than £50 per quarter. On this agreement being submitted to the Inland Revenue authorities for stamping, they claimed that it was properly described as an instrument which was the only, or principal, or primary security for sums of money payable at stated intervals, and claimed duty at the rate of 2*s*. 6*d*. per £100 of the aggregate amount of the minimum quarterly payments for seven years. The plaintiffs

appealed against this decision, but on the case coming before Channell, J., he upheld the contention of the Crown. In the course of his judgment, he said: "I have a strong impression "that such a document as this was not one which the Legislature "had in contemplation when they enacted the clause in the "schedule under discussion. I cannot help thinking that it "was put in for the purpose of taxing instruments of a totally "different character. . . . The case has been argued upon the "footing that it is a supply of goods. If it is, my decision will "have a very curious consequence, for it will involve the con-"clusion that in the case of every contract in writing for the "sale of goods which, by the terms of the contract, are to be paid "for, not in one lump sum, but in instalments at future, stated "times, the instrument is chargeable with an *ad valorem* duty "upon the price." These words seem to indicate that the decision is applicable, *inter alia*, to hire-purchase agreements.

Liability in respect of an unstamped receipt where the amount has already been included in a duly stamped receipt.

The third case comes from Ireland, and is that of *The Attorney-General* v. *Ross* [1909] 2 I.R. 246. This was an appeal from a decision of the King's Bench Division relating to the stamping of a receipt, and arose in the following circumstances: The defendant's father was the owner of certain premises which he let to one Nolan, and he also supplied him with milk, butter and vegetables. Each month the defendant sent in an account to Nolan, including in it both rent and money due for goods supplied, and a duly stamped composite receipt was given for the combined amount. The defendant, at the same time, gave Nolan a formal receipt for the rent alone, but this was unstamped. There were eight of these receipts, and an action was brought by the Crown to recover eight penalties, of £10 each, in respect of them, on the ground that they should have been duly stamped as receipts. The King's Bench Division found in favour of the defendant on the ground that the composite receipts were duly stamped, and that, in the circumstances, the documents relating to the rent alone, "although on their face purporting to be receipts, were not . . . in truth and fact, receipts." The Crown appealed from this decision, and the Court of Appeal allowed the appeal. In giving judgment in favour of the Crown, FitzGibbon, L.J., said: "The real question is whether each of "the unstamped rent receipts was a document which came within

"the statutory description of a receipt which required a stamp. "What it was must be decided upon the document itself, and "by its own purport, contents and nature, and not by the "intention of the party who signed and issued it. Assuming "that a stamped receipt was given for a joint sum of money "which included the amount of the rent for which the "unstamped receipt was given, the question remains whether the "second document answers the description in the Act of "Parliament. If so, it must be stamped. . . . There may often "be two papers which, together, form but one receipt, or one "document may identify and refer to another, so as to show that "one only is the real voucher. In such cases two stamps will "not be necessary—*e.g.*, where a stamped receipt is put in a "covering letter which acknowledges the payment and says 'I "send you herewith a stamped receipt.' But that is not the case "here, and as the printed receipt is a formal and complete "quittance for the debt, a penalty has been incurred by its not "having been stamped, notwithstanding that another acknow-"ledgment of the same with other money, was given with a "stamp upon it."

Imperfect gift of policies of life assurance.

The case of *Vavasseur* v. *Vavasseur* [1909] 25 T.L.R. 250 is of some interest, dealing as it does with a defective title to certain policies of life assurance. The action was brought by two daughters of one James Vavasseur against his executor to recover a sum of £26,000, the proceeds of certain policies of assurance, to which they alleged they were entitled. The following are the facts of the case, as stated on behalf of the plaintiffs: Owing to increasing infirmities, the father, who had two other daughters besides the plaintiffs, in 1900 requested the plaintiffs to manage his estate for him and to give him their assistance generally in the management of his affairs, and this they had done, one of them abandoning a professional career for the purpose. In 1902, after the plaintiffs had undertaken the duties in question, their father told them that in consideration for their services they would be entitled on his death to the proceeds of certain policies of assurance on his life; and on many occasions he assured them that the proceeds of the policies would be theirs, and told them in what way they would be able to obtain them after his death. In 1897 the father made a will in which no mention was made

of the policies, which, in the ordinary course of events, would become part of the residuary estate in which the plaintiffs and the other two daughters of the testator were to participate. Three weeks before he died the father called the plaintiffs into his room and pointed out to them that their sisters were married and provided for, and that the money due on the policies at his death would become the property of the plaintiffs. The testator died without having made a fresh will, and the question arose as to whether there had been a valid gift of the policies to the plaintiffs, or whether they formed part of the residuary estate. Channell, J., gave judgment in favour of the executor, and in doing so, said that it was quite impossible to come to the conclusion that the plaintiffs' case could be supported on any legal ground. After commenting on the absence of corroboration of the plaintiffs' story as to what had happened between their father and themselves, and pointing out that in the circumstances the case might be decided against them on that ground alone, he went on to say that "even accepting literally " the evidence of the plaintiffs, it was quite clear that no legal " effect could be given to the testator's words . . . . They " did not constitute a declaration of trust, but were merely the " expression of a wish; and when to that was added the fact " that the wish was to be acted upon after the testator's death, " it was clear that all they amounted to was a verbal will, and a " verbal will could not be given effect to." He also held that there was no ground for the suggestion that there was a contract whereby the testator promised to do something for his daughters in consideration of their doing something for him.

Power of appointment exercised in forgetfulness of previous appointment. Rescission.

In the case of *Lady Hood of Avalon* v. *Mackinnon* [1909] 1 Ch. 476, it was sought to set aside an appointment made by deed poll, on the ground that it had been made in entire forgetfulness of a previous appointment in favour of the same person. The facts of the case were as follows: Acting under a power contained in her marriage settlement, the plaintiff and her husband had, in April, 1888, executed a deed poll whereby on the occasion of the marriage of their elder daughter, the defendant, they appointed certain property in her favour. Later on, the plaintiff, after the death of her husband, appointed certain shares to her younger daughter. In August 1904, being desirous of equalizing, as she thought,

the shares of her two daughters in the trust funds, and entirely forgetting the original appointment made in 1888 in favour of the elder daughter, the plaintiff appointed a sum of £8,600 to the defendant absolutely. On the facts being brought to her notice, the plaintiff stated that she had no recollection whatever of the joint deed of appointment executed in April 1888 in favour of her elder daughter, although she admitted her signature to the deed; and she sought a declaration that the deed poll of August 1904 should be rescinded and set aside. An order to this effect was made by the Court, and in delivering judgment, Eve, J., after stating that he was quite satisfied that the later appointment in favour of the defendant was made in entire forgetfulness of the earlier appointment, and simply for the purpose of equalizing the shares of the two children, said: "The question is, "whether, in that state of facts, I can, consistently with the law, "say that such a mistake on the part of the appointor entitles "her to have the deed rescinded. . . . I feel bound to "hold . . . that this deed which it is sought to rescind, "was executed by Lady Hood under a mistake brought about by "such circumstances as entitle her to the relief she seeks. "Accordingly, I make an order that the deed be rescinded and "set aside."

Unauthorized representations by agents.

The case of *Refuge Assurance Company* v. *Kettlewell* which has previously been referred to in these Notes in connection with its appearance before the Divisional Court and Court of Appeal (*J.I.A.*, xli, 574, xlii, 401) and which is concerned with the liability of a company for the unauthorized representations of its agents, came before the House of Lords on appeal on 5 March 1909 (25 T.L.R., 395). The appeal was dismissed and the previous decision affirmed without comment.

ORIGINAL TABLES.

English Life Table No. 6.

In response to the suggestion made by the writer of the review of the "Supplement to the 65th Annual Report of the Registrar-General of Births, Deaths and Marriages in England and Wales" (*J.I.A.*, xliii, p. 230), Mr. Vyvyan Marr has computed, and kindly placed at the disposal of the Institute, the appended Tables, based on the English Life Table No. 6.

Mr. Marr has furnished the following statement, as to the graduation of the mortality tables, and the construction of deduced values :—

The tables are given for male lives and for female lives. Mr. George King's graduated values of q_x were adopted in forming the tables relating to male lives—the values chosen being those of his "construction A" (see *J.I.A.*, xliii, p. 150) calculated by second differences. The same method of graduation was used in forming the graduated values of q_x for female lives. The original data relating to females are given below. The figures relating to age groups 75-85, 85-95 and 95 and over, were kindly furnished by Dr. Tatham, while the others were extracted from p. 5 of Part I of the Supplement to the 65th Annual Report of the Registrar-General.

Data, English Life Table No. 6 (Females).

Ages x	Interval n	Mean Population: In interval $T_{x.\overline{n}}$	Mean Population: Total, x and over T_x	Deaths in 10 years: In interval $l_{x.\overline{n}}$	Deaths in 10 years: Total, x and over l_x
0– 5	5	1,822,307	15,810,281	962,127	2,710,149
5–10	5	1,724,889	13,987,974	75,381	1,748,022
10–15	5	1,640,975	12,263,085	42,110	1,672,641
15–20	5	1,557,124	10,622,110	57,075	1,630,531
20–25	5	1,514,357	9,064,986	67,560	1,573,456
25–35	10	2,510,866	7,550,629	152,699	1,505,896
35–45	10	1,877,703	5,039,763	180,127	1,353,197
45–55	10	1,398,218	3,162,060	206,111	1,173,070
55–65	10	955,602	1,763,842	271,758	966,959
65–75	10	569,834	808,240	346,021	695,201
75–85	10	209,491	238,406	273,591	349,180
85–95	10	27,952	28,915	73,134	75,589
95 & over	...	963	963	2,455	2,455

The method of graduation supplies values of q_x from age 17 to age 97 inclusive and, in addition, values of q_x for ages 12 and 102. The values of q_x for ages up to 105 and down to 5 were obtained by finite differences. In order to carry the tables back to age 0 the values of q_x for ages 0 to 4 inclusive, given in the tables of the Registrar-General, were adopted. A juncture with the graduated values was formed by assuming that the function $\log(q_x + \cdot 1)$, from age 4 to age 18 inclusive, formed a series with constant third differences—the differences being found from the values for ages 4, 12, 17 and 18.

The Registrar-General took specially into account the births and deaths of children under five years of age in the years 1886-1900, in ascertaining the rates of mortality for the early years of life. If, however, the published rates for males and females respectively are compared, it will be found that the values of q_x for age three are not in harmony with the values for the other ages. In the tables now submitted these values were arbitrarily adjusted—the values

of q_x for age two being also amended so that at age two the probability of living two years might remain unaltered.

In the Tables submitted the l_x columns were computed from $\log p_x$, seven place logarithms being used. The radix was taken as 508,770 living at age 0 in the case of males and 491,230 living at age 0 in case of females. The life tables accordingly show the number of male and female survivors respectively out of 1,000,000 births. This had the effect of making the limiting age 103 in the case of the male table and 104 in the case of the female table in place of 106 in both tables.

In calculating the D_x, N_x, M_x and R_x columns, however, the values were taken up to age 105 inclusive. The values of $\log D_x$ were obtained by adding $\log v^x$ to $\log l_x$, five place logarithms being used and the natural numbers being taken out to five significant figures. Continuous summations gave N_x. Similarly, in calculating M_x and R_x the values of C_x were previously obtained by adding $\log v^{x+1}$ to $\log d_x$, the values of d_x being computed to four significant figures up to age 105, except at ages 10 to 13 inclusive and five place logarithms being used. The natural numbers of C_x were taken to the nearest unit for ages below 34, and to five significant figures for the remainder of the table.

The values of a_x were obtained from the formula

$$\log a_x = \log N_{x+1} - \log D_x,$$

five figure logarithms being used.

The force of mortality μ_x was computed by the formula

$$\frac{7(d_{x-1}+d_x)-(d_{x-2}+d_{x+1})}{12 l_x}$$

The values of d_x and l_x were respectively those used in the formation of the C_x and D_x columns. Towards the limiting age the formula did not give very satisfactory results, and the values of μ_x for ages over 95 were obtained by adding $\Delta\mu_x$ to μ_x, the values of $\Delta\mu_x$ being derived by differentiating the series $\log(q_x + \cdot 1)$ which, for these ages, has constant fourth differences.

For ages below 6 use was made of the formula

$$\mu_x = -\frac{1}{M}\left\{\delta \log_{10} l_x - \frac{1}{2}\delta^2 \log_{10} l_x + \frac{1}{3}\delta^3 \log_{10} l_x - \&c.\right\}$$

where M is the modulus of common logarithms (*see* *T.F.A.*, vol. i, p. 55). Seven orders of differences of $\log_{10} l_x$ were taken.

The values of $\bar{a}_x$ were obtained by the formula

$$\bar{a}_x = a_x + \frac{1}{2} - \frac{1}{12}(\mu_x + \delta)$$

and $\bar{A}_x$ and $\bar{P}_x$ were obtained from "Rothery and Ryan's Premium Conversion Tables."

The calculations were made in duplicate.

In Table VI, the values of a_x are contrasted with the values according to the English Life Table No. 3.

[Editorial Note.—In view of the great importance of the value of the force of mortality, as deduced from statistics of the general population, at the moment of birth, and in the early years of infancy, and also of the fact that the finite difference formula employed by Mr. Marr in deducing such values gives, as shown by Dr. A. E. Sprague (*T.F.A.*, vol. i, pp. 57, 58), somewhat indeterminate results, it has been thought that the results of an alternative method of experimentally calculating such values, which are on the whole confirmatory of Mr. Marr's figures, might usefully be appended to his Tables.

The method followed is on the general lines of that given in the *Text-Book*, Part II, chap. vi, § 68, and proceeds on the assumption that Makeham's first development of Gompertz's law holds good over the first three years of life, so that, for all values of x, from 0 to 3 inclusive, $l_x = kg^{c^x}s^x$, where the constants k, g, and s, *and also the constant* c, are deduced from the values of $\log l_0$, $\log l_1$, $\log l_2$, and $\log l_3$, as tabulated by Mr. Marr (but taken to seven places) for male, and for female, lives. The values of the logarithms of the constants, and of the force of mortality deduced by the formula

$$\mu_x = \frac{1}{\mathrm{M}}\{-\log_{10}s - (\log_{10}g\log_{10}c)c^x\}$$

where M = the modulus, are as follows:—

Age (x)	Values of Constants			Values of μ_x	
	Common Logarithm of	Males	Females	Males	Females
0	k	5·6025042	5·6013453	·33936	·25491
1	g	0·1040173	0·0899396	·09269	·08198
2	s	$\bar{1}$·9958825	$\bar{1}$·9969237	·03047	·02972
3	c	$\bar{1}$·4018376	$\bar{1}$·4803190	·01478	·01392

It will be observed that the values of μ_x are alternately greater, and less, than those deduced by Mr. Marr on p. 356, but do not, on the whole, greatly differ from his values. The abnormal value, and negative sign, of $\log_{10}c$, as deduced from the experience of the first three years of life, of course arise from the exceptional magnitude and progression of the rate of mortality during that infantile period. It may be added that the experimental deduction of values of μ from the three values $\log_{10}l_0$, $\log_{10}l_1$, and $\log l_2$, with an assumed value of $\log_{10}c = {\cdot}0396$ . . . (following throughout the method adopted in the *Text-Book*, *loc. cit. supra*), gave values of μ_0 materially smaller than those deduced above, and also much smaller than Mr. Marr's tabular values.]

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TABLE I.

English Life Table No. 6. Graduated Values of q_x.

Age	Males	Females	Age	Males	Females	Age	Males	Females
0	·17186	·14066	36	·00955	·00815	72	·08476	·07408
1	·05319	·04949	37	·01008	·00859	73	·09065	·07944
2	·02067	·02033	38	·01063	·00902	74	·09640	·08467
3	·01335	·01315	39	·01120	·00945	75	·10250	·09021
4	·00970	·00955	40	·01180	·00989	76	·10948	·09658
5	·00746	·00740	41	·01240	·01031	77	·11796	·10437
6	·00573	·00573	42	·01301	·01072	78	·12856	·11424
7	·00441	·00446	43	·01360	·01106	79	·14115	·12609
8	.00345	·00354	44	·01416	·01135	80	·15523	·13932
9	·00282	·00293	45	·01475	·01164	81	·17015	·15317
10	·00244	·00257	46	·01541	·01202	82	·18508	·16671
11	·00229	·00243	47	·01620	·01254	83	·19988	·17965
12	·00232	·00245	48	·01714	·01324	84	·21509	·19253
13	·00249	·00261	49	·01820	·01407	85	·23078	·20555
14	·00276	·00285	50	·01936	·01500	86	·24707	·21893
15	·00310	·00313	51	·02057	·01600	87	·26409	·23294
16	·00345	·00343	52	·02182	·01703	88	·28156	·24714
17	·00379	·00368	53	·02305	·01804	89	·29936	·26133
18	·00407	·00386	54	·02427	·01905	90	·31809	·27619
19	·00434	·00100	55	·02558	·02014	91	·33805	·29256
20	·00460	·00413	56	·02706	·02140	92	·36016	·31140
21	·00485	·00428	57	·02880	·02292	93	·38431	·33264
22	·00507	·00445	58	·03088	·02476	94	·41025	·35597
23	·00525	·00467	59	·03324	·02686	95	·43860	·38216
24	·00540	·00493	60	·03581	·02918	96	·47010	·41213
25	·00553	·00519	61	·03853	·03164	97	·50563	·44704
26	·00569	·00545	62	·04130	·03416	98	·54558	·48753
27	·00591	·00568	63	·04402	·03664	99	·59041	·53434
28	·00619	·00585	64	·04673	·03915	100	·64066	·58828
29	·00650	·00598	65	·04963	·04182	101	·69697	·65027
30	·00685	·00611	66	·05289	·04483	102	·76008	·72133
31	·00723	·00627	67	·05673	·04837	103	·83089	·80250
32	·00763	·00651	68	·06134	·05261	104	·91045	·89499
33	·00806	·00684	69	·06665	·05749	105	1·00000	1·00000
34	·00853	·00725	70	·07218	·06283	...	...	...
35	·00903	·00769	71	·07860	·06844	...	...	...

TABLE II.

English Life Table No. 6.

Age	Males				Females				Age
x	l_x	d_x	μ_x	$\log l_x$	l_x	d_x	μ_x	$\log l_x$	x
0	508,770	87,437	·32437	5·70652	491,230	69,096	·24451	5·69128	0
1	421,333	22,411	·09586	·62463	422,134	20,891	·08651	·62545	1
2	398,922	8,242	·02678	·60089	401,243	8,056	·02630	·60341	2
3	390,680	5,216	·01640	·59182	393,187	5,270	·01607	·59449	3
4	385,464	3,739	·01112	·58598	387,917	3,704	·01101	·58874	4
5	381,725	2,847	·00847	·58175	384,213	2,844	·00838	·58457	5
6	378,878	2,171	·00654	·57850	381,369	2,185	·00651	·58135	6
7	376,707	1,662	·00502	·57600	379,184	1,691	·00504	·57885	7
8	375,045	1,294	·00390	·57408	377,493	1,336	·00396	·57691	8
9	373,751	1,054	·00309	·57258	376,157	1,103	·00319	·57537	9
10	372,697	909	·00259	·57136	375,054	963	·00271	·57409	10
11	371,788	851	·00233	·57030	374,091	909	·00247	·57298	11
12	370,937	861	·00228	·56930	373,182	915	·00242	·57192	12
13	370,076	921	·00239	·56829	372,267	971	·00251	·57085	13
14	369,155	1,019	·00261	·56721	371,296	1,059	·00272	·56972	14
15	368,136	1,141	·00293	·56601	370,237	1,158	·00299	·56848	15
16	366,995	1,267	·00328	·56466	369,079	1,266	·00329	·56712	16
17	365,728	1,386	·00363	·56316	367,813	1,354	·00357	·56563	17
18	364,342	1,483	·00394	·56151	366,459	1,414	·00379	·56403	18
19	362,859	1,574	·00421	·55974	365,045	1,460	·00394	·56235	19
20	361,285	1,662	·00448	·55785	363,585	1,502	·00407	·56061	20
21	359,623	1,744	·00474	·55585	362,083	1,550	·00421	·55881	21
22	357,879	1,815	·00498	·55374	360,533	1,604	·00437	·55695	22
23	356,064	1,869	·00518	·55153	358,929	1,676	·00456	·55501	23
24	354,195	1,913	·00534	·54924	357,253	1,762	·00482	·55298	24
25	352,282	1,948	·00548	·54689	355,491	1,845	·00507	·55083	25
26	350,334	1,993	·00562	·54448	353,646	1,927	·00534	·54857	26
27	348,341	2,059	·00581	·54200	351,719	1,998	·00559	·54620	27
28	346,282	2,144	·00606	·53943	349,721	2,046	·00579	·54372	28
29	344,138	2,237	·00636	·53673	347,675	2,079	·00594	·54117	29
30	341,901	2,341	·00669	·53390	345,596	2,112	·00606	·53857	30
31	339,560	2,456	·00706	·53092	343,484	2,153	·00620	·53591	31
32	337,104	2,572	·00747	·52776	341,331	2,222	·00639	·53318	32
33	334,532	2,696	·00797	·52444	339,109	2,320	·00668	·53034	33
34	331,836	2,831	·00832	·52092	336,789	2,441	·00706	·52736	34
35	329,005	2,971	·00882	·51720	334,348	2,572	·00749	·52420	35
36	326,034	3,113	·00933	·51326	331,776	2,704	·00795	·52085	36
37	322,921	3,255	·00986	·50910	329,072	2,826	·00841	·51729	37
38	319,666	3,398	·01041	·50470	326,246	2,943	·00884	·51354	38
39	316,268	3,542	·01097	·50005	323,303	3,055	·00928	·50961	39
40	312,726	3,691	·01157	·49516	320,248	3,168	·00972	·50549	40
41	309,035	3,832	·01218	·49001	317,080	3,269	·01016	·50117	41
42	305,203	3,970	·01279	·48459	313,811	3,364	·01058	·49667	42
43	301,233	4,097	·01340	·47890	310,447	3,433	·01096	·49199	43
44	297,136	4,207	·01398	·47296	307,014	3,485	·01127	·48716	44
45	292,929	4,321	·01455	·46676	303,529	3,533	·01156	·48220	45
46	288,608	4,448	·01518	·46031	299,996	3,606	·01188	·47712	46
47	284,160	4,603	·01591	·45356	296,390	3,717	·01233	·47186	47
48	279,557	4,792	·01679	·44647	292,673	3,875	·01295	·46638	48
49	274,765	5,000	·01781	·43896	288,798	4,063	·01373	·46059	49

TABLE II—*continued.*

English Life Table No. 6.

Age	Males				Females				Age
x	l_x	d_x	μ_x	$\log l_x$	l_x	d_x	μ_x	$\log l_x$	x
50	269,765	5,223	·01894	5·43098	284,735	4,271	·01463	5·45444	50
51	264,542	5.442	·02016	·42249	280.464	4,487	·01561	·44788	51
52	259,100	5,653	·02142	·41347	275,977	4,700	·01665	·44087	52
53	253,447	5,842	·02269	·40389	271,277	4.894	·01769	·43341	53
54	247,605	6,010	·02394	·39376	266,383	5,075	·01871	·42551	54
55	241,595	6,180	·02522	·38309	261,308	5.263	·01977	·41715	55
56	235,415	6,370	·02664	·37183	256,045	5.479	·02095	.40832	56
57	229,045	6,596	·02827	·35992	250,566	5.743	·02236	·39892	57
58	222,449	6,870	·03024	·34723	244,823	6,062	·02408	·38885	58
59	215,579	7,165	·03254	·33361	238,761	6.413	·02611	·37796	59
60	208,414	7,464	·03510	·31893	232,348	6,780	·02839	·36614	60
61	200,950	7,742	·03786	·30309	225,568	7.137	·03087	·35328	61
62	193.208	7,980	·04073	·28602	218,431	7.461	·03345	·33931	62
63	185,228	8,154	·04360	·26771	210,970	7.730	·03604	·32422	63
64	177,074	8,274	·04642	·24816	203.240	7,957	·03862	.30801	64
65	168.800	8,378	·04933	·22737	195.283	8,167	·04129	·29066	65
66	160.422	8,485	·05255	·20526	187,116	8,389	·04422	·27211	66
67	151,937	8,619	·05625	·18166	178,727	8,644	·04762	·25219	67
68	143,318	8,791	·06072	·15630	170,083	8,949	·05169	·23066	68
69	134,527	8,966	·06602	·12881	161,134	9,263	·05653	·20719	69
70	125,561	9,101	·07202	·09885	151,871	9,542	·06198	·18147	70
71	116,460	9,154	·07851	·06618	142,329	9,741	·06786	·15329	71
72	107,306	9,095	·08522	·03063	132.588	9,822	·07395	·12250	72
73	98,211	8,903	·09183	4 99216	122,766	9,753	·07991	·08908	73
74	89,308	8,609	·09819	·95089	113,013	9,569	·08561	·05313	74
75	80,699	8,272	·10465	·90687	103,444	9,331	·09140	·01471	75
76	72,427	7,928	·11182	·85990	94,113	9,090	·09784	4·97365	76
77	64,499	7,610	·12039	·80955	85,023	8,874	·10558	·92954	77
78	56,889	7,313	·13115	·75503	76,149	8,699	·11536	·88167	78
79	49,576	6,998	·14449	·69527	67,450	8.505	·12768	·82898	79
80	42,578	6,609	·16012	·62919	58,945	8,213	·14217	·77045	80
81	35,969	6,120	·17743	·55593	50,732	7,769	·15803	·70529	81
82	29,849	5,525	·19556	·47493	42,963	7,163	·17432	·63309	82
83	24,324	4,861	·21376	·38604	35,800	6,432	·19028	·55388	83
84	19,463	4,187	·23244	·28920	29,368	5,654	·20591	·46788	84
85	15,276	3,525	·25213	·18402	23,714	4,874	·22188	·37501	85
86	11,751	2,903	·27283	·07007	18,840	4,125	·23847	·27508	86
87	8,848	2,337	·29502	3·94682	14.715	3,427	·25597	·16777	87
88	6,511	1,833	·31852	·81365	11,288	2.790	·27444	·05260	88
89	4,678	1.401	·34309	·67004	8,498	2,221	·29337	3·92931	89
90	3,277	1,042	·36902	·51553	6,277	1,734	·31286	·79776	90
91	2,235	755	·39705	·34932	4,543	1,329	·33417	·65738	91
92	1,480	533	·42880	·17014	3,214	1,001	·35883	·50707	92
93	947	364	·46497	2·97621	2,213	736	·38803	·34504	93
94	583	239	·50563	·76558	1,477	526	·42142	·16940	94
95	344	151	·55147	·53624	951	363	·45967	2.97831	95
96	193	91	·60417	·28552	588	242	·50450	·76918	96
97	102	51	·66731	·00971	346	155	·55951	·53846	97
98	51	28	·74353	1·70376	191	93	·62749	·28116	98
99	23	14	·83665	·36122	98	52	·71241	1·99083	99
100	9	6	·95250	0·97357	46	27	·82027	·65890	100
101	3	2	1·10044	·52907	19	12	·96066	·27350	101
102	1	1	1·29702	·01056	7	5	1·15034	0·81723	102
103	...	...	1·57546	$\bar{1}$·39063	2	2	1·42279	·26232	103
104	...	...	2·01895	$\bar{2}$·61880	...	...	1·86148	$\bar{1}$·55789	104
105	...	...	2·96080	$\bar{3}$·57086	...	...	2·79997	$\bar{2}$·57912	105

TABLE III.

English Life Table No. 6 (Males). 3 *per-cent.*

x	D_x	H_x	M_x	R_x	$\log D_x$	a_x	x
0	508,770	10,518,460	202,409·7	3,860,269·3	5·70652	19·675	0
1	409,060	10,009,690	117,518·7	3,657,859·6	·61179	23·470	1
2	376,030	9,600,630	96,393·7	3,540,340·9	·57522	24·532	2
3	357,530	9,224,600	88,851·1	3,443,947·2	·55331	24·801	3
4	342,480	8,867,070	84,216·7	3,355,096·1	·53463	24·891	4
5	329,280	8,524,590	80,991·4	3,270,879·4	·51756	24·889	5
6	317,310	8,195,310	78,607·0	3,189,888·0	·50148	24·828	6
7	306,300	7,878,000	76,841·8	3,111,281·0	·48614	24·720	7
8	296,060	7,571,700	75,529·8	3,034,439·2	·47138	24·575	8
9	286,450	7,275,640	74,538·0	2,958,909·4	·45705	24·399	9
10	277,330	6,989,190	73,753·7	2,884,371·4	·44299	24·202	10
11	268,590	6,711,860	73,097·0	2,810,617·7	·42909	23·989	11
12	260,170	6,443,270	72,500·1	2,737,520·7	·41525	23·766	12
13	252,010	6,183,100	71,913·8	2,665,020 6	·40141	23·536	13
14	244,060	5,931,090	71,304·9	2,593,106·8	·38749	23·302	14
15	236,290	5 687,030	70,650 8	2,521,801·9	·37345	23·067	15
16	228,700	5,450,740	69,939·8	2,451,151·1	·35926	22·834	16
17	221,270	5,222,040	69,173·2	2,381,211·3	·34493	22·600	17
18	214,010	5,000,770	68,359·1	2,312,038·1	·33044	22·367	18
19	206,930	4,786,760	67,513·4	2,243,679·0	·31583	22·132	19
20	200,040	4,579,830	66,641·9	2,176,165·6	·30111	21·895	20
21	193,320	4,379,790	65,748·5	2,109,523·7	·28627	21·656	21
22	186,780	4,186,470	64,838·3	2,043,775·2	·27132	21·415	22
23	180,410	3,999,690	63,918·7	1,978,936·9	·25627	21·169	23
24	174,240	3,819,280	62,999·2	1,915,018·2	·24115	20·919	24
25	168,250	3,645,040	62,085·5	1,852,019·0	·22596	20·664	25
26	162,450	3,476,790	61,182·2	1,789,933·5	·21071	20·402	26
27	156,820	3,311,340	60,285·0	1,728,751·3	·19540	20·134	27
28	151,350	3,157,520	59,385·0	1,668,466·3	·17999	19·862	28
29	146,030	3,006,170	58,475·2	1,609,081·3	·16445	19·585	29
30	140,860	2,860,140	57,553·6	1,550,606·1	·14878	19·305	30
31	135,820	2,719,280	56,617·2	1,493,052·5	·13297	19·021	31
32	130,910	2,583,460	55,663·4	1,436,435·3	·11697	18·735	32
33	126,130	2,452,550	54,693·7	1,380,771·9	·10081	18·445	33
34	121,460	2,326,420	53,706·8	1,326,078·2	·08145	18·153	34
35	116,920	2,204,960	52,700·7	1,272,371·4	·06790	17·858	35
36	112,490	2,088,040	51,675·6	1,219,670·7	·05112	17·562	36
37	108,170	1,975,550	50,632·8	1,167,995·1	·03412	17·263	37
38	103,970	1,867,380	49,574·2	1,117,362·3	·01689	16·961	38
39	99,862	1,763,410	48,501·3	1,067,788·1	4·99940	16·658	39
40	95,867	1,663,548	47,415·5	1,019,286·8	·98167	16·353	40
41	91,977	1,567,681	46,317·0	971,871·3	·96368	16·044	41
42	88,192	1,475,704	45,209·7	925,554·3	·94543	15·733	42
43	84,508	1,387,512	44,095·9	880,344·6	·92690	15·418	43
44	80,932	1,303,004	42,980·0	836,248·7	·90812	15·100	44
45	77,460	1,222,072	41,867·5	793,268·7	·88908	14·776	45
46	74,097	1,144,612	40,758·1	751,401·2	·86980	14·447	46
47	70,829	1,070,515	39,649·4	710,643·1	·85021	14·114	47
48	67,652	999,686	38,535·5	670,993·7	·83028	13·777	48
49	64,557	932,034	37,409·6	632,458·2	·80994	13·438	49

Table III—*continued.*

English Life Table No. 6 (Males). 3 per-cent.

x	D_x	N_x	M_x	R_x	$\log D_x$	a_x	x
50	61,535	867,477	36,269·1	595,048·6	4·78912	13·097	50
51	58,585	805,942	35,112·4	558,779·5	·76779	12·757	51
52	55,710	747,357	33,942·3	523,667·1	·74593	12·415	52
53	52,908	691,647	32,762·2	489,724·8	·72352	12·073	53
54	50,182	638,739	31,578·2	456,962·6	·70055	11·728	54
55	47,538	588,557	30,395·6	425,384·4	·67704	11·381	55
56	44,973	541,019	29,215·0	394,988·8	·65295	11·030	56
57	42,482	496,046	28,033·5	365,773·8	·62820	10·677	57
58	40,056	453,564	26,845·8	337,740·3	·60267	10·323	58
59	37,689	413,508	25,644·8	310,894·5	·57621	9·972	59
60	35,375	375,819	24,428·6	285,249·7	·54870	9·624	60
61	33,115	340,444	23,198·6	260,821·1	·52002	9·281	61
62	30,911	307,329	21,960·0	237,622·5	·49011	8·943	62
63	28,771	276,418	20,720·5	215,662·5	·45896	8·608	63
64	26,704	247,647	19,490·8	194,942·0	·42655	8·274	64
65	24,714	220,943	18,279·4	175,451·2	·39295	7·940	65
66	22,803	196,229	17,088·5	157,171·8	·35800	7·605	66
67	20,969	173,426	15,917·5	140,083·3	·32157	7·271	67
68	19,203	152,457	14,762·6	124,165·8	·28337	6·939	68
69	17,560	133,254	13,619·0	109,403·2	·24304	6·614	69
70	15,858	115,754	12,486·6	95,784·2	·20024	6·300	70
71	14,280	99,896	11,370·6	83,297·6	·15474	5·995	71
72	12,775	85,616	10,280·8	71,927·0	·10635	5·702	72
73	11,351	72,841	9,229·64	61,646·20	·05504	5·417	73
74	10,022	61,490	8,230·58	52,416·56	·00094	5·136	74
75	8,791·8	51,468·0	7,292·67	44,185·98	3·94408	4·854	75
76	7,660·7	42,676·2	6,417·73	36,893·31	·88427	4·571	76
77	6,623·4	35,015·5	5,603·61	30,475·58	·82108	4·287	77
78	5,671·9	28,392·1	4,844·89	24,871·97	·75373	4·006	78
79	4,798·8	22,720·2	4,137·01	20,027·08	·68113	3·735	79
80	4,001·4	17,921·4	3,479·37	15,890·07	·60221	3·479	80
81	3,281·8	13,920·0	2,876·37	12,410·70	·51611	3·242	81
82	2,614·1	10,638·2	2,334·24	9,534·33	·42228	3·023	82
83	2,091·9	7,994·1	1,859·08	7,200·09	·32055	2·821	83
84	1,625·1	5,902·2	1,453·20	5,341·01	·21087	2·632	84
85	1,238·4	4,277·1	1,113·78	3,887·81	·09286	2·454	85
86	924·85	3,038·70	836·35	2,774·03	2·96607	2·286	86
87	676·05	2,113·85	614·53	1,937·68	·82998	2·127	87
88	483·03	1,437·80	441·16	1,323·15	·68397	1·977	88
89	336·92	954·77	309·14	881·990	·52753	1·834	89
90	229·18	617·85	211·171	572·850	·36018	1·696	90
91	151·75	388·67	140·429	361·679	·18113	1·561	91
92	97·526	236·920	90·6240	221·250	1·98912	1·429	92
93	60·583	139·394	56·5210	130·626	·78235	1·301	93
94	36·214	78·811	33·9180	74·105	·55888	1·176	94
95	20·735	42·597	19·4960	40·1874	·31670	1·054	95
96	11·302	21·862	10·6646	20·6914	·05315	·934	96
97	5·8143	10·5606	5·5065	10·0268	0·76450	·816	97
98	2·7907	4·74636	2·6521	4·5203	·44571	·701	98
99	1·2312	1·955668	1·17408	1·86829	·09033	·588	99
100	·48961	·724468	·46851	·69421	$\bar{1}$·68985	·480	100
101	·17081	·234858	·16399	·22570	·23251	·375	101
102	·050253	·064048	·04839	·06171	$\bar{2}$·70116	·274	102
103	·011706	·013795	·01130	·01332	·06840	·178	103
104	·001922	·002089	·00186	·00202	$\bar{3}$·28373	·087	104
105	·000167	·000167	·00016	·00016	$\bar{4}$·22295	...	105

TABLE IV.

English Life Table No. 6 (Males). 3 per-cent.

x	$\bar{a}_x$	$\bar{A}_x$	$\bar{P}_x$	x	$\bar{a}_x$	$\bar{A}_x$	$\bar{P}_x$
0	20·145	·40454	·02008	50	13·592	·59824	·04401
1	23·960	·29177	·01218	51	13·253	·60826	·04589
2	25·027	·26023	·01039	52	12·910	·61840	·04790
3	25·298	·25222	·00997	53	12·569	·62847	·05001
4	25·388	·24956	·00983	54	12·223	·63870	·05225
5	25·386	·24962	·00983	55	11·877	·64893	·05464
6	25·326	·25139	·00993	56	11·525	·65933	·05721
7	25·218	·25458	·01009	57	11·173	·66974	·05995
8	25·072	·25890	·01033	58	10·817	·68026	·06289
9	24·897	·26407	·01061	59	10·467	·69060	·06598
10	24·699	·26992	·01093	60	10·118	·70092	·06927
11	24·487	·27619	·01128	61	9·776	·71103	·07273
12	24·263	·28281	·01165	62	9·437	·72105	·07641
13	24·034	·28958	·01205	63	9·102	·73095	·08031
14	23·799	·29653	·01246	64	8·767	·74085	·08451
15	23·565	·30344	·01288	65	8·433	·75073	·08903
16	23·331	·31036	·01330	66	8·099	·76060	·09391
17	23·098	·31725	·01373	67	7·763	·77053	·09926
18	22·864	·32417	·01418	68	7·432	·78032	·10499
19	22·630	·33108	·01463	69	7·105	·78998	·11119
20	22·392	·33812	·01510	70	6·792	·79924	·11768
21	22·154	·34515	·01558	71	6·485	·80831	·12464
22	21·912	.35231	·01608	72	6·193	·81694	·13191
23	21·667	·35955	·01660	73	5·906	·82542	·13976
24	21·416	·36697	·01714	74	5·626	·83370	·14820
25	21·162	·37448	·01770	75	5·342	·84210	·15764
26	20·899	·38225	·01829	76	5·060	·85043	·16807
27	20·632	·39014	·01891	77	4·774	·85888	·17990
28	20·358	·39824	·01956	78	4·493	·86719	·19301
29	20·082	·40640	·02024	79	4·220	·87526	·20741
30	19 801	·41471	·02095	80	3·964	·88283	·22272
31	19·518	·42307	·02167	81	3·724	·88992	·23898
32	19·231	·43155	·02244	82	3·505	·89639	·25576
33	18·942	·44010	·02324	83	3·300	·90246	·27347
34	18·649	·44875	·02406	84	3·111	·90804	·29188
35	18·355	·45745	·02492	85	2·930	·91339	·31174
36	18·058	·46622	·02582	86	2·761	·91839	·33263
37	17·760	·47504	·02675	87	2·599	·92317	·35521
38	17·457	·48399	·02772	88	2·448	·92764	·37894
39	17·155	·49292	·02873	89	2·302	·93195	·40484
40	16·849	·50196	·02979	90	2·163	·93606	·43275
41	16·541	·51107	·03090	91	2·025	·94014	·46426
42	16·229	·52029	·03206	92	1·891	·94410	·49925
43	15·915	·52957	·03327	93	1·759	·94800	·53893
44	15·596	·53900	·03456	94	1·631	·95179	·58357
45	15·273	·54855	·03591	95	1·504	·95554	·63533
46	14·943	·55830	·03737	96	1·382	·95915	·69403
47	14·611	·56812	·03889	97	1·258	·96281	·76536
48	14·273	·57811	·04050	98	1·137	·96639	·84995
49	13·935	·58810	·04221	99	1·015	·97000	·95565

Table V.

English Life Table No. 6 (Females.) 3 per-cent.

x	D_x	H_x	a_x	x	D_x	H_x	a_x
0	491,220	10,704,314	20·791	53	56,629	792,000	12·986
1	409,840	10,213,094	23·920	54	53,988	735,371	12·621
2	378,220	9,803,254	24·920	55	51,416	681,383	12·252
3	359,730	9,425,034	25·200	56	48,915	629,967	11·879
4	344,660	9,065,304	25·302	57	46,473	581,052	11·503
5	331,420	8,720,644	25·313	58	44,085	534,579	11·126
6	319,400	8,389,224	25·266	59	41,741	490,494	10·751
7	308,310	8,069,824	25·174	60	39,438	448,753	10·379
8	298,000	7,761,514	25·046	61	37,171	409,315	10·012
9	288,300	7,463,514	24·888	62	34,946	372,144	9·649
10	279,070	7,175,214	24·710	64	32,770	337,198	9·290
11	270,250	6,896,144	24·517	64	30,650	304,428	8·932
12	261,740	6,625,894	24·315	65	28,592	273,778	8·576
13	253,500	6,364,154	24·106	66	26,598	245,186	8·218
14	245,470	6,110,654	23·894	67	24,666	218,588	7·862
15	237,640	5,865,184	23·681	68	22,789	193,922	7·509
16	230,000	5,627,544	23·468	69	20,961	171,133	7·164
17	222,540	5,397,544	23·255	70	19,181	150,172	6·829
18	215,260	5,175,004	23·041	71	17,452	130,991	6·506
19	208,180	4,959,744	22·824	72	15,784	113,539	6·193
20	201,310	4,751,564	22·603	73	14,189	97,755·4	5·889
21	194,640	4,550,254	22·378	74	12,682	83,566·4	5·590
22	188,160	4,355,614	22·149	75	11,270	70,884·4	5·290
23	181,870	4,167,454	21·915	76	9,954·5	59,614·4	4·989
24	175,750	3,985,584	21·678	77	8,731·1	49,659·9	4·688
25	169,790	3,809,834	21·439	78	7,592·2	40,928·8	4·391
26	163,980	3,640,044	21·198	79	6,528·9	33,336·6	4·106
27	158,340	3,476,064	20·953	80	5,539·5	26,807·7	3·839
28	152,860	3,317,724	20·705	81	4 628·8	21,268·2	3·595
29	147,530	3,164,864	20·452	82	3,805·7	16,639·4	3·372
30	142,380	3,017,334	20·193	83	3,078·9	12,833·7	3·168
31	137,390	2,874,954	19·926	84	2,452·2	9,754·89	2·978
32	132,550	2,737,564	19·653	85	1,922·4	7,302·69	2·799
33	127,850	2,605,014	19·375	86	1,482·8	5,380·29	2·629
34	123,280	2,477,164	19·094	87	1,124·4	3,897·49	2·466
35	118,820	2,353,884	18·810	88	837·38	2,773·09	2·312
36	114,470	2,235,064	18·525	89	612·07	1,935·71	2·163
37	110,230	2,120,594	18·238	90	438·94	1,323·64	2·016
38	106,100	2,010,364	17·948	91	308·45	884·708	1·868
39	102,080	1,904,264	17·654	92	211·86	576·258	1·720
40	98,175	1,802,184	17·357	93	141·64	364·398	1·573
41	94,371	1,704,009	17·056	94	91·770	222·758	1·427
42	90,680	1,609,638	16·751	95	57·381	130·988	1·283
43	87,094	1,518,958	16·441	96	34·420	73·6076	1·139
44	83,622	1,431,864	16·122	97	19·645	39·1876	·995
45	80,264	1,348,242	15·798	98	10·547	19·5426	·853
46	77,021	1,267,978	15·463	99	5·2474	8·99560	·714
47	73,877	1,190,957	15·121	100	2·3724	3·74820	·580
48	70,826	1,117,080	14·773	101	·94829	1·37580	·451
49	67,853	1,046,254	14·419	102	·32198	·427511	·328
50	64,950	978,401	14·064	103	·087114	·105531	·211
51	62,113	913,451	13·706	104	·016704	·018417	·103
52	59,338	851,338	13·348	105	·001713	·001713	...

TABLE VI.

Comparative Values of a_x. 3 per-cent.

Age	Males: English Life Table No. 6	Males: English Life Table No. 3	$\frac{(2)}{(3)}$	Females: English Life Table No. 6	Females: English Life Table No. 3	$\frac{(5)}{(6)}$
(1)	(2)	(3)	(4)	(5)	(6)	(7)
0	19·675	18·151	1·084	20·791	18·850	1·103
15	23·067	22·105	1·044	23·681	22·193	1·067
30	19·305	19·014	1·015	20·193	19·337	1·044
45	14·776	15·010	·984	15·798	15·658	1·009
60	9·624	10·018	·961	10·379	10·527	·986
75	4·854	5·145	·944	5·290	5·483	·965
90	1·696	2·179	·779	2·016	2·328	·866

SIXTH INTERNATIONAL CONGRESS OF ACTUARIES.

THE Sixth International Congress was held most successfully in Vienna, from June 7th to 12th, and was largely attended by delegates and members from twenty-six countries. The following Governments nominated official delegates: Austria, Hungary, Australia, Belgium, France, Germany, Greece, Holland, Italy, New Zealand, Norway, Servia, Sweden, Switzerland, United States of America, and United States of Mexico. Official delegates also attended on behalf of Insurance Institutions in Belgium, Denmark, England, France, Germany, Holland, Italy, Norway, Russia, Scotland, Sweden, Switzerland, and the United States of America and Canada: those attending officially on behalf of the Institute of Actuaries being Messrs. G. F. Hardy (President) and W. P. Phelps (Joint Honcrary Secretary).

The transactions of the Congress are contained in three bulky volumes, including the Reports and Memoirs submitted on the several subjects comprised in the programme (see *J.I.A.*, vol. xlii, pp. 221, 222), and a fourth volume, to be published later, will include an abstract of the discussions.

The following Officers of the Congress were duly elected at the inaugural meeting on Monday morning, June 7th:

President:
HOFRATH E. CZUBER.

Vice-President:
DR. JAS. KLANG.

General Secretaries:
DR. J. GRAF, C. NOSKE, AND R. S. B. SAVERY.

	Vice-President:	*Secretary:*
BELGIUM	A. BÉGAULT.	F. HANKAR.
DENMARK	DR. J. P. GRAM.	DR. C. BURRAU.
ENGLAND	G. F. HARDY.	W. P. PHELPS.
FRANCE	L. MARIE.	A. QUIQUET,
GERMANY ...	DR. K. VON RASP.	DR. D. BISCHOFF.
HOLLAND ...	DR. M. C. PARAIRA.	DR. D. P. MOLL.
HUNGARY	DIR. W. VON ORMODY.	J. ALTENBURGER.
ITALY ...	COM. MARCO BESSO.	COM. G. TOJA.
NORWAY ...	DR. A. S. GULDBERG.	DIR. H. ALMÉ.
RUSSIA	PROF. S. DE SAVITCH.	DR. SCHETALOW.
SCOTLAND	J. J. MCLAUCHLAN.	G. C. STENHOUSE.
SWEDEN	S. PALMÉ.	DR. F. LUNDBERG.
SWITZERLAND	DR. F. TREFZER.	DR. J. RIEM.
UNITED STATES & CANADA	A. HUNTER.	W. C. MACDONALD.

The meetings of the Congress were continued on the mornings of Tuesday, June 8th, to Saturday, June 12th, both inclusive, when the several subjects included in the official Programme, and the Papers contributed thereon, were fully discussed, the British members taking a prominent part in the discussions.

The Social engagements of the Congress included (i) Reception at the Chamber of Commerce and Industry of Lower Austria, on Sunday evening, June 6th: (ii) Reception and Banquet in the New Town Hall, by the Municipal Corporation, preceded by an inspection of the City collections, on Monday evening, June 7th; (iii) Gala performance at the Royal Opera House, on Tuesday evening, June 8th: (iv) Excursion to Schönbrunn (Gloriette), followed by concert and afternoon tea, on Wednesday, June 9th; (v) Promenade Concert in the Stadtpark, and evening visit to the Prater, on Thursday, June 10th: (vi) Congress Banquet in the "Sofiensaal", on Friday evening, June 11th; (vii) Excursion to the Semmering Pass, on Sunday, June 13th.

At the close of the official proceedings on Saturday, June 12th, cordial votes of thanks were unanimously accorded to the Imperial and Municipal authorities, and to the Organizing Committee and Officers of the Congress, for the hearty welcome given to the delegates and members, and the arrangements made for the conduct of the meetings, and for the social enjoyment of those attending the Congress; and Mr. G. F. Hardy gave expression to the hearty appreciation of the members and friends from Great Britain in these respects.

The invitation of the representatives from Holland, to hold the Congress in 1912 at Amsterdam, was cordially and unanimously accepted.

OBITUARY NOTICE.

Mr. Frederick Hendriks, F.I.A.

We regret to announce the death of Mr. Frederick Hendriks, which took place on 8 May 1909, at the advanced age of eighty-two years.

Mr. Hendriks was born in 1827, and commenced his insurance career when appointed, in his 20th year, as actuary of the Globe Life Assurance Company, an office which he held until the transfer of the business of that office to the Liverpool and London Assurance Company in 1864. He then took up the appointment of actuary to the Universal Life Assurance Society, of which he was subsequently appointed a director, retaining this position until the transfer of its business to the North British and Mercantile Insurance Company in 1901. He was also connected, as consulting actuary, with several assurance institutions, and a large number of superannuation funds and benefit and provident societies, and retained several of these offices until shortly before his death.

His contributions to the *Journal* of the Institute of Actuaries were mainly spread over the years 1848–1862, the earliest, a "Memoir of the early history of Auxiliary Tables for the computation of Life Contingencies", written in his 21st year, opening the first number of the first volume. Later contributions, dealing mainly with the historical side of life assurance, appeared in the first ten volumes of the *Journal*. Mr. Hendriks took part in an early movement of secession from the Institute, and was for some years out of sympathy with its proceedings; but, on the occasion of the grant of the Royal Charter in 1884, these old differences were happily adjusted, and Mr. Hendriks was, with others, appointed an Honorary Fellow of the Institute. His last contribution in the *Journal*, a review of a volume published in Holland on the History of Life Assurance in the Netherlands, appeared in 1899, in vol. xxxiv.

Mr. Hendriks had, at the date of his death, held for many years a seat on the Council of the Royal Statistical Society, of which he was Vice-President on several occasions between 1875 and 1906. He was also a contributor to the *Journal* of the Society, and to other financial and economic periodicals.

He was a man of large accomplishments, in European languages, literature, and art, of exceptionally quick intelligence, and of sound actuarial knowledge, and was greatly respected and honoured by all who had the privilege of his friendship.

JOURNAL

OF THE

INSTITUTE OF ACTUARIES.

Notes on Mortality and Life Assurance in India.

By A. T. WINTER, F.I.A., *Assistant Actuary of the Phoenix Assurance Company, Limited.*

[Read before the Institute, 26 April 1909.]

Introduction. AT the Congress of Actuaries in Berlin in 1906 I contributed a paper on Life Assurance in India. With certain modifications and additions, a considerable part of that paper is reproduced in that which I am now reading. My excuse for going over much of the same ground again is that the subject has not been before the Institute for many years past, and that the discussion here will no doubt elicit further information which will be useful to the profession.

At the present time India is one of the largest and most rapidly developing fields for insurance business in which the rates of mortality differ materially from those in temperate climates. Not only are there a greater number of Europeans from year to year engaging in the civil and military administration of the country and its mercantile pursuits who require the benefits of assurance, but the more Europeanized Natives themselves are also seeking the same protection.

As evidence of the extent of the interests of British insurance companies in India, I find that in the case of those offices which

submit to the Board of Trade separate returns of their Indian business the sums assured in force amount to about £7,000,000. But this, of course, does not nearly represent the total amount covered by policies on lives in India, as many English companies do not make separate returns of such business, and a considerable and increasing amount is being written by American, Canadian and local companies.

No legislative restrictions.

India, as a field for life assurance, is free from legislative enactments such as those which restrict the operations of life offices in some of our colonies. Nevertheless, if such extremes be avoided, some Government supervision there would probably be advantageous. Under existing conditions there is no proper safeguard against native companies starting operations on unstable lines and without sufficient capital. The evils likely to result from such a state of affairs are, I understand, being urged upon the Indian Government, and it is hoped legislation similar to our 1870 Act may result.

The notes I am reading to-night are classed under five headings, namely, (1) Indian Mortality; (2) Native Life Assurance; (3) Premiums; (4) Policy-values; (5) Conclusion.

Indian Mortality.

Under this heading I am including—

I. The experience of the "British Empire" Company (1872-1902).

II. A comparison of the "British Empire" Experience with that of the "Standard" (1870-1900) and "Oriental" (1874-1891).

III. The experience of the Indian Postal Insurance Scheme (1884-1904).

IV. The mortality of British military officers (1875-1905).

I.—"*British Empire*" *Experience.*

In 1904 I had occasion to make an investigation into the mortality experienced by lives assured in India in the British

Empire Mutual Life Assurance Company and the Positive Government Security Life Assurance Company (up to the date the latter office was transferred to the former, namely, 1895). The experience (to which, for brevity, I shall refer as the "British Empire" experience) extends over the years 1872 to 1902 and comprises the following data :

	Years of Life	Deaths
Europeans	19,867	389
Eurasians. . . .	4,961	94
Asiatics	12,594	192
Total	37,422	675

A table of the graduated rates of mortality and also of D_x and N_x at 3 per-cent interest are given at the end of paper.

Construction of "Combined Races" Table.

As it was considered desirable to observe the effect of selection, the policy year method was adopted in tabulating the facts. The rates of mortality were graduated by the graphic method up to age 65, to which age the data were considered sufficiently large to give fairly reliable results. After age 65 an assumption had to be made as to the rates of mortality which would hold, as the facts were too meagre to give proper averages, and, guided by the rates for ages immediately preceding age 65, the most reasonable basis on which to proceed appeared to be to adopt rates 50 per-cent higher than the H^M (text-book graduation); and this plan was accordingly resorted to. For similar reasons, a corresponding method was used by Messrs. Rothery and Hardy in their experience of the Barbadoes Mutual (*J.I.A.*, vol. xxvii), where the assumption made was that after age 70 the rates of mortality would be 30 per-cent higher than those of the H^M Table. During short periods of furlough to Europe, Europeans lives were kept under observation. This was found to be advisable as several deaths occurred during these periods, the inference being that the visits to Europe were often not in the nature of holidays, but were due to ill-health.

The expected deaths by the graduated rates of mortality and the actual deaths are compared in the following table "A":

TABLE A.

"British Empire" Indian Experience (combined races).

Comparison of Actual Deaths with Expected Deaths by graduated rates of mortality.

Ages (x)	Exposed to Risk	Actual Deaths	Expected Deaths by graduated q_x	Accumulated Deviation
16–24	736	4	4	0
25–29	3,078	22	22	0
30–34	6,014	48	56	+8
35–39	7,260	95	82	−5
40–44	6,916	92	100	+3
45–49	5,753	120	115	−2
50–54	3,962	115	115	−2
55–59	2,247	87	91	+2
60–64	993	55	53	0
65–69	376	26	28	+2
70–74	76	5	8	+5
75–79	11	6	2	+1
All Ages	37,422	675	676	+1

Mortality according to Race.

The exposed to risk, the actual deaths and the expected deaths according to the O^{M} Table are given for quinquennial groups of ages in Table "B", separate particulars being recorded for Europeans, Eurasians and Asiatics. It will be found from the table that the following ratios exist between the actual and expected deaths, the special feature noticeable being the relatively light mortality of Asiatics up to age 45.

Ratios of Actual to Expected Deaths by O^{M} Table.

	Europeans	Eurasians	Asiatics	Total
Ages below 45 . .	1·60	1·32	1·16	1·40
Ages 45 and over .	1·55	2·15	1·67	1·65
Totals, all ages . .	1·57	1·74	1·43	1·54

TABLE B.

"British Empire" Experience, India (1872–1902).

Actual Deaths and Expected Deaths.

Ages	EUROPEANS				EURASIANS			ASIATICS				TOTALS			
	Exposed to Risk	Actual Deaths	Expected Deaths by		Exposed to Risk	Actual Deaths	Expected Deaths by $O^M q_x$	Exposed to Risk	Actual Deaths	Expected Deaths by		Exposed to Risk	Actual Deaths	Expected Deaths by	
			O^M $q_{x+5}+{\cdot}003$	$O^M q_x$						O^M q_{x+6}	$O^M q_x$			O^M q_{x+7}	$O^M q_x$
16–19	18	0	·1	·0	5	0	·0	19	1	·1	1	42	1	·1	·1
20–24	310	1	2·5	1·3	80	0	·3	304	2	1·6	1·3	694	3	3·9	2·9
25–29	1,370	15	13·0	7·2	476	1	2·5	1,232	6	8·4	6·4	3,078	22	21·7	16·1
30–34	2,854	26	31·5	18·5	951	11	6·2	2,209	11	18·6	14·3	6,014	48	52·7	39·0
35–39	3,641	53	47·3	29·3	1,063	19	8·5	2,556	23	26·9	20·6	7,260	95	79·8	58·4
40–44	3,668	54	58·0	36·7	890	4	8·9	2,358	34	31·6	23·6	6,916	92	98·3	69·2
16–44	11,861	149	152·4	93·0	3,465	35	26·1	8,678	77	87·2	66·3	21,004	261	256·5	185·7
45–49	3,136	58	62·5	40·2	672	15	8·6	1,915	47	34·9	24·9	5,753	120	110·4	73·7
50–54	2,412	61	63·6	40·7	411	15	6·9	1,139	39	28·5	19·3	3,962	115	106·5	66·9
55–59	1,461	60	53·4	34·1	256	15	6·0	530	12	19·1	12·4	2,247	87	87·3	52·5
60–64	686	41	35·7	22·9	111	7	3·7	196	7	10·4	6·5	993	55	57·0	33·1
65–69	252	13	19·1	12·3	46	7	2·3	78	6	6·2	3·8	376	26	32·1	18·4
70–74	53	4	5·9	3·9	...	...	...	23	1	2·7	1·7	76	5	9·7	5·6
75–79	6	3	1·0	·7	...	...	...	5	3	1·0	·6	11	6	2·1	1·3
45–79	8,006	240	240·9	154·8	1,496	59	27·5	3,916	115	102·8	69·2	13,418	414	405·1	251·5
Totals	19,867	389	393·3	247·8	4,961	94	53·9	12,594	192	190·0	135·5	37,422	675	661·6	437·2

Adaptation of O^M Table to represent mortality of races.

As the separate experiences of Europeans and Natives were too small to graduate, an attempt was made to find some adaptation of the O^M Table which would approximately represent the respective rates of mortality. On examination, it appeared that, taking the Native rate of mortality as $O^M q_{x+6}$ and the European rate $O^M q_{x+5} + \cdot 003$, the deviation of the actual from the expected deaths was small. The expected deaths on these assumed rates of mortality are included in Table B. It was also noticed that the mortality of the combined races, including Eurasians, approximated to $O^M q_{x+7}$, and the expected deaths on that assumption are also given.

The following figures show how the above assumptions accord with the facts :

Age	Europeans		Natives		Combined Races including Eurasians	
	Expected Deaths	Actual Deaths	Expected Deaths	Actual Deaths	Expected Deaths	Actual Deaths
16–29	15·6	16	10·1	9	25·7	26
30–49	199·3	191	112·0	115	341·2	355
50–69	171·5	175	64·2	64	282·9	283
70–79	6·9	7	3·7	4	11·8	11
Totals	393·3	389	190·0	192	661·6	675

At the end of the paper is given a table of rates of mortality based on $q'_x = (O^M) q_{x+5} + \cdot 003$, together with the corresponding D_x and N_x columns at 3 per-cent interest.

Select Experience.

Table "C", which follows, gives deaths, classified in races, and analyzed according to

(*a*) first and second years of assurance;

(*b*) third, fourth and fifth years of assurance;

(*c*) subsequent years;

and compares these deaths with those expected by

(1) Select $O^{[M]}$ for the first five years of assurance and $O^{M(5)}$ for subsequent years;

(2) $O^{M(5)}$ throughout.

TABLE C.

"British Empire" Indian Experience. Comparison of Actual Deaths with the Expected, by $O^{[M]}$ Select and $O^{M(5)}$ Table.

Ages	Europeans				Eurasians				Asiatics				Total			
	1st and 2nd years	3rd, 4th 5th years	1st to 5th years	After 5 years	1st and 2nd years	3rd, 4th 5th years	1st to 5th years	After 5 years	1st and 2nd years	3rd, 4th 5th years	1st to 5th years	After 5 years	1st and 2nd years	3rd, 4th 5th years	1st to 5th years	After 5 years
Ages 16–44—																
(1) Actual Deaths	35	52	87	62	6	5	11	21	25	22	47	30	66	79	145	116
Expected Deaths by—																
(2) $O^{[M]}$ (Select) & $O^{M(5)}$	15·60	23·83	39·43	45·39	4·52	6·87	11·39	12·90	13·57	19·29	32·86	25·26	33·69	49·99	83·68	83·55
(3) $O^{M(5)}$ throughout	29·32	30·41	59·73	45·39	8·49	8·77	17·26	12·90	25·75	24·61	50·36	25·26	63·56	63·79	127·35	83·55
Ratios—																
(1)/(2)	2·21	2·19	2·21	1·36	1·33	·73	·97	1·87	1·84	1·14	1·43	1·19	1·96	1·58	1·74	1·40
(1)/(3)	1·20	1·72	1·47	1·36	·71	·57	·64	1·87	·97	·89	·93	1·19	1·04	1·24	1·15	1·40
Ages 45–72—																
(1) Actual Deaths	7	35	42	195	6	4	10	12	12	32	44	67	25	71	96	304
Expected Deaths by																
(2) $O^{[M]}$ (Select) & $O^{M(5)}$	6·94	13·97	20·91	126·79	1·74	3·23	4·97	18·49	5·26	10·53	15·79	45·36	13·94	27·73	41·67	190·64
(3) $O^{M(5)}$ throughout	12·32	18·10	30·42	126·79	3·08	4·20	7·28	18·49	9·38	13·67	23·05	45·36	24·78	35·97	60·75	190·64
Ratios—																
(1)/(2)	1·02	2·50	2·01	1·54	3·45	1·24	2·02	2·28	2·28	3·05	2·80	1·48	1·80	2·57	2·30	1·59
(1)/(3)	·57	1·93	1·38	1·54	1·95	·95	1·38	2·28	1·29	2·34	1·91	1·48	1·01	1·97	1·58	1·59
All Ages 16–72																
(1) Actual Deaths	42	87	129	257	12	9	21	66	37	54	91	97	91	150	241	420
Expected Deaths by																
(2) $O^{[M]}$ (Select) & $O^{M(5)}$	22·54	37·80	60·34	172·18	6·26	10·10	16·36	31·39	18·83	29·82	48·65	70·62	47·63	77·72	125·35	274·19
(3) $O^{M(5)}$ throughout	41·64	48·51	90·15	172·18	11·57	12·97	24·54	31·39	35·13	38·28	73·41	70·62	88·34	99·76	188·10	274·19
Ratios—																
(1)/(2)	1·87	2·30	2·14	1·50	1·93	·89	1·29	2·10	1·97	1·81	1·88	1·37	1·91	1·93	1·93	1·53
(1)/(3)	1·01	1·79	1·43	1·50	1·04	·69	·86	2·10	1·06	1·41	1·24	1·37	1·03	1·50	1·29	1·53

The following observations are suggested by an examination of this table :

Europeans and Selection.

1. Amongst European lives, the mortality is nearly as high in the first five years of assurance as in subsequent years, the ratios of actual to expected deaths by the $O^{M(5)}$ Table being:

1·43 for the first 5 years of assurance ; and
1·50 after the first 5 years of assurance.

During the first few years' residence in India, Europeans are more likely to become victims to enteric fever and similar diseases than subsequently, and, as assurances are frequently effected when a man first goes out to the country, this period of acclimatization is often concurrent with the first five years of assurance. This, I think, explains to a large extent the heavy mortality of Europeans during that period.

2. When the mortality rates of Europeans are compared with $O^{M(5)}$ Table, it is seen that these rates are lowest in the first and second years, then in the third, fourth and fifth years higher than the ultimate rates. The figures are :

Ages	1st and 2nd years	3rd, 4th and 5th years	After 5 years
16–44	1·20	1·72	1·36
45–72	·57	1·93	1·54
16–72	1·01	1·79	1·50

Asiatics and Selection.

3. Up to age 45 the mortality of Asiatics is more favourable than that of Europeans, and the effect of selection is much more evident in the former race. Beyond that age up to age 72, the mortality rates in the first five years of assurance are higher for Asiatics than Europeans. The ratio of actual to expected deaths by the $O^{M(5)}$ Table is as follows :

Assurance Years	Ages 16–44	Ages 45–72
Europeans.		
First 5 years of Assurance . .	1·47	1·38
After „ „ . .	1·36	1·54
Asiatics.		
First 5 years of Assurance . .	·93	1·91
After „ „ . .	1·19	1·48

Eurasians and Selection.

4. The Eurasian experience is perhaps too small to base any definite conclusions upon, but it will be noticed that the mortality of this race is lighter in the first five years than in the case of Europeans and Asiatics, and heavier after the first five years. The mortality is also relatively much heavier after age 44 than up to that age.

5. As was, of course, anticipated, the actual deaths are very much heavier in the first five years than the expected by the $O^{[M]}$ analyzed tables, and as far as I am aware there is only one published table with which this mortality could properly be compared, namely, that given by Mr. Arthur Hunter, F.I.A., in a paper read before the Congress of Actuaries in 1903. He there gives the experience of Native lives assured in India and Ceylon by two American and one Canadian Company, with the following results, which are compared with those brought out in the above Table "C".

Native Lives. Ratio of Actual to Expected Deaths.

Assurance Years	British Empire Experience, by $O^{[M]}$ and $O^{M(5)}$	Two American and one Canadian Office. By Actuarial Society's (of America) Table
1–5	1·55	2·41
6th and thereafter	1·37	1·90

Relatively the "British Empire" mortality appears to have been favourable.

II.—*Comparison with other Mortality Experiences.*

"Standand" and "Oriental."

It will, I think, be useful to institute a comparison between the experience we have been describing and the results of similar investigations. The most important are those of the "Standard" and "Oriental" Offices. Particulars of the "Standard" mortality for the years 1870 to 1885 and 1885 to 1900 are given in Tables VII and VIII in a paper contributed by Mr. S. C. Thomson, B.A., F.I.A., F.F.A., to the Congress in 1903; and the "Oriental" experience from 1874 to 1891 was given in a paper read by Mr. James Chatham, F.I.A., F.F.A., before the Congress in 1900.

In the "Standard" investigations, the second period, 1885–1900 (Table VIII), shows more favourable rates of mortality than the first. This is no doubt due to a certain extent to a general improvement in mortality, but, without information as to the relative amount of "select" business in the two experiences, it is impossible to determine the weight that should be given to this improvement. In fact, Mr. Thomson expresses the opinion "that it would not be safe without considerable modification "to base premiums upon them, owing to the paucity of numbers "and the somewhat select character of the lives forming the subject "of the observations." As the period covered by the "Standard" Tables, 1870–1885 and 1885–1900, is close to that of the "British Empire", namely, 1872–1902, I have combined the "Standard" figures for purposes of comparison.

In the following Table "D" are given estimates of the expected deaths by the O^{M} Table, and by the O^{M} Table taking ages 7 years older, and the actual deaths for each of the above-mentioned experiences; also similar information in regard to the "British Empire" experience. The aggregate expected deaths for all ages and races by the O^{M} Table with 7 years' addition to ages do not differ appreciably from the actual deaths. This is more especially the case with the "Oriental" and "British Empire." Combining the totals for these two offices, the expected deaths by this method exceed the actual by just over 1 per-cent. In the total of the three offices the expected deaths by this method are about 3 per-cent less than the actual.

Extra mortality increases with period of exposure.

It has before been remarked in papers on tropical mortality that the rates of extra mortality increase with the age of the life and the period of exposure; and the investigations referred to above support these contentions.

Mr. G. F. Hardy (*J.I.A.*, xxv, 235) says in regard to European lives in India:

"The effect of exposure on acclimatized lives appears to "increase with its duration, and is by no means in the nature "of a constant addition to the rate of mortality."

Again, Messrs. G. F. Hardy and H. J. Rothery, in reference to mortality in the West Indies (Barbadoes Mutual), state:

"It will be observed that the extra mortality increases very "rapidly with the age, and is more nearly a constant percentage "upon the H^{M} rates than a constant addition." (*J.I.A.*, vol. xxvii, p. 178).

TABLE D.

Mortality Experience of "Standard", "Oriental" and "British Empire" (Combined Races).

Ages	"STANDARD" 1870-1900 Actual Deaths	*Estimated Expected Deaths O^M	*Estimated Expected Deaths O^M with 7 years added to ages	"ORIENTAL" 1874-1891 Actual Deaths	Estimated Expected Deaths O^M	Estimated Expected Deaths O^M with 7 years added to ages	"BRITISH EMPIRE" 1872-1902 Actual Deaths	Estimated Expected Deaths O^M	Estimated Expected Deaths O^M with 7 years added to ages	TOTAL OF THREE COMPANIES Actual Deaths	Estimated Expected Deaths O^M	Estimated Expected Deaths O^M with 7 years added to ages
20–24	32	13·2	17·4	8	9·3	12·2	3	2·9	3·9	43	25·4	33·5
25–29	118	58·2	78·6	56	48·5	65·5	22	16·1	21·7	196	122·8	165·8
30–34	144	101·0	136·8	115	94·4	127·8	48	39·0	52·7	307	234·4	317·3
35–39	226	128·9	176·2	159	111·9	156·8	95	58·4	79·8	480	302·2	412·8
40–44	177	134·3	190·8	143	108·9	154·0	92	69·2	98·3	412	312·4	443·1
45–49	191	123·6	185·7	125	81·8	121·9	120	73·7	110·4	436	279·1	418·0
50–54	157	99·6	158·2	88	53·6	84·9	115	66·9	106·5	360	220·1	349·6
55–59	87	56·2	95·7	47	31·0	51·5	87	52·5	87·3	221	139·7	234·5
60–64	...	...	...	28	14·1	24·7	55	33·1	57·0	83	47·5	81·7
65–69	...	...	...	8	3·9	7·0	26	18·4	32·1	34	22·3	39·1
70–74	...	...	...	...	...	...	5	5·6	9·7	5	5·6	9·7
75–79	...	...	...	...	...	...	6	1·3	2·1	6	1·3	2·1
Totals	1,132	715·0	1,039·4	777	560·7	806·3	674	437·1	661·5	2,583	1,712·8	2,507·2

* The Exposed to Risk have been multiplied by q_x and q_{x+7}, where x is the central age of the group.

The same feature will be noticed in the Tropical Experience of the New York Life Office, particulars of which are given in a paper by Mr. A. Hunter, F.I.A., read before the Actuarial Society of America in 1908. (See *Transactions* of that Society, vol. x, p. 395).

In the New York Life Office the same scale of premiums applies to a number of tropical countries, including India. The rates of mortality experienced in these countries are given below.

I have also added the mortality rates derived from the experience of the Uncovenanted Service Family Pensions Fund (India), 1837-1872 (*J.I.A.*, xviii, p. 153). The members of this Fund, comprising Europeans, Eurasians, and Native Christians, are medically examined at entry. The period covered, 1837-1872, is somewhat remote, and the mortality shown is probably higher than that which rules in the fund at the present time. I am giving the rates of mortality here, however, not only for purposes of comparison, but as a further example of the fact, that in India the extra mortality increases with age.

The mortality in the Barbados Mutual Office is referred to above, and as a comparison of the rates of mortality in that experience with those of the "British Empire" Office in India may be of interest, I give the relative rates hereunder.

Rates of Mortality.

Age	British Empire. India (combined races)	New York Life Tropical Experience (excluding first two years of Assurance)	Uncovenanted Service Family Pension Fund 1837—1872	Barbadoes Mutual. West Indies (excluding first two years of assurance)*
27	·0073	·0108	·0094	·0123
37	·0113	·0133	·0187	·0164
47	·0200	·0196	·0312	·0219
57	·0405	·0354	·0451	·0344
67	·0758	·0738	·0926	·0658

* Including the first two years of assurance, the total rate of mortality was 10 per-cent less than when those years were excluded.

It may here be mentioned that Messrs. Hardy and Rothery considered the rates of mortality of the "Barbados Mutual" as "being of the character of a standard of minimum tropical mortality."

It would have been of considerable interest if a table could have been added showing the mortality experienced by

European lives after their permanent retirement to Europe, but, in the case of the "Positive" and "British Empire" offices, there are not sufficient data on which to base any definite conclusions. The only way to obtain a sufficiently large number of facts relating to such lives would be for the offices doing business in India jointly to collect the facts bearing on this point.

III.—*Indian Postal Insurance Scheme.*

Through the kindness of the Actuary of the India Office, Mr. Willis Browne, F.I.A., I have been supplied with data which have enabled me to investigate the mortality rates amongst lives assured under the Indian Government Postal Insurance Scheme. This scheme embraces all members of the Civil Service, but is taken advantage of almost exclusively by Natives.

The annual reports on the administration of the Fund are very complete and give much valuable information as to the class of policies effected and the ranks from which the policyholders are drawn. The report for the year ending 31 March 1904 (the last year included in the experience), tells us that 1,062 policies were issued for amounts under Rs. 1,000, and 506 policies for larger sums, the average policy being for Rs. 1,342 (about £89). We are also informed that 1,491 of the new entrants were Asiatics and 77 were non-Asiatics. It appears that the lives are medically examined before being admitted to the benefits of the scheme.

The information supplied has enabled me to complete an investigation of the mortality experienced in the Fund from its inauguration in February 1884 to 31 March 1904.

Whole Life and Endowment Assurance policies are granted under the scheme, the latter plan having been introduced in February 1898. Policies issued under this plan already preponderate in number, the existing policies on the 31 March 1904 consisting of:

	3,074 Whole Life Policies and
	5,340 Endowment Assurance Policies.
Total	8,414

It will be observed that, as all the Endowment Assurances, as well as a certain proportion of the Whole Life Policies, were effected within six years of the close of the observations, the experience includes a large percentage of recently selected lives.

The following Table "E" gives the Rates of Mortality for quinquennial groups of ages; and the expected deaths by the "British Empire" combined Indian Table, the "British Empire" (Natives) Table, and the "British Empire" (European) Table.

It will be seen that the mortality under the "British Empire" (European) Table is heavier, and, under the "British Empire" (Native) Table, lighter than that experienced under the Indian Postal Insurance Scheme.

Table E.

Indian Postal Insurance Scheme.

Mortality Experience, 1884—1904.

Age	Exposed to risk	Actual Deaths	Rates of Mortality quinquennial groups (3)÷(2)	Expected Deaths by "British Empire" India (Combined) Races Table	Expected Deaths by "British Empire" (Native) Mortality	Expected Deaths by "British Empire" India (European) Mortality
(1)	(2)	(3)	(4)	(5)	(6)	(7)
21–24	1,649	12	·0073	9·7	9·0	13·6
25–29	8,628	83	·0096	63·0	58·4	81·8
30–34	11,842	101	·0086	110·1	99·2	130·7
35–39	10,676	116	·0109	120·6	111·8	138·9
40–44	7,719	116	·0150	111·9	103·8	121·7
45–49	4,475	78	·0175	89·5	80·5	89·2
21–49	44,989	506	...	504·8	462·7	575·9
50–54	1,939	76	·0392	56·3	48·5	51·0
55–59	516	29	·0563	20·9	18·6	18·8
60–64	103	2	·0194	5 5	5·6	5·4
65–69	11	2	·1818	·8	·9	1·0
50–69	2,569	109	...	83·5	73·6	76·2
All Ages	47,558	615	...	588·3	536·3	652·1

IV.—*Mortality of British Military Officers in India.*

In a paper read before the Institute by Mr. J. J. McLauchlan, F.F.A. (*J.I.A.*, xxxiv, p. 251), are given the following particulars,

relating to Commissioned Officers in British Regiments in India, taken from the Army Medical Report, 1886:

STATEMENT F.

Period	Average Strength	Average No. of Deaths in a year	Average No. invalided in a year	Death Rate per 100	Invaliding Rate per 100
10 years 1875-1886	2,292	38·80	132·30	1·7	5·8

Similar information, obtained from the same source, up to and including the year 1905 is as follows—

Period	Average Strength	Average No. of Deaths in a year	Average No. invalided in a year	Death Rate per 100	Invaliding Rate per 100
10 years 1887-1896	2,007	34·3	137·8	1·70	6·86
9 years 1897-1905	1,988	27·5	129·8	1·38	6·52

Expeditionary forces.

The deaths do not include those occurring in expeditionary forces, and, unfortunately, complete information as to such deaths is not forthcoming. Returns have, however, been published by the India Office of such deaths from wounds occurring during the years 1891–1903 inclusive, and they are as follows:

Killed, or Died of Wounds	Wounded
20	70

These casualties resulted from the Miranzai and Manipur Expeditions in 1891, the attack on Wana 1894, the Defence and Relief of Chitral 1895, the Tirah Expedition 1897–8, the Waziris Expeditions 1901 and 1902, and various other small frontier expeditions, &c. Their effect would be to increase the above rates of mortality of officers during the period by about ·8 per 1,000 per annum. The recorded rate of mortality would, of course, be still further increased if information were forthcoming regarding deaths from disease in these expeditions.

Compared with "British Empire" experience.

As the deaths of Army officers given in Statement "F" do not include those resulting from warlike expeditions, it will be useful to compare them with the "British Empire" experience of Europeans in India, which consists almost entirely of lives engaged in civil occupations. The age at which officers generally retire from India is about 50, and Statement F may be assumed to deal with lives up to that age.

From the "British Empire" European Experience (Table B) it will be seen that the exposed to risk and deaths under age 50 amount to 14,997 and 207, respectively, giving a rate of mortality of 1·38 per 100, which exactly corresponds to the above experience of Army officers for the nine years 1897–1905, and is about 20 per-cent lower than the experience of Army officers for the period 1875–1896.

In view of this, and that the somewhat heavy invaliding list indicates that there are a considerable number of impaired lives returning home, it appears that, on a peace footing, military officers should be charged premiums at least as high as civilians in India. In addition such extras should, of course, be charged as may be considered necessary to cover war risks.

Military Service Family Pensions scheme.

I have recently taken out the experience of officers under the Indian Military Service Family Pensions Scheme for the period 1873–1902, the data having been kindly furnished by Mr. Willis Browne, F.I.A. This experience, which includes all British officers joining Indian regiments (*i.e.*, Indian Staff Corps) since 1873, covers up to age 50 (the usual age of retirement) 57,180 years of life and 663 deaths, and shows for those ages the very low rate of mortality of 1·16 per 100.

I do not propose to give details of this experience, as I do not think it would serve any useful purpose, for the following reasons—

(1) British officers joining the Indian Army must have served for one year at least (and frequently have served several years) in a British regiment in India. They then have to pass a medical test before entering the Indian Army. Consequently a part at least of the period of acclimatization is excluded, and the lives coming under observation have passed through that period unimpaired.

(2) A part of the experience is in respect of officers, retired before age 50, who continue their membership of the fund.

Before leaving this subject it will be interesting to refer to the rates of mortality in regard to military lives in India, given by Mr. Samuel Brown in *J.I.A.*, vol. xvi, p. 206. These were based on the combined experience of

(*a*) Madras Military Fund...	...	1808–1858
(*b*) Bombay „ „ ...	...	1816–1865
(*c*) Bengal „ „ ...	...	1800–1847

The rates of mortality averaged about 2·8 per 100 under age 50, *i.e.*, about twice the rate shown in the above Statement "F" for the period 1897–1905.

Although the improved conditions of life in India probably account for a considerable reduction in the mortality amongst military officers, it must be borne in mind, in comparing the earlier experience with that given in Statement "F", that the former includes deaths from all causes, whereas the latter excludes deaths resulting from military expeditions.

NATIVE LIFE ASSURANCE.

The mortality experience of Native assured lives in India is not of very general interest to Actuaries, as comparatively few Companies have embarked in that class of business. Actuaries have, however, in recent years been asked to advise Native Assurance Companies on questions of rates and reserves, and Members of the Institute have also been consulted from time to time on other subjects involving the rates of mortality of Oriental races.

Whilst admitting that the experiences of Native assured lives which I am able to give are not large, and that results might differ considerably with the varying conditions under which the business is transacted, any light that can be thrown on this matter will, I hope, be of some use to the profession.

Eligible Natives.

The vast majority of the Indian Natives are quite ineligible for assurance on account of their poverty and illiteracy. According to the Census Report of 1901, less than one per-cent. of the male population over 20 years of age were literate in English—that is less than 750,000. As, generally speaking, only the class of Natives who are educated in English would be acceptable to British companies for assurance, it will be seen that the scope for Native assurances is very much smaller than might be imagined from the enormous population of the

country; and companies can only look to a very limited part of the Native community for acceptable business. Experience shows that this class is subject to a much lighter mortality than the general population, the difference being considerably greater than would be found in a comparison of assured lives and the general population in western communities. Famine and epidemics are two of the chief causes of the very high rates of mortality ruling among the Indian Native population. From the former cause the comparatively well-to-do and literate class of Natives referred to are practically immune, and, although they still have much to learn in hygienics, their means enable them to live under more comfortable and less unsanitary conditions than the rank and file of the population. This renders them less liable to succumb to plague and other epidemics, and, in case of illness, they have the advantage of medical assistance which often is not available to the poorer members of the community. These facts are mentioned more especially to show why mortality rates based on census statistics of the whole population are no criterion of the rates which may be expected to rule amongst assured lives.

The "Oriental" office in Bombay, which was established in 1874, was, I believe, the first company to insure Native lives at moderate premiums. At that time, very little information was available as a guide to the rates of mortality, which were likely to rule, amongst the class from which Native policyholders would be drawn. It was a new development for an Oriental race, and only experience could teach companies the weight to be attached to special features of the business—amongst the more important of these being the degree of probity, in their relations to a life office, which might be expected from Natives of the class referred to.

Age and class limitations.

As from time to time further evidence of the rates of mortality amongst assured Native lives has been forthcoming, additional companies have entered the field, and there are several British companies, having branches in India, who now extend the benefits of life assurance at moderate premiums to certain classes of the Natives of India, subject in most cases to restrictions as to the class of assurance and limitations as to age at entry. Some of these companies specifically state the classes of Native lives whom alone they are prepared to assure. Some, again, restrict Natives to endowment assurance policies, and most of them exclude Native lives whose ages exceed 45 years. As the rates of mortality amongst assured Natives are, until the later

ages, relatively light compared with Europeans, it appears that endowment assurances are the class of contract which can most advantageously be granted on these lives.

A comparison of the rates of mortality in the general Native population and amongst Native assured lives is appended. Much higher rates rule in the former throughout the table. The disparity is, however, specially noticeable up to age 45, and this feature may partly be accounted for by the greater proportion of select lives amongst the assured lives up to that age.

TABLE G.

Natives. Rates of Mortality.

Ages	Natives (Males) Census 1881 G. F. Hardy (a)	"British Empire" Natives ungrad. (b)	Indian Govt. Postal Assce. ungrad. (c)	"Standard" 1895–1900 Natives ungrad. (d)	OM
20–24	·0191	·0066	·0073	·0103	·00431
25–29	·0207	·0049	·0096		·00523
30–34	·0226	·0050	·0086	·0090	·00648
35–39	·0248	·0090	·0109		·00804
40–44	·0281	·0144	·0150	·0170	·01001
45–49	·0328	·0242	·0175		·01277
50–54	·0403	·0342	·0392	·0256	·01693
55–59	·0480	·0226	·0563		·02338
60–64	·0672	·0357	·0194	·0662	·03344
65–69	·0990	·0770	·1818		·04900

(*a*) *J.I.A.*, vol. xxv., p. 234.

(*b*) Derived from Table B, p. 369.

(*c*) Table E, p. 378.

(*d*) Proceedings of the Congress of Actuaries, 1903, p. 113.

PREMIUMS.

As a result of enquiries made of British Companies I am able to furnish the following statements in regard to the extras charged for lives resident in India, Burma, Ceylon and the Straits Settlements. I would take the opportunity of thanking the Offices for their courtesy in answering my enquiries.

Dealing first with the offices which charge a uniform extra for all ages, I have prepared the following Table "H."

Table H.

Annual Rates per-cent. of Extra Premium charged for India, Burma, Straits Settlements and Ceylon by Companies adopting the same rate of extra premium at all ages.

Number of Offices charging rates referred to.

Rate of extra charged	Whole Life Policies				Endowment Assurance Policies			
	India	Burma	Straits Settlements	Ceylon	India	Burma	Straits Settlements	Ceylon
£ s. d.								
2 2 0	...	...	1	...	...	...	1	...
2 0 0	...	...	1	...	...	...	1	...
1 10 0	2	3	3	3	2	2	3	2
1 5 0	1	1	1	1	1	1	1	1
1 1 0	1	1	1	1	1	1	1	1
1 0 0	18	19	20	18	14	17	16	16
0 15 0	3	1	1	4	6	3	4	6
0 12 6	1	1	...	1	1	1	...	1
0 10 0	...	...	...	1	...	...	...	1
*0 6 0	1	1	1	1	1	1	1	1

Thirteen of the offices referred to above charge the extra for a limited term of years only.

Nine of these charge a uniform extra of 1 per-cent per annum for ten years only.

One of these charges a uniform extra of 1 per-cent per annum for five or ten years only, according to circumstances.

One of these charges a uniform extra of 1 per-cent per annum for five years only.

One of these charges a uniform extra of 25*s.* per cent. per annum for five years only.

One of these charges a uniform extra of 30*s.* per cent. per annum for five years only.

One office charges 30*s.* for first five years (if life unacclimatized) reducible to 15*s.* afterwards.

Another office in the same circumstances charges 30*s.* per-cent. first five years, 20*s.* per-cent. second five years, and 10*s.* per-cent. afterwards.

* The Company charging 6*s.* per-cent extra per annum makes this rate permanent, and takes only a special class of lives, subject generally to favourable mortality.

It will be noticed that more than half of these offices charge 1 per-cent per annum extra for all the countries referred to, and about half limit the extras charged to a term of years only.

Graduated extras. Seventeen of the offices giving me information charge premiums based on graduated extras to home rates increasing with the age at entry. Of these I have selected seven

companies having agencies in the East and actively working there, and give below the average extras charged by them.

STATEMENT J.

Description of Assurance	Average yearly extra over home rate Per £100 assured		
	£	*s.*	*d.*
Whole-Life Assurance—			
Age at entry 20	0	11	9
,, ,, 30	0	14	1
,, ,, 40	0	17	3
,, ,, 50	1	5	6
Endowment Assurance. Death or 60—			
Age at entry 25	0	12	0
,, ,, 35	0	14	10
,, ,, 45	1	2	0

New York Life.

In Mr. A. Hunter's paper, already referred to, dealing with mortality in tropical countries, are given the rates of premium charged by the New York Life in certain parts of the tropics, including India. These rates are said to be "similar to those charged by other American and Canadian Companies."

The annual extras over home rates are as follows—

STATEMENT K.

Description of Assurance	Annual extra over home rate. Per £100 assured		
	£	*s.*	*d.*
Whole-Life Assurance—			
Age at entry 25	1	0	10
,, ,, 35	1	4	0
,, ,, 45	1	9	10
,, ,, 55	2	0	4
20 Year Endowment Assurance—			
Age at entry 25	0	19	2
,, ,, 35	1	1	2
,, ,, 45	1	5	9
,, ,, 55	1	17	9

The British offices chiefly concerned with Indian risks, namely those whose average rates of extra premium are given in the above Statement "J", appear now to recognize generally that the extra should be an increasing one with age at entry; and none of these

offices, I believe, restrict the extra to a fixed term of years only. Their practice in both these respects appears to be warranted by the mortality experience of assured lives, which shows the extra risk to increase with advancing age.

Premiums for Natives. As several offices are charging the same premiums for Natives as for Europeans it will be interesting to ascertain how far this appears to be justified.

The net premiums for whole-life and endowment assurance policies have been calculated:

1. According to the "British Empire" Experience of combined races, *i.e.*, Europeans, Eurasians and Natives.
2. According to the "British Empire" Experience of Europeans where $q'_x = \mathrm{O}^{\mathrm{M}}q_{x+5} + {\cdot}003$.
3. According to the "British Empire" Experience of Natives where $q'_x = \mathrm{O}^{\mathrm{M}}q_{x+6}$.

STATEMENT L.

Annual Net Premiums per £100. Interest 3 per-cent.

I.—*Combined Races.* "*British Empire*"—*India.*

Age	Whole-Life: Combined Races. India	Whole-Life: Excess over OM = Net extra	15-Year Endowment Assurance: Combined Races. India	15-Year Endowment Assurance: Excess over OM = Net extra	25-Year Endowment Assurance: Combined Races. India	25-Year Endowment Assurance: Excess over OM = Net extra
	£ s. d.	£ s. d.	£ s. d.	£ s. d.	£ s. d.	£ s. d.
20	1 12 11	0 6 9	5 11 5	0 2 0	3 2 5	0 2 8
30	2 5 10	0 10 0	5 15 5	0 3 4	3 8 0	0 4 8
40	3 6 1	0 15 7	6 3 0	0 6 5	4 0 0	0 10 0
50	5 0 10	1 6 2				

II.—*Europeans.* "*British Empire*"—*India.*

Age	Whole-Life: Europeans. India.	Whole-Life: Excess over OM = Net extra	15-Year Endowment Assurance: Europeans. India	15-Year Endowment Assurance: Excess over OM = Net extra	25-Year Endowment Assurance: Europeans. India	25-Year Endowment Assurance: Excess over OM = Net extra
	£ s. d.	£ s. d.	£ s. d.	£ s. d.	£ s. d.	£ s. d.
20	1 15 0	0 8 10	5 13 10	0 4 5	3 4 11	0 5 2
30	2 6 9	0 10 11	5 17 3	0 5 2	3 9 10	0 6 6
40	3 5 4	0 14 10	6 3 8	0 7 1	3 19 11	0 9 11
50	4 17 1	1 2 5	...	...	...	...

III.—*Natives.* "*British Empire*"—*India.*

Age	Whole-Life		15-Year Endowment Assurance		25-Year Endowment Assurance	
	Natives. India	Excess over O^M = Net extra	Natives. India	Excess over O^M = Net extra	Natives. India	Excess over O^M = Net extra
	£ *s.* *d.*	£ *s.* *d.*	£ *s.* *d.*	£ *s.* *d.*	£ *s.* *d.*	£ *s.* *d.*
20	1 11 6	0 5 4	5 10 10	0 1 5	3 1 8	0 1 11
30	2 3 10	0 8 0	5 14 5	0 2 4	3 6 9	0 3 5
40	3 3 5	0 12 11	6 1 4	0 4 9	3 17 4	0 7 4
50	4 17 2	1 2 6	...	...	...	...

The excess over the O^M premium (or net extra) is greater in the case of Europeans than that of Natives, for ages at entry 20 to 40, but in practice offices do not, in any instance, charge lower rates for the latter. They are probably fully justified in adopting this course, as the experience of Native mortality is meagre compared with that of Europeans, and a margin in premiums is required to cover possible adverse variations in the mortality rates. Then again, Europeans can generally furnish absolute proof of age, whereas the evidence on this point which the Native can produce is much less reliable.

The fact that the different experiences of Native assured lives may vary considerably is evidenced by the comparison of the rates of mortality of "British Empire" Natives with those assured under the Government Postal Assurance Scheme, Table "E", p. 378, where the latter show considerably heavier mortality than the former. Under the Government Postal Assurance Scheme, however, the actual deaths are less than those expected by the "British Empire" (India) European rates of mortality.

Age Limit for Natives.

Assuming that Native lives are not accepted at higher ages at entry than 40, and that only the best classes of Natives are regarded as eligible at all, the "British Empire" experience does not appear to warrant higher rates of premium for Natives than for Europeans.

Considerable additions have, for the following reasons, to be made to the net extras shown under heading II in arriving at the corresponding office extra premiums.

First, it is customary for offices to allow a rebate of extra premiums for periods of home residence. In the "British Empire" experience lives were kept exposed for these periods, and, consequently, the premiums under heading II do not

provide for any rebate. The net extra has, therefore, to be increased to provide this benefit. Assuming an assured is on an average at home for six months in 5 years, *i.e.*, that he pays $4\frac{1}{2}$ years' extra during 5 years' assurance, the net extra above would require to be increased by $\frac{1}{9}$th, or (say) 11 per-cent.

Second, this increased net extra would require to be loaded for expenses, commission, profits and contingencies (including possible deterioration after retirement to England)—a further 25 per-cent (say)—making the total loading on the extras under heading II approximately 39 per-cent.

In the following table are given the net extras increased by 40 per-cent :

STATEMENT M.

Office Annual Extra Premiums per £100 Assurance.

Age	Whole-Life	15-Year Endowment Assurance	25-Year Endowment Assurance
	£ *s.* *d.*	*s.* *d.*	*s.* *d.*
20	0 12 5	6 3	7 3
30	0 15 4	7 3	9 1
40	1 0 10	9 11	13 11
50	1 11 5	...	...

The above extras are, under the whole-life table, slightly higher, and under endowment assurance tables lower, than those of the British companies given on page 385 (Statement J.).

Extras on home rates.

Under the whole-life table the extras approximate to those found by rating a life up 5 years and charging in addition a constant of 10*s.* per-cent. This method of rating appears to be a suitable one for India under that table.

It will be noticed that the extra premiums under endowment assurances are lower than those under whole-life assurances, and that they increase with the age at entry and length of the term of the policy.

POLICY-VALUES.

I should preface the following remarks by saying that they relate specially to companies working actively in India, and particularly to those insuring Native lives. In an office which has comparatively few Indian risks and which does not assure Native lives, the matter would not, of course, be of sufficient importance to warrant a departure from its general method of valuing policies subject to climate or occupation extras.

It is the practice of some offices to value such policies on the same basis as home contracts, and to reserve part of the extra premium for the current year to meet the unexpired risk, regarding the extra risk as a constant one from year to year. On the basis of the "British Empire" experience, however, such reserves would appear to be, after the first few years' duration, inadequate for whole-life policies.

Taking age 30 at entry, for example, it will be seen from Statement "X", given at the end of the paper, that the additions required to the O^M reserves per £100 whole-life policy are as follows—

Duration years	For Policies on European Lives in India	For Policies on Native Lives in India
	£	£
1	·12	·24
3	·39	·73
5	·68	1·24
10	1·49	2·58
20	3·23	5·13
30	4·43	6·62
40	4·52	5·47

Lives should be rated up in valuation. The values of policies on Native lives correspond to O^M values taking lives 6 years older than true ages at entry, and those for European lives approximate closely to O^M values taking the lives 4 years older than true ages at entry, and these would appear to be suitable methods of valuation. These methods not only more truly represent the liability under the sum assured than is the case where part or the whole of the year's extra is reserved in addition to the ordinary home value, but they also have the advantage of making proper provision for increased liability—

(*a*) under bonuses;

(*b*) under paid-up and limited payment policies, if the extra premium ceases with the ordinary premium.

In the case of European lives, the basis of reserves would require to be modified on a life permanently returning to Europe.

With regard to Native lives, the adoption of the ordinary English reserve, with, perhaps, a part of the extra for current risk, is especially unsuitable, as the rates of mortality appear to correspond more closely to a constant addition to age, and the

question of modification of reserves on retirement does not arise as with English lives.

Endowment assurance values. It will be seen from Statement "N" that under endowment assurances the O^M policy-values are generally slightly greater than those under the Indian Tables, but the differences are always small. If any considerable amount of bonuses attached, the differences would be in the opposite direction; but, except in cases where the bonus was, relatively to sum assured, very large, the deficiency in the O^M value would be small.

At the end of the paper are given specimen policy-values based on the three tables of mortality referred to on page 386.

Conclusion.

It will be convenient to summarize the more important points referred to in the paper.

As regards the "British Empire" Experience.

Selection. 1. The effect of selection on European mortality in India is not apparent, the mortality rates in the first five years of assurance being approximately the same as those ruling for lives of the same age which have been insured for longer periods. This may, perhaps, be accounted for by the fact that the trying period of acclimatization is frequently concurrent with the first five years of assurance.

Mortality rates. 2. The rates of mortality of Europeans, Asiatics and combined races, respectively, are approximately represented by the following modifications in the O^M Table—

For Europeans, $q'_x = q_{x+5} + \cdot 003$
For Asiatics, $q'_x = q_{x+6}$
For combined races, including Eurasians, $q'_x = q_{x+7}$

x being the normal age.

This last rate of mortality is brought out approximately in the "British Empire" experience, but would, of course, vary with the relative number of the different races combined in any experience.

Premiums. From the foregoing, it appears that in the case of Europeans in India an addition to age at entry, supplemented by a constant, is a more suitable way of deriving the Indian rate

of premium from the home rate than increasing the latter by a uniform addition at all ages at entry.

Natives. 3. The most eligible class of Natives are assurable at the same rates as Europeans in India, provided their age at entry does not exceed 40.

Policy-values. 4. With regard to policy-values, the plan of increasing the ordinary home reserve by the whole or part of the current year's extra appears to give insufficient reserves for whole-life policies after the first few years' duration, more especially where the amounts of reversionary bonuses attaching are relatively large.

In the case of Native lives the reserves on the above basis are less adequate than in that of European lives.

A more suitable basis, if the valuation is under the O^{M} Table, would apparently be to treat European lives as 4 years older and Native lives as 6 years older than their true age at entry.

Under endowment assurances, the ordinary O^{M} reserves for true ages at entry are apparently sufficient.

As regards the Experience of the Indian Postal Assurance Scheme.

5. This experience, which consists almost entirely of Native lives, shows heavier mortality than the "British Empire" (Native) Experience, but lighter mortality than the "British Empire" (European) Experience.

As regards the Experience of British Military Officers in India.

6. The mortality from 1875–1905 shows a very great improvement over the earlier experiences.

Without taking into account war risks, British military officers should be charged premiums at least as high as civilians in India. In addition, such extras for occupation as may be considered necessary should also be imposed.

Statement N.

Comparison of Policy-Values. £100 Assured. Interest 3 %.

I.—*"British Empire" Indian Experience (Combined Races). Whole Life Policies.*

Duration	Age at Entry 20: "British Empire" Indian Combined Races (1)	Age at Entry 20: O^M (2)	Age at Entry 20: (1) - (2)	Age at Entry 30: "British Empire" Indian Combined Races (3)	Age at Entry 30: O^M (4)	Age at Entry 30: (3) - (4)	Age at Entry 40: "British Empire" Indian Combined Races (5)	Age at Entry 40: O^M (6)	Age at Entry 40: (5) - (6)	Age at Entry 50: "British Empire" Indian Combined Races (7)	Age at Entry 50: O^M (8)	Age at Entry 50: (7) (8)	Duration
1	1·15	·91	·21	1·52	1·26	·26	2·13	1·70	·43	2·80	2·37	·43	1
3	3·51	2·89	·62	4·62	3·83	·79	6·46	5·18	1·28	8·21	7·18	1·03	3
5	5·96	4·92	1·04	7·81	6·49	1·32	10·85	8·76	2·09	13·49	12·06	1·43	5
10	12·39	10·30	2·09	16·33	13·50	2.83	21·85	18·16	3·69	26·33	24·38	1·95	10
20	26·69	22·40	4·29	34·61	29·20	5·41	42·43	38·11	4·32	51·72	48·12	3·60	20
30	42·71	36·49	6·22	51·83	46·46	5·37	62·27	57·54	4·73	...	...	...	30
40	57·80	51·98	5·82	68·43	63·27	5·16	...	...	...	...	...	...	40

Endowment Assurances.

Duration	Age at Entry 20: "British Empire" Indian Combined Races (1)	Age at Entry 20: O^M (2)	Age at Entry 20: (1) - (2)	Age at Entry 30: "British Empire" Indian Combined Races (3)	Age at Entry 30: O^M (4)	Age at Entry 30: (3) - (4)	Age at Entry 40: "British Empire" Indian Combined Races (5)	Age at Entry 40: O^M (6)	Age at Entry 40: (5) - (6)	Duration
25-Year Term										25-Year Term
1	2·68	2·68	·00	2·68	2·68	·00	2·86	2·71	+ ·15	1
3	8·26	8·29	− ·03	8·23	8·26	− ·03	8·75	8·34	+ ·41	3
5	14·18	14·23	− ·05	14·08	14·15	− ·07	14·86	14·26	+ ·60	5
10	30·53	30·72	− ·19	30·33	30·46	− ·13	31·07	30·49	+ ·58	10
20	72·30	72·69	− ·39	71·73	72·21	− ·48	70·45	71·54	−1·09	20
15-Year Term										15-Year Term
1	5·21	5·25	− ·04	5·14	5·21	− ·07	5·10	5·13	− ·03	1
3	16·16	16·27	− ·11	15·91	16·12	− ·21	15·78	15·90	− ·12	3
5	27·84	28·01	− ·17	27·42	27·76	− ·34	27·17	27·41	− ·24	5
10	60·68	60·93	− ·25	60·12	60·57	− ·45	59·42	60·03	− ·61	10

Comparison of Policy Values. £100 Assured. Interest 3 %.

II.—"*British Empire*" *Indian Experience (Europeans)* $\left| q'_x \quad q^{O^M}_{x+5} + {\cdot}003 \right|$. *Whole Life Policies.*

Duration	Age at Entry 20: "British Empire" India, Europeans (1)	Age at Entry 20: O^M (2)	Age at Entry 20: (1)–(2)	Age at Entry 30: "British Empire" India, Europeans (3)	Age at Entry 30: O^M (4)	Age at Entry 30: (3)–(4)	Age at Entry 40: "British Empire" India, Europeans (5)	Age at Entry 40: O^M (6)	Age at Entry 40: (5)–(6)	Age at Entry 50: "British Empire" India, Europeans (7)	Age at Entry 50: O^M (8)	Age at Entry 50: (7)–(8)	Duration
1	1·02	·94	·08	1·38	1·26	·12	1·94	1·70	·24	2·72	2·37	·35	1
3	3·13	2·89	·24	4·22	3·83	·39	5·90	5·18	·72	8·20	7·18	1·02	3
5	5·33	4·92	·41	7·17	6·49	·68	9·96	8·76	1·20	13·69	12·06	1·63	5
10	11·16	10·30	·86	14·99	13·50	1·49	20·52	18·16	2·36	27·33	24·38	2·95	10
20	24·48	22·40	2·08	32·43	29·20	3·23	42·24	38·11	4·13	52·33	48·12	4·21	20
30	39·97	36·49	3·48	50·89	46·46	4·43	62·11	57·54	4·57	...	...	...	30
40	56·38	51·98	4·40	67·79	63·27	4·52	...	...	...	...	...	...	40

Endowment Assurances.

Duration	Age at Entry 20: "British Empire" India, Europeans (1)	Age at Entry 20: O^M (2)	Age at Entry 20: (1)–(2)	Age at Entry 30: "British Empire" India, Europeans (3)	Age at Entry 30: O^M (4)	Age at Entry 30: (3)–(4)	Age at Entry 40: "British Empire" India, Europeans (5)	Age at Entry 40: O^M (6)	Age at Entry 40: (5)–(6)	Duration
25-Year Term										25-Year Term
1	2·59	2·68	–·09	2·59	2·68	–·09	2·69	2·71	–·02	1
3	7·99	8·29	–·30	7·98	8·26	–·28	8·26	8·34	–·08	3
5	13·74	14·23	–·49	13·69	14·15	–·46	14·12	14·26	–·14	5
10	29·81	30·72	–·91	29·63	30·46	–·83	30·06	30·49	–·43	10
20	71·79	72·69	–·90	71·23	72·21	–·98	70·38	71·54	–1·16	20
15-Year Term										15-Year Term
1	5·12	5·25	–·13	5·05	5·21	–·16	4·99	5·13	–·14	1
3	15·89	16·27	–·38	15·71	16·12	–·41	15·49	15·90	–·41	3
5	27·45	28·01	–·56	27·13	27·76	–·63	26·76	27·41	–·65	5
10	60·26	60·93	–·67	59·81	60·57	–·76	59·11	60·03	–·92	10

STATEMENT N—*continued.*

Comparison of Policy-Values. £100 Assured. Interest 3 %.

III.—"*British Empire" Indian Experience (Natives)* $\left[\begin{matrix} O^{M} \\ q'_{x} = q_{x+6} \end{matrix}\right]$. *Whole-Life Policies.*

Duration	Age at Entry 20: "British Empire" India. Natives (1)	Age at Entry 20: O^M (2)	Age at Entry 20: (1) − (2)	Age at Entry 30: "British Empire" India. Natives (3)	Age at Entry 30: O^M (4)	Age at Entry 30: (3) − (4)	Age at Entry 40: "British Empire" India. Natives (5)	Age at Entry 40: O^M (6)	Age at Entry 40: (5) − (6)	Duration
1	1·13	·94	·19	1·50	1·26	·24	2·08	1·70	·38	1
3	3·13	2·89	·54	4·56	3·83	·73	6·31	5·18	1·13	3
5	5·81	4·92	·89	7·73	6·49	1·24	10·63	8·76	1·87	5
10	12·09	10·30	1·79	16·08	13·50	2·58	21·74	18·16	3·58	10
20	26·23	22·40	3·83	34·33	29·20	5·13	44·09	38·11	5·98	20
30	42·27	36·49	5·78	53·08	46·16	6·62	63·94	57·54	6·40	30
40	58·76	51·98	6·78	69·74	63 27	6·47	...	...	...	40

Endowment Assurances.

Duration	Age at Entry 20: "British Empire" India. Natives (1)	Age at Entry 20: O^M (2)	Age at Entry 20: (1) − (2)	Age at Entry 30: "British Empire" India. Natives (3)	Age at Entry 30: O^M (4)	Age at Entry 30: (3) − (4)	Age at Entry 40: "British Empire" India. Natives (5)	Age at Entry 40: O^M (6)	Age at Entry 40: (5) − (6)	Duration
25-Year Term										25-Year Term
1	2·68	2·68	−·00	2·69	2·68	+·01	2·80	2·71	+·09	1
3	8·28	8·29	−·01	8·27	8·26	+·01	8·59	8·34	+·25	3
5	14·20	14·23	−·03	14·16	14·15	+·01	14·62	14·26	+·36	5
10	30·58	30·72	−·14	30·41	30·46	−·05	30·85	30·49	+·36	10
20	72·42	72·69	−·27	71·84	72·21	−·37	70·92	71·54	−·62	20
15-Year Term										15-Year Term
1	5·23	5·25	−·02	5·16	5·21	−·05	5·10	5·13	−·03	1
3	16·20	16·27	−·07	15·99	16·12	−·13	15·77	15·90	−·13	3
5	27·89	28·01	−·12	27·56	27·76	−·20	27·17	27·41	−·24	5
10	60·74	60·93	−·19	60·27	60·57	−·30	59·54	60·03	−·49	10

Interest 3 %.

"*British Empire*" *Indian Experience* (1872-1902).

x	COMBINED RACES			EUROPEANS*			x
	q_x	D_x	N_x	q'_x	D_x	N_x	
20	·0055	55,363	1,159,756	·0078	55,368	1,132,206	20
21	·0057	53,459	1,106,297	·0080	53,335	1,078,871	21
22	·0059	51,606	1,054,691	·0082	51,367	1,027,504	22
23	·0061	49,807	1,004,884	·0084	49,460	978,041	23
24	·0064	48,061	956,823	·0087	47,615	930,429	24
25	·0067	46,363	910,460	·0089	45,826	884,603	25
26	·0070	44,711	865,749	·0092	44,093	840,510	26
27	·0073	43,104	822,645	·0095	42,416	798,094	27
28	·0077	41,543	781,102	·0098	40,790	757,304	28
29	·0081	40,022	741,080	·0101	39,215	718,089	29
30	·0085	38,543	702,537	·0104	37,689	680,400	30
31	·0089	37,102	665,435	·0107	36,213	644,187	31
32	·0093	35,701	629,734	·0110	34,781	609,406	32
33	·0097	34,338	595,396	·0114	33,395	576,011	33
34	·0101	33,015	562,381	·0118	32,053	543,958	34
35	·0105	31,729	530,652	·0121	30,754	513,204	35
36	·0109	30,482	500,170	·0126	29,495	483,709	36
37	·0113	29,271	470,899	·0130	28,276	455,433	37
38	·0118	28,096	442,803	·0135	27,096	428,337	38
39	·0124	26,956	415,847	·0140	25,952	402,385	39
40	·0130	25,848	389,999	·0145	24,843	377,542	40
41	·0137	24,769	365,230	·0151	23,769	353,773	41
42	·0145	23,718	341,512	·0158	22,728	331,015	42
43	·0154	22,693	318,819	·0164	21,718	309,327	43
44	·0164	21,693	297,126	·0172	20,739	288,588	44
45	·0175	20,717	276,409	·0180	19,788	268,800	45
46	·0187	19,761	256,648	·0189	18,865	249,935	46
47	·0200	18,827	237,821	·0199	17,968	231,967	47
48	·0216	17,913	219,908	·0210	17,097	214 870	48
49	·0232	17,015	202,893	·0222	16,251	198,619	49
50	·0249	16,137	186,756	·0234	15,428	183,191	50
51	·0268	15,276	171,480	·0248	14,627	168,564	51
52	·0290	14,434	157,046	·0264	13,848	154,716	52
53	·0312	13,607	143,439	·0280	13,091	141,625	53
54	·0334	12,798	130,641	·0299	12,353	129,272	54
55	·0357	12,011	118,630	·0319	11,634	117,638	55
56	·0381	11,245	107,385	·0340	10,936	106,702	56
57	·0405	10,501	96,884	·0364	10,255	96,447	57
58	·0430	9,782	87,102	·0390	9,593	86,854	58
59	·0455	9,089	78,013	·0419	8,951	77,903	59

$^{*}q'_x = q^{OM}_{x+5} + \cdot 003$

Interest 3 %.

"*British Empire*" *Indian Experience* (1872-1902)—*continued.*

x	Combined Races			Europeans*			x
	q_x	D_x	N_x	q'_x	D_x	N_x	
60	·0481	8,423	69,590	·0450	8,327	69,576	60
61	·0509	7,784	61,806	·0483	7,720	61,856	61
62	·0539	7,173	54,633	·0520	7,133	54,723	62
63	·0571	6,588	48,045	·0560	6,566	48,157	63
64	·0607	6,031	42,014	·0603	6,017	42,140	64
65	·0649	5,500	36,514	·0651	5,490	36,650	65
66	·0701	4,993	31,521	·0702	4,983	31,667	66
67	·0758	4,508	27,013	·0758	4,498	27,169	67
68	·0820	4,045	22,968	·0819	4,036	23,133	68
69	·0888	3,605	19,363	·0885	3,597	19,536	69
70	·0961	3,190	16,173	·0956	3,184	16,352	70
71	·1041	2,799	13,374	·1034	2,795	13,557	71
72	·1129	2,435	10,939	·1118	2,433	11,124	72
73	·1224	2,097	8,842	·1209	2,098	9,026	73
74	·1328	1,787	7,055	·1308	1,791	7,235	74
75	·1441	1,504	5,551	·1414	1,511	5,724	75
76	·1563	1,250	4,301	·1530	1,260	4,464	76
77	·1695	1,024	3,277	·1654	1,035	3,429	77
78	·1839	826	2,451	·1787	839	2,590	78
79	·1996	654	1,797	·1931	670	1,920	79
80	·2164	508	1,289	·2087	524	1,396	80
81	·2347	387	902	·2251	403	993	81
82	·2544	287	615	·2430	303	690	82
83	·2755	208	407	·2619	222	468	83
84	·2984	146	261	·2818	160	308	84
85	·3228	100	161	·3037	111	197	85
86	·3493	66	95	·3256	75	122	86
87	·3772	41	54	·3509	49	73	87
88	·4074	25	29	·3742	31	42	88
89	·4392	14	15	·4030	19	23	89
90	·4737	8	7	·4277	11	12	90
91	·5098	4	3	·4609	6	6	91
92	·5452	2	1	·4858	3	3	92
93	·5902	1	...	·5030	2	1	93
94	...	...	...	·5363	1	...	94
95	...	...	...	...	...	...	95
96	...	...		.	...	...	96
97	...	...	...	...	...	...	97

$*q'_x = q^{\text{OM}}_{x+5} + {\cdot}003$

ABSTRACT OF THE DISCUSSION.

MR. A. W. TARN, in opening the discussion, said that in the course of his paper Mr. Winter had made a very complete collection of the various tables that had been published with regard to the experience of life assurance companies, both British and American, which had transacted business in India, and the results shown in those tables were no doubt a valuable contribution to the knowledge of the comparative duration of life among certain classes in that country, but of course, as the author pointed out, they could not be said to represent Indian mortality as a whole. What they did represent was what might be called the middle classes of India, professional men, merchants, tradesmen, and Government employees, who, it should be remembered, formed but a very small proportion of the natives. For the general mortality in India, only the Census Returns were available, and they gave a very inadequate idea of the real incidence of the mortality. In an interesting paper read before the Royal Statistical Society in April last, entitled "Peradventures of an Indian Life Table", by Sir Athelstan Baines, the Honorary Foreign Secretary of that Society, that gentleman very graphically described the difficulties of obtaining statistics from which could be constructed reliable life tables of Indian mortality. He there mentioned that the registration of births and deaths was of very little value for that purpose, and also that the age distribution was of an irregular character, owing to the marriage and birth rates being affected by periodic famines. Then there was a tendency to state ages in multiples of five, and also to set down favourite numbers, while in advanced life the age was considerably over-stated, even to the extent, according to Sir Athelstan Baines, of from 25 to 30 years. In addition to famines, the natives of India were subject to periodic intervals of cholera and bubonic plague, causing an exceptionally heavy mortality. For instance, during the epidemic of plague in India from 1896 to 1903, there were reported more than two million deaths from this cause alone, and the number might easily have been increased to three millions, because a large number of deaths were not reported, as no evidence could be obtained.

In Table G of the paper, Mr. Winter compared the rate of mortality in the different experiences which he tabulated with the rate of mortality brought out in the Census of 1881. He thought the author would considerably add to the value of the table, if he would also include the results of the Census of 1901, and thus bring the table more up-to-date. The figures of the last Census were certainly more reliable than those of 1881, and also showed a marked decrease in the mortality at the younger ages. There was another feature in the tabular results of the Census of 1901 which was of interest. The mean duration of life was set out for six different provinces of India, viz., Bengal, Bombay, Madras, the North-West Provinces (now the United Provinces), the Punjaub, and Burma; and it seemed very curious that throughout the table

there should be such a wide difference, between the mean duration of life in Bombay and the North West Provinces on the one hand, and in Burma on the other. For example, at age 30, the figures were 22 and 22·3 for the former provinces respectively, and 27·7 for Burma. Perhaps the President, who constructed the table, would be able to throw some light upon that curious circumstance, and explain whether the mortality was much lighter in Burma, or whether the method of taking the census there was of a more scientific character.

He believed that the heavy mortality amongst the troops stationed in India was due to a great extent to malarial fever, which was known to be caused entirely by mosquitoes. With regard to the mortality among Europeans resident in India, he submitted, in view of the great improvements made in the conditions of life, and in the increased facilities for residence in the hill country during the hot months, within the last ten or twelve years, that any experience going back to the early seventies was of comparatively little value, in determining what the rate of mortality amongst Europeans in India was at the present time. It seemed to him, also, that Mr. Winter was hardly consistent in his views as to the effect of acclimatisation upon Europeans. In one part of the paper, he stated that during the first five years the mortality was exceptionally heavy, and, later on, that the effect of exposure of acclimatised lives increased with the duration of the risk, so that in course of time the mortality must be very considerable indeed. Nobody seemed to know what the effect of acclimatisation really was: it was a matter which was of a somewhat nebulous character, and was, he thought, not so much due to the climate as to the care which residents took of themselves, and to their avoidance of mosquitoes and other insects. The longer they were in India the more careful, of course, they became in those matters. The Indian statistics as to the effect of acclimatisation were insufficient at present to enable any definite law to be formulated, but it was to be hoped that some day more accurate knowledge would be obtained upon so important a subject.

Mr. Tarn referred in conclusion to the work of the Oriental Company, which he was very glad Mr. Winter had mentioned, because, in a paper he himself had the honour of reading before the Institute some ten years ago (*J.I.A.*, vol. xxxiv, p. 517) he was one of the first to point out the highly important work that that company was then doing and had been doing now for thirty-five years. The company was formed by the late Mr. McLauchlan Slater, in the face of very great difficulties, but he succeeded in conferring upon the native population of India very great benefits, charging them the same rates of premium as were charged to Europeans. He felt convinced that had Mr. Slater been alive at the present time he would have been much gratified with the substantial progress that life assurance had been making amongst the native population throughout the length and breadth of our great Indian Empire.

Mr. C. W. KENCHINGTON thought that the main interest in the present essay centred round the mortality of Europeans resident in India, because that was the section of the subject which touched British actuaries most closely in their daily practice. At the same time, he thought that the mortality of the native assured lives was also of interest, especially in view of vast possibilities of extension of life assurance in that direction, and it was particularly on that section of the paper that he wished to offer a few remarks. Rather more than a year ago, he was engaged with Mr. Rusher on the valuation of a Native Widows' Fund, and during the course of that work they made an investigation into the mortality experience of the Society from 1849 down to 1905, a period of fifty-six years. The Fund was the Church Missionary Society's Tinnevelly Native Catechists' Widows' Fund. Tinnevelly was in the Madras Presidency. Membership was compulsory on all the native agents of the society in the district—the pastors, catechists and schoolmasters—and the contributions were graduated according to scale in three broad groups of ages, under 25, 25—35, and 35—50, the contributions providing a maximum pension of 5 rupees per month to the widows on the death of members. It would appear that considerable care was exercised in keeping the records of the society, and although there might have been some slight tendency to under-statement of age, especially when the ages were on the border line between two groups, he thought very considerable reliance might be put on the trustworthiness of the data. The number of members enrolled was 841, and the number of years of life exposed to risk 17,719, as compared with 12,594 in the British Empire experience mentioned by Mr. Winter. The deaths were 369, as against 192, *i.e.*, there were nearly twice as many deaths as those given by the author in his experience. The exposed to risk was not quite so high a proportion, due to the difference in the age distribution. The ungraduated rates, which were exceedingly rough, were in the first instance graduated by Mr. Spencer's twenty-one term formula, but the experience was too small to admit of entirely satisfactory rates being brought out, and it was then that recourse was had by Mr. Rusher and himself to the valuable methods suggested by Mr. Winter in his Berlin paper, namely, rating up the O^{M} Table a certain number of years. Various trials were made in that direction, but it was eventually found that the British Offices' Non-Profit Male Experience Ultimate Table (O^{NM}), rated up five years, fitted the experience better than any simple modifications of the O^{M} Table. At no point was there a large accumulated deviation, although there was a slight tendency to understate the mortality up to age 54, and to overstate it subsequently.

There was a reverse tendency to be observed in Table B, given by Mr. Winter, where a positive accumulated deviation of 10·2 up to age 44 became a negative deviation of 2 over the whole table. Since the present paper had been in his hands, he had regraduated the experience by a new 27-term graduation formula, with very high graduating power, having a smoothing co-efficient of $\frac{1}{525}$ as

compared with Mr. Spencer's $\frac{1}{100}$, and the application of that formula had brought out a curve which he ventured to think was a satisfactory one from age 19, the youngest age brought into the table, up to 77, above which the data was too scanty to allow of any reliable results. At age 77, it was of interest to note that the rates would run into the British Offices' O^{NM} Ultimate Table for the same age, without any break in the continuity of the curve; but, considering the fact that the number exposed to risk at every age above 65 was less than 100, it was deemed advisable to run the rates into the O^{NM} Table at age 66, rated up 4 years. That had the effect of somewhat overstating the mortality at the end of the table, but he thought it was probably more in accord with fact than running them into the O^{NM} Table at age 77. In order to compare the rates so brought out with those given in the paper, he had calculated the rates of premium at 3 per-cent interest for whole-life and endowment assurances, which would exactly compare with the figures given by Mr. Winter in statement L—Natives. The comparison was as follows:—

Net Annual Premiums per £100. Interest 3 per-cent.

Age. (1)	Tinnevelly catechists. (2)	British Empire natives. (3)	Excess of (2) over O^M (4)	Excess of (3) over O^M. (5)
		Whole-Life Assurance.		
	£ s. d.	£ s. d.	s. d.	£ s. d.
20	1 14 10	1 11 6	8 8	5 4
30	2 4 1	2 3 10	8 3	8 0
40	3 4 4	3 3 5	13 10	12 11
50	4 14 0	4 17 2	19 4	1 2 6
		15-year Endowment Assurance.		
20	5 15 8	5 10 10	6 3	1 5
30	5 14 2	5 14 5	2 1	2 4
40	6 4 1	6 1 4	7 6	4 9
		25-year Endowment Assurance.		
20	3 5 6	3 1 8	5 9	1 11
30	3 7 4	3 6 9	4 0	3 5
40	3 19 9	3 17 4	9 9	7 4

The figures he had given were sufficient, he ventured to think, to show the considerable degree in which they confirmed those given by the author. They were in substantial agreement at ages 30 to 40, although they were considerably in excess at age 20. It had to be borne in mind, in that comparison, that the Tinnevelly experience did not refer to medically-selected lives, whilst Mr. Winter's

experience was derived from an aggregate table which included the period of selection, and that would have the tendency to reduce the premium in the same way that the H^M Table, for instance, produced smaller premiums at the younger ages than the $H^{M(5)}$ Table. He had also calculated the reserves for policies, to compare with those given by Mr. Winter, and it would appear that the reserves required would be slightly greater than those given in the paper for the shorter durations, but smaller for the longer durations. In the case of endowment assurances, there would be a small increase also. In conclusion, he mentioned that an investigation was also made into the mortality experience of the widows, 311 lives, and that a very light rate was brought out; it had to be remembered that they were pensioners, and perhaps the light mortality was to be expected. The British Female Annuitants Ultimate O^{af} Table, rated up two years only, apparently met that experience.

Mr. H. E. W. LUTT said a point occurred to him upon which Mr. Winter might be able to throw some light. The experience of the British Empire, he imagined, applied to those who insured whilst they were in India, but he was not quite certain whether any lives assured in Great Britain before going out to India were included?

Mr. WINTER said they were all assurances effected in India.

Mr. LUTT said that, in Table H, Mr. Winter gave the extra premiums charged under whole-life policies and endowment assurance policies, and he supposed that most of the offices, referred to in that table as adopting the same rate for all ages, were companies that did not actively pursue business in India, and the rates were applicable consequently to people who assured before they went out, and were charged the extra when incurred. The mortality experience, however, was for lives assured whilst in India, and in any experience taken from the lives assured in India it would be found rather hard to study the question of acclimatisation unless the papers were looked at very carefully to see how long each life had resided there. It seemed really that there were two different kinds of experience which had to be compared one with the other—that of lives accepted by offices in Great Britain, and afterwards going out to India (being charged an extra which varied from 1 per-cent throughout to 1 per-cent for five years only), and that of lives under policies effected in India. Mr. Winter had given very valuable information with reference to the latter, and if he could give any information with regard to the former it would be most useful.

Mr. F. J. VINCENT asked Mr. Winter whether his attention had been drawn to a work that Mr. S. B. Neill, the Actuary of the China Mutual Life Assurance Company, had lately completed for the Opium Commission? Mr. Neill had taken out the experience of that Company, both with regard to opium smokers and non-opium smokers, and brought out the same general characteristics deduced by Mr. Winter in the native race mortality, namely that

there was a very steady rate of mortality up to about age 40 or 45, and then a very rapidly increasing mortality.

Mr. E. A. Rusher: The subject of mortality in India is of importance to the members of this Institute, not only because vital statistics of all kinds must ever be of interest to us, but also because the occasions upon which members are called upon to advise on questions involving rates of mortality in that Empire are increasing rapidly. We feel, therefore, under a debt of gratitude to Mr. Winter for the admirable manner in which he has placed the most recent figures before us, and for the further light he thereby gives upon a difficult and complicated subject. The evolution of our knowledge of Indian mortality is well shown by comparing the references in the first volume of the Journal with the paper just read. In the former is given (*J.I.A.*, vol. i, p. 79-83, 166) the very rough and crude results of a census taken in Bombay in 1849, and also some statistics as to the mortality among European troops in India, results not to be despised for their crudeness, but of value as showing that even the roughest of figures may be of use in starting us on the path that leads in the end to scientific accuracy, such as we may fairly claim for the paper before us.

Probably the part of the paper that will most interest members is that referring to European lives. The facts Mr. Winter brings out may not be entirely novel to many of us, but, whereas we have hitherto partly guessed at them from general consideration, partly had our considered guesses confirmed by previous compilations, such as the Standard, New York, and others, we have them more generally confirmed by the somewhat ampler experience of the British Empire, now before us. With reference to European lives, one or two facts seem worth calling special attention to. In the first place, Mr. Winter shows that the effect of selection is counterbalanced by the effect of climate, and that this latter seems to act with increasing force—or, at all events, that the rates of extra mortality increase—with the age of the life and the period of exposure. From this it follows, as pointed out by the author, that the system of treating the extra premium as a term risk extra is unsound, and gives insufficient policy reserves.

Another fact to be noted is that the extras at present charged by a large number of offices are insufficient to meet the risk. As, generally, the extra premium is not charged during residence in Europe, it would seem preferable, taking all things into consideration, to insist upon endowment assurances, in all cases where it is known at the time of proposal that the life is about to proceed to India. The mortality amongst native lives has not hitherto received much attention at the hands of the Institute, if we may except our President's masterly paper on the 1881 census (*J.I.A.*, vol. xxv, p. 217). The results shown by Mr. Winter are distinctly interesting, and seem to prove that, up to age 40 or 45, carefully selected native lives can be taken at the same rate as Europeans, the mortality corresponding roughly to that of the O^M Table, with an addition of

six years to the age. Here, again, it would seem that these facts point to a restriction of the class of assurance to endowment assurances.

Presumably the class of lives accepted is of the very best and most select. I take it that only those in acknowledged sound financial positions would be admitted, and that practically all the lives are of the merchant class—as Mr. Tarn has pointed out. As is to be expected, the rates differ widely from those of the Census Returns, but it is interesting to note that, in the experience to which Mr. Kenchington has referred, containing about double the number at risk of the British Empire experience, the results are in practical agreement. It should be remembered that the experience is based on the lives of the Native Christian Clergy and Teachers in one large district of India, drawn from all castes, many of them of the Brahmin or highest, but a large proportion being of the lowest castes. The majority live, not in the great towns and cities, but in remote outlying districts where they spend the whole of their lives—certainly by no means immune from the risks of cholera and similar diseases. Although the experience ranges over some 50 or 60 years, it is hardly probable that there has been any appreciable change in the rate of mortality. That such a class of lives should undergo practically the same rate as the very select native lives in the British Empire experience, shows what great possibilities there are of improvement in the general rate shown in the census figures.

In dealing with both European and Native mortality, however, it seems to me a word of caution is necessary. We have by no means arrived, even in Mr. Winter's paper, at anything approaching scientific accuracy as to the rates of premium necessary. India is rather a Continent than a Country, and the rates of mortality amongst Europeans and Natives may be expected to differ in different parts, as much as the rates in the United Kingdom, say, differ from those in European Turkey, and I do not look for any satisfactory basis for a large development in business until sufficient facts have been collected to provide fairly reliable tables for different provinces. To give an instance which occurs to one from Mr. Winter's paper, the mortality of European Officers serving with the native troops, one would suppose, must differ materially from that of the same class serving with the white troops. The whole conditions of service are different, and so much harder, that probably only the very strongest come through the ordeal unscathed. The effect of the harder service may be obscured somewhat by the fact mentioned by Mr. Winter on p. 380 that, before joining, a stringent health selection is exercised, and part of the acclimatisation period has been successfully passed, and one cannot help thinking that, could we compare like with like, *i.e.*, the mortality amongst the same men if they had not joined native regiments with that actually experienced, we should find the latter considerably the heavier.

I make these remarks, not in any disparagement of Mr. Winter's able and enlightening work, but to emphasise that extreme care must be exercised in basing extra rates for European lives

temporarily or permanently resident in India upon the figures before us. I should be disposed to think that, at all events for the military profession, we should add a margin of considerably more than the 40 per-cent he suggests, seeing that we have no control over their movements in India, nor could we well increase the extra charge upon an officer joining a native regiment.

The President, in proposing a vote of thanks to Mr. Winter for his very interesting paper, said he had listened to the communication with a very great deal of interest, and he thought the Institute was indebted to the author, not only for collecting the statistics and carefully analysing them, but also for the succinct manner in which he had conveniently summarised the results at the close of the paper. What mainly interested the members, no doubt, was not the question of general mortality in India, but the question of mortality of insured lives, whether European or native. With regard to Europeans, there was no doubt there must be still a great deal of unpublished data which would be very interesting and valuable, if it could be made available for general use. The question of Europeans was very much complicated by the fact of the varying time for which the lives under observation were exposed to risk in a tropical climate. They returned to England at varying periods of life; and when that retirement took place, the whole of the story had not been told, because, as Mr. Winter pointed out, it was not only necessary to deal with the extra mortality while the lives were immediately subject to the extra risk, but also to take into account the effect on the mortality afterwards in England. That was a most difficult point to determine, in settling the necessary extra premiums to cover Indian risks.

One of the most notable features in respect to Indian mortality of assured European lives was the extraordinary improvement that had taken place in the last generation, or certainly within the last fifty years. The author had referred, for example, to the experience of the Uncovenanted Service Pension Fund, and it might be interesting to add one or two facts to those which Mr. Winter had given. That Fund was a fairly homogeneous body of data, consisting practically entirely of European lives, and the experiences were of some value, because the exposure to the Indian climate was on the whole longer than that of most European lives in India. The author gave a short table which, amongst other data, contained the rates of mortality observed in the Uncovenanted Service Family Pension Fund between the years 1837 and 1872, and it would be interesting to add the rates that had been observed during the thirty years that had elapsed since that time. At age 27 Mr. Winter's figures gave a rate of mortality of ·0094 while the rate for the thirty years following was ·0069; at age 37 Mr. Winter's rate was ·0187, whereas the rate at the more recent period was ·0150; at age 47 the rate fell in the last thirty years from ·0312 to ·0195; at 57 from ·0451 to ·0333; and at 67 from ·0926 to ·0626. That was an extraordinary reduction in the

rates of mortality of a body of men one would have supposed to be practically subjected to the same climatic and other conditions in both periods. It might be somewhat closely represented by saying that the mortality of the former period had been reduced by about one-third at all ages. It was also of interest to point out that, in the last seven years of the later period, from 1894 to 1901, the mortality showed a further reduction, as compared with the whole period, of nearly 10 per-cent. Those rates also had a certain amount of interest and value, because they furnished what was somewhat difficult to obtain with respect to assured lives in India, something like adequate data at the older ages. There were rather more than 100 deaths above 70.

With regard to native lives, he thought it was only within recent years that very much interest had been taken in the subject, but the experience of one or two native offices that had taken up the work and made a speciality of the business showed that it could be made to pay without charging prohibitive rates, provided the proper conditions were present. The main conditions appeared to him to be great care in the selection of the lives, and a knowledge, in those who were responsible for the management of the society, of the natives and the character of their lives. The Census Returns that had been referred to by Mr. Tarn were unfortunately of very little use in throwing any light on the mortality of assured lives in India, though they were of use in showing the general mortality of the population. There was no doubt, a greater difference between the general population and the assured lives than there was in this country. As an illustration, he would refer to two well-known native companies working in Bombay, where, for many years past, there has been prevalent an extraordinary amount of plague, the number of persons dying in that city from plague having been appalling: yet one of those companies had not had a single death from plague amongst the native assured lives. That simple fact showed the difference between the class of lives carefully selected by the insurance company and that of the average native, which, of course, predominated in the census returns.

With regard to the various Census Tables, the 1881 Tables purported to show the average mortality of the natives in India, giving due weight to the periodic famines and plagues, but the 1891 Returns were intended to show the rate of mortality prevailing between 1881 and 1891, which was a period on the whole very favourable, not containing any severe famines. The 1901 Census Returns were intended to be average tables, giving the general mortality of the population. As Mr. Tarn had pointed out, they showed there had been some slight improvement, although he could not attach much weight to that conclusion, as regards the general rate of mortality amongst the natives of India. With regard to Burma, he had been very much struck with the results in that province, showing a rate of mortality so much lighter than the remaining provinces of India, but, of course, it had to be remembered that there was a difference of race between the Burmese and the

Indians, and no doubt that, as well as the difference of climate, had a great deal to do with it.

He was very interested in the form in which Mr. Winter threw his whole conclusions, and also interested in the second conclusion, in which the author pointed out that the native mortality might be well represented by an addition of six years in the age to the O^M Table. About twelve years ago he had to consider, in the light of the data then available, on what terms it was possible to insure native lives in India with a prospect of success, and he then concluded, from an examination of the facts that were then available, that an addition of about five years to the H^M Table and $\frac{1}{4}$ per-cent constant would meet the case. That would be seen to be extremely near to the conclusion which Mr. Winter had arrived at, especially as he believed the $\frac{1}{4}$ per-cent was put on merely to cover contingencies. So far as his own experience went since that time, he believed that the author's general conclusions represented very closely the present state of knowledge as to Indian mortality, and he was inclined to think that, notwithstanding the few criticisms which had been made on the figures in the paper, the rates which Mr. Winter gave were rates which might be said to depend upon fairly reliable experience, and which were safe to employ in practice. At all events, whatever might be the result of a more careful study of the paper, he was quite sure it would be found to contain a great deal of very valuable material and that it would repay very careful and close scrutiny.

The vote of thanks was unanimously agreed to.

Mr. A. T. Winter, in reply, expressed his thanks at the kind manner in which the paper had been received, and the attention that had been paid to it. One of his main purposes, he said, in bringing forward the essay was that it should serve as the introduction to a discussion that might elicit some very useful facts. To that extent the paper had, he thought, been justified, and he heartily thanked all who had participated in the debate. Mr. Tarn thought that the remarks as to what is generally regarded as the select period, *i.e.*, the period of the first five years after assurance, giving about the same rates of mortality as subsequent years of exposure on lives of the same age, was inconsistent with the statement that the extra mortality increased with age and duration. His (Mr. Winter's) meaning was that the difference between Home and Indian rates of mortality would be greater, for example, at age 40 than age 30, but the Indian rate of mortality in the case of European lives would be approximately the same at any specified age for all durations of the insurance. He had been very much interested in Mr. Kenchington's remarks, and the figures he gave in regard to the Widows' Fund of the Church Missionary Society at Tinnevelly, which figures apparently confirmed approximately the conclusions come to in the paper with regard to the experience of the British Empire (natives) in India. The natives forming the Church Missionary Society readers were apparently drawn from a rather different class than assured lives. Mr. Kenchington stated that the

widows are entitled to pensions of 5 rupees per month, or about £4 a year, and that revealed the fact that that class of lives must on an average be less well to do than those who effected insurances with British companies.

Mr. Rusher practically assumed that the class of native lives included in the experience was the very best class, and that was so. One of the most important points in regard to Indian life assurance was to satisfy oneself thoroughly that the best class of native lives was being dealt with. If proper precautions were not taken in that direction, the experience of native assured lives would, he believed, be very far from satisfactory. He quite agreed with Mr. Rusher in the statement that India should be regarded as a continent rather than a country, and that in different parts there would no doubt be very wide differences in the rates of mortality; but, of course, there was not much available information on that point. With regard to the suggestion that possibly 40 per-cent loading on the net extra premium was insufficient, he had to admit that the 40 per-cent was somewhat arbitrary, but he hoped it might prove sufficient, because several of the British offices were charging rates which were lower than those he had given. He had to thank the President for communicating the figures up to date, in connection with the Uncovenanted Service Pension Fund. He had not seen those figures published, and, when in India, he attempted to get possession of the up-to-date mortality figures, as he anticipated these would show much lighter rates of mortality than the earlier ones, but the office was not disposed to divulge the actual facts. He was also very much interested to hear that as long as twelve years ago the President made an assumption that the rate of mortality among the natives of India could be represented by taking the H^M rates of mortality for normal ages, plus five years; and had then added $\frac{1}{4}$ per-cent constant for contingencies. That assumption was very close to the results in the British Empire (native) experience.

LEGAL NOTES.

By ARTHUR RHYS BARRAND, F.I.A., *Barrister-at-Law.*

Rights of a participating policyholder in respect of profits.

THE case of *Baerlein* v. *Dickson* 1909, 25 T.L.R. 585, has attracted considerable attention from all interested in life assurance matters, dealing as it does with the power of the directors of a life assurance company to allocate the surplus shown in the working of the company in accordance with their discretion, even though certain policyholders should suffer loss by reason of the method adopted. The action was

brought against the defendant in his capacity of manager of the Standard Life Assurance Company, and was in reality brought against that company. The facts of the case are as follow:—The plaintiff, in 1889, took out an endowment assurance policy for £10,000 with the defendant company. The policy was expressed to be with participation in profits, and the premium paid was 11*s.* 3*d.* per £100 assured in excess of that for a corresponding policy without profits. The plaintiff paid the premiums due on the policy, and from time to time, up to and including 1900, received certain sums representing his share of the profits. He alleged, however, that from 1900 to 1905 inclusive, the company made large profits, amounting to £827,362, and distributed thereout £100,000 in dividends and bonuses among the shareholders of the company, and also distributed a sum of £50,166 as interim bonus during the period in question in respect of those profit-sharing policies which became claims by death or maturity before 15 November 1905. On this date the company decided not to distribute any of the balance of the profits, amounting to £677,196, among the profit-sharing policyholders, but to apply the amount to the strengthening of the valuation basis, to providing for the earlier payment of claims, and to the readjustment of the value of their securities. The plaintiff alleged that by reason of the profits of the company having been dealt with in this manner, he had derived no benefit in respect of the extra premium paid by him over the period in question, and that all consideration for such extra premiums had therefore totally failed. He therefore claimed (1) a declaration that the payments made to shareholders during the period in question were improper and a breach of contract and that he was entitled to a reasonable proportion thereof; (2) a similar declaration in respect of the transfer of the balance of the profits to the insurance fund of the company; (3) a declaration that the company was not entitled to divide any future profits among the shareholders unless and until a reasonable proportion of such profits was allocated to pay the bonus due to him for the period in question; (4) Payment of a sum representing a reasonable share of the profits made by the company during the said period; (5) Damages for breach of contract, and (6) Return of the excess premium paid over and above that payable for a corresponding non-profit policy.

The defendant company objected that no cause of action was disclosed by the statement of claim. They further alleged that

the distribution of the funds objected to by the plaintiff was made in fulfilment of the obligations of the directors and in the *bonâ fide* exercise of their powers, and was reasonable, proper and necessary, and in the interests of the company, its shareholders and policyholders. They objected that the plaintiff was not entitled in law to question the methods employed by the defendants in their valuations, and they relied upon the statutes incorporating the company.

Section 51 of the Standard Life Assurance Company Act 1832 provides that "It shall be in the power of the ordinary "directors to appoint dividends to be made among the partners "in proportion to their respective shares of the company's stock "from the profits and emoluments of the company; the ordinary "directors being always entitled to lay aside and accumulate "such part of the profits of the company as they shall judge "proper and expedient and to dispose of the same from time to "time as may appear to them best for the advantage and security "of the said company; provided always that it shall be in the "power of the ordinary directors to make such regulations as "they shall think fit for the purpose of allowing persons who "shall effect policies or transact other species of business with "the company to participate in the profits arising from the class "of business in which they may be respectively concerned, and "that to such extent and upon such terms and conditions as "the ordinary directors may from time to time think proper for "encouraging the business of the company."

The case was heard at the Manchester Assizes, and judgment was given in favour of the defendant company. In delivering his judgment Walton, J., said: "In the heading of the policy "these words occur, 'Sum Assured £10,000, payable on "'3 September 1907, or at death if previous thereto. With "'participation in profits.' I have to decide what is the true "construction of these latter words 'With participation in "profits' . . . I think, although it does not say so in words, "that must be read as referring back to section 51. It must, "because . . . section 51 is the only power the directors have to "allow participation in profits. What does that mean? I have "come to the conclusion that it means this: Where a policy-"holder takes a policy without participation in profits he, in "effect, gives up any claim which he might have to the benefit "which would come to him by the directors allowing him, "under section 51, some participation in profits; in fact, he

"agrees with the directors that when they consider this question "of the distribution of profits among policyholders, they may "leave him out, and they may distribute profits among other "policyholders without regard to him . . . In consideration of "that he pays a lower premium than that which is paid by those "who take the ordinary rights, privileges and benefits which "policyholders derive from the fact of their being policyholders. "The others pay a higher premium because they are to be taken "into account when the directors exercise the power which they "have under section 51. I have come to the conclusion that "that is the true meaning of 'With participation in profits' "and 'Without participation in profits' . . . I think, therefore, "that all the plaintiff is entitled to is this, that he was one of "those who were to take the benefit of any distribution, if "distribution there were; but whether there was to be a "distribution seems to me to be a matter entirely within the "discretion of the directors, and I need not say that if they "thought there was to be a distribution, it would be entirely "within their discretion how much should be distributed among "the policyholders. I cannot come to any other conclusion "than that that must be a matter within the discretion of the "directors, to be dealt with by them; but of course to be dealt "with by them in the exercise of their duties and always in "good faith; in other words, honestly. There is no suggestion "here that the directors have not been acting according to the "best of their belief in doing what they thought right and "proper in the exercise of their duties. That being so, I think "this action fails."

When is the contract to surrender a policy complete?

The case of *Ingram-Johnson* v. *Century Insurance Company, Limited* [1909] 46 S.L.R. 746, came before the Scottish Courts recently, and is concerned with an interesting point in connection with the surrender of policies. Here the plaintiff was assured with the defendant company under a sickness and accident policy, of a form known as a Full Return Policy. It contained, *inter alia*, the following clause: "At any time, after five years' premiums have been paid, "this policy may be surrendered for a cash payment which in "no case will be less than one-third of the whole premiums "received, and will increase with the duration of the policy." After five premiums had been paid, the assured wrote to the

office enquiring the amount of the surrender-value, and on being informed as to this, wrote on 12 December 1906 as follows: "I "have decided that I will accept the surrender-value of my Full "Return Policy, and shall be glad to have the money as soon as "possible. If there are any special forms to fill up, kindly "forward them to me." In reply to that communication, he was asked to forward his policy by return, and was informed that a cheque would be sent in the course of a day or two. On December 18 the policy was returned to the assured, with an endorsement for his signature, and he was informed that on receipt of same, duly completed, the amount of the surrender-value would be forwarded to him. The assured did not return the endorsed policy, and nothing was done for two months. He then fell ill and made a claim under the policy, contending that the surrender of the policy was not completed, he having neither signed the form of surrender-receipt nor received the surrender-value. The company resisted the claim on the ground that the policy had been surrendered. The Lord Ordinary found in favour of the plaintiff on this issue, but on appeal this decision was reversed, it being held that the clause in the policy relating to surrender was a standing offer by the company which the plaintiff had accepted, that there was therefore a completed contract between the parties for the surrender of the policy, and that the company was therefore only liable for the surrender-value.

In giving judgment to this effect, the Lord President said: "It seems to me that the letter of 12 December 1906 . . . . "was an acceptance of what I have described as a standing offer "on the part of the company to pay the surrender-value, and "that after that there was no more to be said. It does not "seem to me to make the slightest difference that as a matter of "fact the documents, which it was quite proper to have duly "completed—I mean the documents acknowledging the receipt "of the money, and acknowledging that the policy was no "longer in force—had not been sent. It would seem to me "just as little to the point to say that if anyone made a contract "with his stockbroker for the buying or selling of shares, he "might say that the contract had not been completed because "the transfer had not been signed . . . I think the question "simply comes to this—Was there a concluded contract or was "there not? I think there was a concluded contract, and after "that, the only money which the insurance company were due

"was the surrender-value, which has been tendered in the "present action; and accordingly, I think that is the end of the "matter." Lord Kinnear said: "I quite agree with your "Lordship. I think this is a case of a completed contract "between the parties, concluded by the pursuer's letter of "12 December, and that the policy containing an offer to "accept a surrender on certain terms, and the letter, together, "announcing that the insured wishes to surrender and "demanding the surrender-value, make a completed bargain from "which neither party could afterwards withdraw . . . It does not "affect the completion of a bargain that something remains "to be done in order to carry it out . . . No doubt the pursuer "changed his mind, but then it was too late, for he had made "his bargain."

Effect of incorrect statements in proposals not signed by the assured.

Two cases recently came before the Divisional Court in which the question arose as to the effect of incorrect statements in a proposal when such proposal purported to be signed by the assured, but as a matter of fact was not signed by him or by anyone authorized by him to do so. The cases referred to were those of the *Pearl Life Assurance Company* v. *Johnson* and *The Same Company* v. *Greenhalgh* [1909] 2 K.B. 288, and were heard together, the point involved being practically the same in the two cases. They came before the Divisional Court by way of appeal from decisions given in Courts of Summary Jurisdiction in Yorkshire and Lancashire respectively, under section 7 of the Collecting Societies and Industrial Assurance Companies Act 1896. In the first case, Sarah Ann Johnson, the respondent, insured the life of her husband with the appellant company, the proposal being dated 26 July 1907. It was admitted that material facts as to the health of the life assured were withheld, and the company contested the policy on this ground. It was, however, found as a fact by the magistrate that the proposal form was not signed by the proposer, and that she had no knowledge of its existence; and further, that neither the appellants' agent nor any other person had been authorized by the respondent to sign it on her behalf. He accordingly held that the policy itself alone formed the contract between the parties, and that it should be construed as if the words "Such proposal being the basis of the contract," which it contained, were omitted. He therefore held that the policy was valid, and

that the matter could not be determined by a return of the premiums paid, which had been offered by the company. Similar facts were proved with regard to the second case, except that the incorrect information related to the age of the life assured and not to the state of health, and the dispute was as to the amount payable and not as to the cancelling of the policy by the return of the premiums paid. The justices found that the respondent had not signed the proposal form, and had no knowledge of its existence; that it was not proved that the respondent had incorrectly stated the age of the deceased; that if a mistake was made as to age, such mistake was not induced or contributed to by the respondent; and that the appellants were estopped by the issue of the policy from contending that there was no contract.

The company appealed from these decisions and the two cases were heard together. The Divisional Court dismissed both appeals, and in delivering judgment to this effect, Lord Alverstone, C.J., said "In our opinion the assurance company "are liable for the amount insured on both policies, and both "appeals must be dismissed. In both cases they have issued "policies and received premiums. It is quite true that these "policies purport to be made on the basis of proposals, and "the policy in each case contains a condition that if any "statement in the proposal is untrue, the policy is void and "all premiums are forfeited. But in both cases the Justices "have found that the alleged proposals were not in fact signed "by the proposers, nor had they any knowledge of the contents "or given any authority to make the statements. In our "opinion, therefore, if the assurance company attempt to rely "on breach of the condition they must fail, because no proposal "was in fact made which can be said to be the basis of the "contract; and the company, having issued the policy and "received the premiums, are estopped from saying that there "is no policy, unless they can prove that an untrue proposal "was made by, or with the authority of, the proposer."

ACTUARIAL NOTE.

On formulas for the Force of Mortality. *By* Duncan C. Fraser, M.A., F.I.A., *Actuary to the Royal Insurance Company, Limited.*

(1). Three distinct formulas can be written down for u_n, each beginning with u_0 and ending with a second difference:

$$\left.\begin{aligned} u_n &= u_0 + n\Delta u_0 + (n)_2 . \Delta^2 u_0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (a) \\ u_n &= u_0 + n\Delta u_0 + (n)_2 \Delta^2 u_{-1} \quad . \quad . \quad . \quad . \quad . \quad . \quad (\beta) \\ u_n &= u_0 + n\Delta u_{-1} + (n+1)_2 \Delta^2 u_{-2} \quad . \quad . \quad . \quad (\gamma) \end{aligned}\right\} \quad \text{I}$$

These formulas can be extended to the fourth difference $\Delta^4 u_{-2}$ by adding the following expressions:

$$\left.\begin{aligned} & (n)_3\Delta^3 u_{-1} + (n+1)_4 \Delta^4 u_{-2} \text{ added to} \quad . \quad . \quad (a) \\ & \left.\begin{aligned} & (n+1)_3\Delta^3 u_{-1} + (n+1)_4\Delta^4 u_{-2} \\ \text{or} \quad & (n+1)_3\Delta^3 u_{-2} + (n+2)_4\Delta^4 u_{-2} \end{aligned}\right\} \text{added to} . \quad (\beta) \\ & (n+2)_3\Delta^3 u_{-2} + (n+2)_4\Delta^4 u_{-2} \text{ added to} . \quad (\gamma) \end{aligned}\right\} \quad \text{II}$$

When so extended the three formulas begin with the same term and end with the same difference, and are therefore, as pointed out in the notes on p. 235, identities.

The expressions (II) are consequently the respective errors of the formulas (I) considered with reference to the fourth difference formula ending at $\Delta^4 u_{-2}$.

(2). In the formulas (I), transfer u_0 to the left side, divide by n, and then make n diminish indefinitely, so that in the limit the left side is $\dfrac{du}{dn}$. Then the formulas become

$$\left.\begin{aligned} \frac{du}{dn} &= \Delta u_0 - \frac{1}{2}\Delta^2 u_0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (a) \\ \frac{du}{dn} &= \Delta u_0 - \frac{1}{2}\Delta^2 u_{-1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (\beta) \\ \frac{du}{dn} &= \Delta u_{-1} + \frac{1}{2}\Delta^2 u_{-2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (\gamma) \end{aligned}\right\} \quad \text{III}$$

and the expressions for the error become

$$\left.\begin{array}{ll}
\frac{1}{3}\Delta^3 u_{-1}+\frac{1}{12}\Delta^4 u_{-2} & \ldots\ldots\ (a)\\
\left.\begin{array}{l}-\frac{1}{6}\Delta^3 u_{-1}+\frac{1}{12}\Delta^4 u_{-2}\\ \text{or}\quad -\frac{1}{6}\Delta^3 u_{-2}-\frac{1}{12}\Delta^4 u_{-2}\end{array}\right\} & \ldots\ldots\ (\beta)\\
\frac{1}{3}\Delta^3 u_{-2}-\frac{1}{12}\Delta^4 u_{-2} & \ldots\ldots\ (\gamma)
\end{array}\right\}\ \text{IV}$$

Let $\qquad u_n=-l_{x+n}$

Then $\qquad \frac{du}{dn}=\mu_x.l_x\,;\quad \Delta u_0=d_x\,;\quad \Delta^2 u_0=\Delta d_x=(d_{x+1}-d_x)\,;$

Therefore,

$$\left.\begin{array}{ll}
\mu_x.l_x=d_x-\frac{1}{2}(d_{x+1}-d_x)=\frac{1}{2}(3d_x-d_{x+1}) & \ldots\ (a)\\
\mu_x.l_x=d_x-\frac{1}{2}(d_x-d_{x-1})=\frac{1}{2}(d_x+d_{x-1}) & \ldots\ (\beta)\\
\mu_x.l_x=d_{x-1}+\frac{1}{2}(d_{x-1}-d_{x-2})=\frac{1}{2}(3d_{x-1}-d_{x-2}) & (\gamma)
\end{array}\right\}\ \text{V}$$

with respective errors.

$$\left.\begin{array}{ll}
\frac{1}{3}\Delta^2 d_{x-1}+\frac{1}{12}\Delta^3 d_{x-2} & \ldots\ldots\ (a)\\
\left.\begin{array}{l}-\frac{1}{6}\Delta^2 d_{x-1}+\frac{1}{12}\Delta^3 d_{x-2}\\ \text{or}\quad -\frac{1}{6}\Delta^2 d_{x-2}-\frac{1}{12}\Delta^3 d_{x-2}\end{array}\right\} & \ldots\ldots\ (\beta)\\
\frac{1}{3}\Delta^2 d_{x-2}-\frac{1}{12}\Delta^3 d_{x-2} & \ldots\ldots\ (\gamma)
\end{array}\right\}\ \text{VI}$$

The formulas V can be combined in various ways to diminish the errors shown in VI.

Combination used	Formula for $\mu_x . l_x$		Error
$\frac{a+\beta}{2}$	$d_x - \frac{1}{4}(d_{x+1} - d_{x-1})$	(1).	$\frac{1}{12}(\Delta^2 d_{x-1} + \Delta^3 d_{x-2})$
$\frac{\beta+\gamma}{2}$	$d_{x-1} + \frac{1}{4}(d_x - d_{x-2})$	(2).	$\frac{1}{12}(\Delta^2 d_{x-2} - \Delta^3 d_{x-2})$
$\frac{a+2\beta}{3}$	$\frac{2d_{x-1} + 5d_x - d_{x+1}}{6}$	(3).	$\frac{1}{12}\Delta^3 d_{x-2}$
$\frac{2\beta+\gamma}{3}$	$\frac{2d_x + 5d_{x-1} - d_{x-2}}{6}$	(4).	$-\frac{1}{12}\Delta^3 d_{x-2}$
$\frac{a+4\beta+\gamma}{6}$	$\frac{7(d_x + d_{x-1}) - (d_{x+1} + d_{x-2})}{12}$.	(5).	True to 3rd differences of d_x

VII

In the last of these formulas the error is eliminated; that is, the formula is true to third differences of d or fourth differences of l, and its defect from absolute accuracy can only be expressed in terms of higher differences.

The errors stated against formulas (1) to (4) are the exact differences between those formulas and formula (5).

The most useful formulas for μ are those derived from V (β), and from (1), (3), and (5) in VII. They are

$$\mu_x = \frac{d_x + d_{x-1}}{2l_x} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{A}$$

$$\mu_x = \frac{4d_x - (d_{x+1} - d_{x-1})}{4l_x} = q_x - \frac{1}{4} \cdot \frac{d_{x+1} - d_{x-1}}{l_x} \quad . \quad . \quad . \quad \text{B}$$

$$\mu_x = \frac{2d_{x-1} + 5d_x - d_{x+1}}{6l_x} = q_x - \frac{1}{6} \frac{d_{x+1} + d_x - 2d_{x-1}}{l_x} \quad . \quad . \quad \text{C}$$

$$\mu_x = \frac{7(d_x + d_{x-1}) - (d_{x+1} + d_{x-2})}{12l_x} \quad . \quad . \quad . \quad . \quad . \quad \text{D}$$

A and D are the well-known *Text-Book* formulas. B and C were given by Mr. Spencer, *J.I.A.*, vol. xxxiii, p. 353.

(5). The important formula, D, can be obtained by a more direct process as follows:

Let F_n denote $\frac{l_{-n} - l_n}{2n}$ so that F_0, being the limit to which F_n approaches as n is indefinitely diminished, is equal to $-\frac{dl}{dn}$ or to $\mu . l$.

It will be noticed that

$$F_{-1}=F_1=\frac{l_{-1}-l_1}{2}$$

and
$$F_{-2}=F_2=\frac{l_{-2}-l_2}{4}$$

Take the fourth difference of F_{-2}:

$$\Delta^4F_{-2}=F_2-4F_1+6F_0-4F_{-1}+F_{-2}$$

$$=F_2+F_{-2}-4(F_1+F_{-1})+6F_0$$

$$=\frac{(l_{-2}-l_2)-8(l_{-1}-l_1)}{2}+6\mu.l$$

Therefore, if fourth differences are small enough to be disregarded,

$$\mu.l=\frac{8(l_{-1}-l_1)-(l_{-2}-l_2)}{12}$$

$$=\frac{8(d_{-1}+d_0)-(d_{-2}+d_{-1}+d_0+d_1)}{12}$$

$$=\frac{7(d_{-1}+d_0)-(d_{-2}+d_1)}{12}$$

(6). *Mr. Spencer's Formula.*—Formula B is one of great simplicity and power, and, if taken generally, Mr. Spencer's remark (*J.I.A.*, vol. xxxiii, p. 353) that it "is not quite so simple or accurate (except at high ages) as the well-known formula" (A), does not, I think, do it justice. The error to second differences in (B) is $\frac{1}{12}\frac{\Delta^2 d_{x-2}}{l_x}$, and in (A) is $-\frac{1}{6}\frac{\Delta^2 d_{x-2}}{l_x}$, and we should, therefore, expect to obtain more accurate results from (B) than from (A). Mr. Spencer's remark was no doubt intended to refer to the numerical examples given by him in applying the formula to the *Text-Book* Life Table at ages 30, 40, &c., and it is instructive to trace the cause of its apparent deficiency at the younger ages of this series.

(7). The special convenience of Mr. Spencer's formula lies in the fact that it derives μ from q by a small and easily calculated

correction, but if there is any error in q that error will be repeated in μ. The values of q as given on pages 495 and 497 of the *Text-Book* represent accurately the ratios of the tabular values of d to the tabular values of l, but the values of l having been cut down and otherwise modified to a slight extent, the values of d are affected by small tabular errors, which are sufficient in several instances to influence the values of q if taken to 5 decimal places.

The point can be tested by recalculating the values of q from the values of $\log_{10} p$ derived directly from the formula of the table and given on pages 88 and 89 of the *Text-Book*.

Age x	q_x Values in Life Tables	q_x Recalculated
30	·00771	·007719
40	·01001	·010020
50	·01572	·015734
60	·02983	·029829
70	·06410	·064084
80	·14126	·144280
90	·31579	·315495

It will be observed that the tabular values of q are not strictly true in the 5th decimal place in 6 of the 7 instances here given.

When Mr. Spencer's correction is applied to the true values of q, it gives values of μ true to 5 decimal places at ages 30, 40, 50, and 60, and from this it is clear that his formula can be used with confidence at the younger as well as at the older ages of the table.

(8). The discussion of Mr. Spencer's formula suggests the remark that the values of d and l as given in an ordinary life table do not permit a very precise calculation of μ to be made. The values of d are usually given to 3 or 4 significant figures, the last of which may be in error to the extent of a unit, and this limits the number of significant figures which can be obtained correctly in calculating μ.

(9). The formulas discussed up to this point are derived from the expression $-\frac{1}{l_x}\frac{dl_x}{d_x}$ by stating $-\frac{dl}{d_x}$ in terms of $-\Delta l_x$, or of d_x. A similar series of formulas can be derived from the

expression $-\frac{d\log l_x}{d_x}$ by stating it in terms of $-\Delta \log l$ or of $\operatorname{colog} p$, and, corresponding to the formulas A, B, C, and D, we obtain

$$\mu_x = \frac{1}{2}(\operatorname{col} p_x + \operatorname{col} p_{x-1}) \quad . \quad . \quad . \quad . \quad \mathrm{A}'$$

$$\mu_x = \operatorname{col} p_x - \frac{1}{4}(\operatorname{col} p_{x+1} - \operatorname{col} p_{x-1}) \quad . \quad . \quad . \quad \mathrm{B}'$$

$$\mu_x = \operatorname{col} p_x - \frac{1}{6}(\operatorname{col} p_x + \operatorname{col} p_{x+1} - 2\operatorname{col} p_{x-1}) \quad . \quad . \quad \mathrm{C}'$$

$$\mu_x = \frac{1}{12}\left\{7(\operatorname{col} p_x + \operatorname{col} p_{x-1}) - (\operatorname{col} p_{x+1} + \operatorname{col} p_{x-2})\right\} \quad . \quad \mathrm{D}'$$

These formulas imply the use of logs to the base *e*. When the logs are to the base 10, the expressions for μ must each be multiplied by $\frac{1}{M}$ or 2·30258509.

They are convenient of application because the values of $\operatorname{col}_{10} p$ are nearly always available. It should be particularly noticed that for the greatest accuracy the values of $\operatorname{col}_{10} p$ used in the construction of l should be employed, in preference to the values stated in the life table. Thus, in the O^{M} and $\mathrm{O}^{\mathrm{M}(5)}$ Tables, the construction values of $\operatorname{col}_{10} p$, given on p. 153 of "Principles and Methods", are the values which should be employed.

(10). The numerical examples of the application of the formulas given below are based on the $\mathrm{O}^{\mathrm{M}(5)}$ Table. The values of μ according to that table, calculated from the values of the constants given on p. 149 of "Principles and Methods", are for decennial ages as follows:

Age	μ	Age	μ
20	·0065143	60	·0285965
30	·0074241	70	·0616295
40	·0096574	80	·1427160
50	·0151395	90	·3417597

Values of μ derived by formulas A, B, C, D from the values of d and l stated in the Life Table, and the errors in the values, are given in Table I:

TABLE I.

Age	Approximate Values of μ			
	Formula A	Formula B	Formula C	Formula D
20	·0065099	·0065074	·0065082	·0065074
30	·0074269	·0074296	·0074287	·0074251
40	·0096597	·0096540	·0096559	·0096559
50	·0151466	·0151368	·0151401	·0151401
60	·0285955	·0285915	·0285928	·0285888
70	·0615833	·0616510	·0616284	·0616223
80	·1425214	·1428273	·1427253	·1427307
90	·3436717	·3410088	·3418964	·3417920
Age	Errors in the above Values			
	$(\mu-A)\times 10^7$	$(\mu-B)\times 10^7$	$(\mu-C)\times 10^7$	$(\mu-D)\times 10^7$
20	44	69	61	69
30	− 28	− 55	− 46	− 10
40	− 23	34	15	15
50	− 71	27	− 6	− 6
60	10	50	37	77
70	462	− 215	11	72
80	1946	−1113	− 93	−147
90	−19120	7509	−1367	−323

Values of μ derived by formulas A′, B′, C′, D′ from the values of $\operatorname{col}_{10}p$ given to 7 decimal places on p. 153 of "Principles and Methods", and the errors in the values, are shown in Table II.

(11). If the algebraical expressions which have been given for the errors in the respective formulas be examined, it will be found that the following relation holds between them as far as third differences of d's:

$$(\text{Error in A})-(\text{Error in C})=2\{(\text{Error in C})-(\text{Error in B})\}$$

with a similar relation between the errors in A′, B′, and C′. The errors shown in Table I do not answer this test except at the higher ages, the effect of the tabular errors being greater at the

TABLE II.

Age	Approximate Values of μ			
	Formula A′	Formula B′	Formula C′	Formula D′
20	·0065152	·0065138	·0065143	·0065143
30	·0074262	·0074230	·0074240	·0074241
40	·0096625	·0096545	·0096572	·0096574
50	·0151519	·0151324	·0151389	·0151395
60	·0286269	·0285789	·0285949	·0285963
70	·0617041	·0615864	·0616256	·0616290
80	·1428991	·1426104	·1427066	·1427148
90	·3422094	·3415005	·3417368	·3417571
Age	Errors in the above Values			
	$(\mu-A')\times 10^7$	$(\mu-B')\times 10^7$	$(\mu-C')\times 10^7$	$(\mu-D')\times 10^7$
20	− 9	5	0	0
30	− 21	11	1	0
40	− 51	29	2	0
50	− 124	71	6	0
60	− 304	176	16	2
70	− 746	431	39	5
80	−1831	1056	94	12
90	−4497	2592	229	26

younger ages than the effect of the errors of the formulas. Up to age 60, the simpler formulas A and B give as good results as the more complex formulas C and D; and these results are true to 5 decimal places, the number of significant figures given accurately in the approximate values being equal to the number of significant figures in the values of *d*, from which they are derived.

The errors shown in Table II answer the test referred to above, and the superiority of formula B′ over formula A′ is evident. Formula D′ gives results true to 7 decimal places up to age 60, and at the higher ages results of great accuracy, which are systematically true to one place further than those given by C′.

In order to illustrate the effect of the tabular errors, the values of l have been recalculated for ages 58–62 from the fundamental formula of the Table, the tabular value of l_{60} being

taken as the radix; and values of d to six significant figures have been obtained as under:

Age	d_x Tabular Value	d'_x Recalculated	$d_x - d'_x$
58	1669	1669·06	$-$·06
59	1742	1742·88	$-$·88
60	1819	1818·90	$+$·10
61	1897	1896·60	$+$·40

The values of μ at age 60 derived from the recalculated values are:

Formula	μ	Error $\times 10^7$
A	·0286018	-53
B	·0285950	$+15$
C	·0285973	$-\ 8$
D	·0285966	$-\ 1$

The errors in the approximate values for age 60 given in Table I can therefore be accounted for as follows:

Formula	Error due to use of Tabular Values $\times 10^7$	Error due to Formula $\times 10^7$	Total Error $\times 10^7$
A	63	-53	10
B	35	$+15$	50
C	45	$-\ 8$	37
D	78	$-\ 1$	77

(12). *General Formulas.*—Since $\dfrac{du}{dn}$ is the limit of $\dfrac{u_n - u_0}{n}$ when n is indefinitely diminished, it follows that, corresponding to every formula for u_n which begins with the term u_0, there is a formula for $\dfrac{du}{dn}$ in terms of the differences of u.

The diagram on p. 236, giving the general formulas for u_n, can be easily modified to give formulas for $\dfrac{du}{dn}$, coefficients being inserted according to the following scheme, which shows the

limits of the values of $\frac{(n+k)_r}{n}$, when n is indefinitely diminished, for integral values of k and r, k being less than r.

k	Values of $\frac{(n+k)_r}{n}$ when $n=0$					
	$r=1$	$=2$	$=3$	$=4$	$=5$	$=6$
5	...	..	...	...	...	$\frac{1}{6}$
4	...	...	...	...	$\frac{1}{5}$	...
3	...	..	...	$\frac{1}{4}$	...	$-\frac{1}{30}$
2	...	...	$\frac{1}{3}$	...	$-\frac{1}{20}$	...
1	...	$\frac{1}{2}$	..	$-\frac{1}{12}$	...	$\frac{1}{60}$
0	1	...	$-\frac{1}{6}$	...	$\frac{1}{30}$	...
-1	...	$-\frac{1}{2}$	..	$\frac{1}{12}$	...	$-\frac{1}{60}$
-2	...	..	$\frac{1}{3}$	...	$-\frac{1}{20}$	...
-3	...	...	...	$-\frac{1}{4}$	...	$\frac{1}{30}$
-4	...	...	...	...	$\frac{1}{5}$	...
-5	...	...	...	...	...	$-\frac{1}{6}$

In this scheme each diagonal line of figures running upwards to the right consists of the differences of the line above it. The uppermost line, being the values of $\frac{1}{r}$, can be extended automatically, and the scheme can be completed to any extent required. The formulas already discussed can be very readily obtained by using this scheme of coefficients in conjunction with the diagram on p. 236.

The advantage of following the central line of differences is obvious on account of the small values of the coefficients involved.

(13). *Mr. Lidstone's Formula.*—Mr. Lidstone has given (*J.I.A.*, vol. xxxii, p. 390) an interesting proof of a general

formula which had been previously suggested, but without proof, by Mr. H. N. Sheppard. An alternative proof of a more elementary character can be obtained by following the method of section (5).

Denoting $\dfrac{l_{-n}-l_n}{2n}$ by F_n

$$\Delta^{2n}F_{-n}=(F_n+F_{-n})-2n\{F_{n-1}+F_{-(n-1)}\}+(2n)_2\{F_{n-2}-F_{-(n-2)}\}$$
$$-\ \ldots\ +(-1)^n(2n)_nF_0.$$

Assuming that $\Delta^{2n}F_{-n}$ may be neglected, and writing the series on the right in reverse order, it follows that approximately, since $F_1=F_{-1}$, &c.,

$$(2n)_n.F_0-(2n)_{n-1}.2F_1+(2n)_{n-2}.2F_2-\ \ldots\ +(-1)^n.2F_n=0$$

But $\quad (2n)_n=\dfrac{\underline{|2n}}{(\underline{|n})^2}$; $(2n)_{n-1}=\dfrac{\underline{|2n}}{\underline{|n-1}\ \underline{|n+1}}$; &c., and $F_0=\mu.l$.

Therefore,

$$\mu.l=(\underline{|n})^2\left\{\frac{l_{-1}-l_1}{\underline{|n-1}\ \underline{|n+1}}-\frac{1}{2}\,\frac{l_{-2}-l_2}{\underline{|n-2}\ \underline{|n+2}}+\frac{1}{3}\,\frac{l_{-3}-l_3}{\underline{|n-3}\ \underline{|n+3}}\ \&c.\right\}\quad \text{VIII}$$

(14). The above theorem can be generalized still further.

Let $\qquad F_t^{(2)}$ denote $\dfrac{l_{2t}-2l+l_{-2t}}{(2t)^2}$

$\qquad\quad F_t^{(3)}$,, $\dfrac{l_{3t}-3l_t+3l_{-t}-l_{-3t}}{(2t)^3}$

and so on.

When $t=0$ these expressions become $\dfrac{d^2l}{dt^2}$, $\dfrac{d^3l}{dt^3}$, &c. That this is so is seen most clearly by observing that the above expressions are of the form $\dfrac{\Delta^\kappa}{(2t)^\kappa}\,l_{-\kappa t}$, where Δ means the difference for the interval $2t$. I am indebted to Mr. Lidstone for the remark that the above series of expressions can be stated in this concise form, which, making $t=0$, becomes at once $\dfrac{d^\kappa l}{dt^\kappa}$.

Observing that $F_{t}^{(\kappa)}=F_{-t}^{(\kappa)}$, and following the procedure of the last section,

$$\frac{d^{\kappa}l}{dt^{\kappa}}=2(\underline{|n})^{2}\left\{\frac{F_{1}^{(\kappa)}}{\underline{|n-1}\,\underline{|n+1}}-\frac{F_{2}^{(\kappa)}}{\underline{|n-2}\,\underline{|n+2}}+\&c.\right\} \quad . \quad . \quad . \quad \text{IX}$$

We can thus obtain the values of the successive differential coefficients of l in terms of simple symmetrical functions of l.

(15). If $-\log l$ be substituted for l in the last section, the values of μ and of its successive differential coefficients are obtained in terms of simple symmetrical functions of $\log l$.

(16). *The relation between* μ *and colog* p.—The above notes suggest a close connection between these two functions, and when the table follows Makeham's law the connection can be very simply stated.

With the usual notation, and using λ for $\log_{e}$

$$\mu_{x+a}=\lambda s+c^{x+a}.\lambda c.\lambda g.$$

$$\text{col}\,p_{x}=\lambda s+c^{x}.(c-1).\lambda g.$$

These expressions are identical if $c^{a}.\lambda c=c-1$. Thus, a depends only on c. To find a,

$$c^{a}=\frac{c-1}{\lambda c}$$

Therefore,

$$1+a\lambda c+\frac{a^{2}}{2}\cdot\overline{\lambda c}|^{2}+\ .\ .\ .\ =1+\frac{1}{2}\lambda c+\frac{1}{6}\overline{\lambda c}|^{2}+\&c.$$

Whence,

$$a=\frac{1}{2}+\frac{1}{24}\lambda c,\ \text{approximately},$$

$$=\frac{1}{2}+\frac{1}{24}\frac{\log_{10}c}{M} \qquad M=\cdot 434\ .\ .\ .$$

When $\log_{10}c=\cdot 039$,

$$a-\frac{1}{2}=\frac{\cdot 09}{24}\ \text{of a year = about 33 hours.}$$

$\text{Log}_{10}\,c$ has, I think, never been found to be so great as $\cdot 0434$ at the adult ages. Thus, in tables following Makeham's law $a-\frac{1}{2}$ is never so great as $\frac{1}{240}$ of a year or, say, $1\frac{1}{2}$ days.

In such tables, colp_x is therefore the force of mortality at an age exceeding $x+\frac{1}{2}$ by a constant period depending on c and not exceeding $1\frac{1}{2}$ days. In a table which does not follow Makeham's law, we may assume that successive small sections follow that law, and that for the adult ages of aggregate tables of male lives a relation similar to that stated above exists between μ and col p.

(17). It may be proved similarly that

$$\text{col}\,({}_tp_x)=\mu_{x+a}$$

where a depends on c and t, and is approximately

$$=\frac{1}{2}t+\frac{1}{24}t^2.\frac{\log_{10}c}{M}.$$

When $\log_{10}c=\cdot039$,

$$a=\frac{1}{2}t+\frac{3}{80}t^2=5\cdot375 \text{ when } t=10.$$

THE INSTITUTE OF ACTUARIES.

EXAMINATIONS, April 1909.

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Examination for Admission to the Class of Associate (Part I).

First Paper.

1. Solve the equations

$$(1)\quad \frac{x}{a}+\frac{a}{x}=\frac{y}{b}+\frac{b}{y}=\frac{x}{y}+\frac{y}{x}$$

$$(2)\quad yz=a^2,\ zx=b^2,\ xy=c^2.$$

2. Without assuming the knowledge of any formula, prove that in a quadratic equation, where the coefficient of the first term is unity, (1) the sum of the roots is equal to the coefficient of x with its signs changed, (2) the product of the roots is equal to the third term.

Show that if $a(b-c)x^2+b(c-a)xy+c(a-b)y^2$ be a perfect square, the quantities a, b and c are in harmonical progression.

3. A, B and C start from Cambridge at 3, 4 and 5 o'clock respectively to walk, drive and ride respectively to London. C overtakes B at 7 o'clock and C overtakes A $4\frac{1}{2}$ miles further on at 7.30. When and where will B overtake A?

4. Find the number of homogeneous products of r dimensions that can be formed out of the n letters a, b, c . . and their powers.

Prove that the result can be expressed in the form

$$(-1)^r[{}^{n+r}C_0 - {}^{n+r}C_1 + {}^{n+r}C_2 \ldots + (-1)^r \; {}^{n+r}C_r].$$

5. The streets of a city are arranged like the lines on a chessboard. There are m streets running north and south and n east and west. Find the number of ways in which a man can travel from the N.W. corner to the S.E. corner going the shortest possible distance.

6. Assuming the truth of the exponential theorem, obtain the expansion of $\log_e(1+x)$.

* Find the value of e to four places of decimals and use the tables to prove that $\log_e 2 = \cdot 6931$.

7. If p_1, p_2, p_3 . . . are the respective chances that a number of independent events will separately happen, find the chance that some one at least of the events will happen.

A can hit a target four times in 5 shots; B three times in 4 shots; C twice in 3 shots; and D once in 2 shots. They fire a volley: what is the probability that 2 shots at least hit?

8. From twenty cards, consisting of the ten, knave, queen, king and ace of each suit, four cards are drawn at random. From these four cards two are drawn and found to be hearts. What is the chance that the remaining two are neither of them hearts?

* "A Short Collection of Actuarial Tables" will be supplied for use in answering this question.

Second Paper.

1. If n be a positive integer and $h = \Delta x$, find the value of u_{x+nh} in terms of u_x and its successive differences, without using the method of separation of symbols.

Find the nth term of the series

$$1 + 37 + 61 + 77 + \text{\&c.}$$

2. Given the following table

$$x = 1,\ f(x) = 0$$

$$x = 2,\ f(x) = 1{\cdot}12$$

$$x = 3,\ f(x) = 2{\cdot}87$$

$$x = 5,\ f(x) = 6{\cdot}12$$

find $f(4)$, and then using $f(1)$, $f(2)$, $f(3)$ and $f(4)$, show how to find x when $f(x) = 2{\cdot}7$.

3. If $\Delta v_x = u_x$, obtain an expression for the value of $\Sigma_1^n u_x$.

Apply this method to find the sum of n terms of

(1) A geometrical progression, whose first term is a and common ratio r.

(2) The series whose nth term is $\dfrac{1}{n(n+1)(n+2)(n+3)}$.

4. Find the sum of the following series—

(1) $1 + \dfrac{4}{\underline{|1}} \cdot \dfrac{1}{4} + \dfrac{4\,.\,7}{\underline{|2}} \cdot \dfrac{1}{4^2} + \dfrac{4\,.\,7\,.\,10}{\underline{|3}}\,\dfrac{1}{4^3} + \ldots$ to ∞.

(2) $\dfrac{1}{9\,.\,18} - \dfrac{1\,.\,3}{9\,.\,18\,.\,27} + \dfrac{1\,.\,3\,.\,5}{9\,.\,18\,.\,27\,.\,36} - \ldots$ to ∞.

(3) $5 + 10 + 17 + 28 + 47 + 82 + \ldots$ to 20 terms.

5. If y be a function of x, explain clearly what is meant by the differential coefficient of y with regard to x.

Write down the following differential coefficients—

(1) $\dfrac{d}{dx}(x\,.\log x)$; (2) $\dfrac{d}{dx^2}(a + bx)^n$; (3) $\dfrac{d}{dx}(5 + 4x)^{\log x}$

6. Expand $f(x)$ in a series of ascending powers of x, stating under what conditions this is possible.

Use this expansion to obtain the first three terms of the expansion of $(1+4x+x^2)^7$.

7. Explain what is meant by $\int \phi(x)dx$ and $\int_a^b \phi(x)dx$.

Evaluate the following integrals—

$$(1)\ \int \frac{dx}{x.\log x};\quad (2)\ \int \frac{2x^3+5x+3}{x^2(2x+3)}dx;\quad (3)\ \int_a^b x^3e^x dx.$$

8. Assuming $y_x = a+bx+cx^2$, prove that

$$\int_1^3 y_x dx = 2y_2 + \frac{1}{12}[y_0 - 2y_2 + y_4],$$

and hence show that

$$\int_{-\frac{1}{2}}^{\frac{1}{2}} e^{-\frac{1}{10}x^2}dx = \cdot 9921 \text{ approximately.}$$

EXAMINATION FOR ADMISSION TO THE CLASS OF ASSOCIATE

(PART I) *Third Paper,*
(PART II) *First Paper.*

1. Find the value of an annuity payable m times a year at an effective rate of interest i, and state what tables you would require to enable you to find the numerical value in any particular case.

2. If i and d are corresponding effective rates of interest and discount; j and f the corresponding nominal rates of interest and discount convertible m times a year: and δ, δ' the corresponding forces of interest and discount, prove that

$$(1)\ i-d=id;\quad (2)\ j-f=\frac{jf}{m};\quad (3)\ \delta=\delta'.$$

3. Show that the rate of interest involved in $a_{\overline{n|}}$ when n and $a_{\overline{n|}}$ are known is approximately equal to

$$i + \frac{2h(a_{\overline{n|}}^{(i)} - a_{\overline{n|}})}{a_{\overline{n|}}^{(i-h)} - a_{\overline{n|}}^{(i+h)}},$$

where i is a trial rate near the true rate of interest, h is small in comparison with i, and $a_{\overline{n|}}^{(x)}$ represents the value of the annuity at rate of interest x.

*4. A bond, bearing interest at 6 per-cent payable half-yearly on 1 January and 1 July in each year, and redeemable at par on 1 January 1927, is bought on 13 November 1908 for $105\frac{1}{2}$. Find approximately the nominal rate, convertible half-yearly, which the investment yields.

*5. The debentures of a company, which bear interest at 5 per-cent, payable half-yearly, are redeemable in 10 years' time at 110 and are quoted at a price which yields a nominal rate of 5 per-cent. It is proposed to change these debentures into a perpetual $4\frac{1}{2}$ per-cent debenture stock, with interest payable half-yearly. How much debenture stock in even pounds should, in your opinion, be given for each £100 of the old issue in order that the proposal may be a fair one?

*6. A loan of £1,000 is to be repaid with 5 per-cent interest by a yearly annuity for five years calculated at 3 per-cent. Draw up a schedule showing to one decimal place the amounts of interest and of capital repaid in each year.

7. A loan of £10,000 is to be paid off in 30 years by quinquennial instalments, the first of which is to be made at the end of five years, the second at the end of ten years, and so on. Given that the rate of interest is 4 per-cent, and the annual amount set aside for service of the loan is £583, obtain an equation for finding the rate of interest at which the annual amounts set aside must be accumulated in order that the quinquennial payments may repay the whole loan at the end of 30 years.

8. A table of the values, correct to five places of decimals, of v^n and $s_{\overline{n}|}$, for all integral values of n from 1 to 100 at $3\frac{1}{4}$ per-cent interest, is required. Assuming that a table of logarithms, to as many places of decimals as you may desire, is available, explain clearly how you would employ it to save labour, and show how many places of decimals you would use; what precautions you would take to avoid errors; and what checks you would apply.

Given that $\log 1{\cdot}0325 = {\cdot}01389006$.

Examination for Admission to the Class of Associate (Part II).

Second Paper.

1. Express the value of an endowment assurance policy in commutation symbols, by the retrospective and prospective methods.

* "A Short Collection of Actuarial Tables" will be supplied for use in answering these questions.

Show that

$$A_{x-n:\overline{t-n}|} - P_{\overline{x:t}|}\, \mathbf{a}_{x-n:\overline{t-n}|} = A_{x-n} - P_{\overline{x:t}|}\, \mathbf{a}_{x-n} + (P_{\overline{x:t}|} - P_x)\frac{N_x}{D_{x-n}}$$

and give a verbal interpretation of the right-hand expression.

2. Interpret, and show how to obtain the values of

$$a_{x:\overline{yz}|} \text{ and } a_{xy:\overline{wz}|}.$$

Express A^{3}_{xyz} in terms of single premiums for assurances payable on the first death.

3. Show that $Q^1_{xy} = \mu_x \mathring{e}_{xy} + \frac{1}{4}(\mathring{e}_{x-2:y} - \mathring{e}_{x+2:y})$ approximately.

4. Find, without making any assumption as to distribution of deaths, the value of $\bar{a}_x^{(m)}$.

5. How would you find the annual premium for an assurance payable only in the event of (x) dying within n years, the sum assured commencing at 1, and increasing by k per annum so long as (x) and (y) are jointly alive?

6. Assuming an equal distribution of deaths throughout the year of age, prove that $P_{x:\overline{n}|}^{(m)} = P_{x:\overline{n}|} + \frac{m-1}{2m} P_{x:\overline{n}|}^{(m)}(P^1_{x:\overline{n}|} + d)$, and give a verbal interpretation of the result. Deduce a similar expression for ${}_tP_x^{(m)}$.

7. Find an expression, in terms of the net annual premiums, for the amount of the paid-up policy which can be granted, in respect of an assurance effected n years since; and apply the formula to the case of an assurance payable only if (x) die after (y).

In what cases is the formula not applicable?

8. Find the value of n annual payments of 1 each, payable during the life of (x), the first payment to be made $\frac{1}{t}$th of a year hence.

Third Paper.

1. Assuming an equal distribution of deaths throughout the year of age, deduce the probabilities (1) that (x) will die after (y) within (n) years; (2) that (x) will die in the nth year, and (y) predecease him by at least t years, and (z) survive him by at least t years.

2. Obtain expressions for the average age at death of

(i) l_x persons of the exact age x;

(ii) A stationary population aged x and upwards.

What does $\dfrac{T_x - T_{x+n} - nl_{x+n}}{l_x - l_{x+n}}$ represent?

3. From the formula $\mu_x = -\dfrac{1}{l_x}\cdot\dfrac{dl_x}{dx}$, deduce an expression for μ_x in terms of q_x, ${}_1q_x$, ${}_2q_x$, &c.

Under what conditions would $\mu_x = q_x$?

4. In a certain community, free from immigration and emigration, the birth-rate (*i.e.*, the ratio of births to total population) amongst males, previously stationary, has been increasing, during the last 10 years, at the rate of k per-cent. per annum. On reaching age 20, each male has to give 5 years' military service. How would you proceed to estimate the numerical strength, 12 years hence, of the army, which now musters 500,000 men?

What will be the numbers at the respective ages?

5. Deduce a finite difference formula for interpolation between values of a function of two variables, and show how you could obtain an approximation to the value of a joint-annuity on two lives from two values only of the table on page 27 of "A Short Collection of Actuarial Tables."*

Show how you would apply your method to find the value of $a_{37:33}$ (Carlisle 3 per-cent).

6. Assuming that $\log_{10} l_{[x]+t} = \log_{10} l_{x+t} - f_t - \beta c^x \psi_t$, and that $l_{x+t} = \kappa s^{x+t} g^{c^{x+t}}$, show that the value of a joint annuity on two or more joint lives, $a_{[x]+t:[y]+t\ldots}$ is equivalent to an annuity on a single life aged $[w]+t$, calculated at a different rate of interest, and with varying payments during the period of selection.

7. Show how to obtain, by the use of Gauss's tables, the logarithm of $(a+b)$, the logarithms of a and b being given; and apply the method to construct a table of curtate life annuities. State any objections to this method which occur to you.

8. What method would you adopt to construct a table of policy values for whole-life assurances, with premiums limited to t payments? How would you verify the results?

* A copy of these Tables will be supplied to the Candidate for use in answering this question.

Examination for Admission to the Class of Fellow (Part III).

First Paper.

1. Discuss the arguments for and against the adoption of lives, policies, and amounts assured, respectively, as the basis of a Standard Mortality Table deduced from the experience of Life Assurance Companies. On which of these classes of data were the 17 Offices', H^M, 30 American Offices' and O^M Tables respectively based?

2. It was assumed in the construction of the English Life Table No. 3 that the ratio of the average annual deaths in a group of ages to the mean population of the group was equal to the force of mortality at the central point of age of the group. Criticize this assumption, and explain briefly an alternative process by which you would expect to produce more satisfactory results.

3. In preparing tables of sickness rates with a view to allowing for accurate apportionment of the claims over the customary periods of attack, what information would you ask for in dealing with (*a*) a large number of societies or branches, and (*b*) a single society (without branches)? State in what respect the sickness data of the Manchester Unity Experience, 1866-1870, were incomplete, and show how the defect might have been overcome.

4. Show how the principle of uniform seniority may be utilized in the case of the British Offices' Annuity Tables in deducing annuity-values on two joint lives. (N.B.—It will be sufficient to limit the investigation to the case where one life is a male and the other a female, and to deal with ultimate values only.)

5. State generally what methods of graduation are available in dealing with the following statistics:

(*a*) Mortality tables of the aggregate type;

(*b*) Sickness tables;

(*c*) Secession rates;

(*d*) Population data tabulated for groups of ages; and

(*e*) Marriage rates.

Indicate in each case which method you prefer and on what grounds.

6. Explain the rationale of the method of graduation by reference to a standard table; and mention any instances known to

you in which that method has been employed. Show that the use of a particular summation formula for the purpose of graduation by reference to a given standard table leads to a standard series of corrective quantities applicable to all tables which have been graduated by the formula in question.

7. On closing its books for the quinquennial valuation, a life office finds that some of its Stock Exchange securities stand in the ledger above, and others below, the market prices of the day. How do you think these investments should be valued? Discuss briefly the principles which should guide the Directors.

8. What is the "select and ultimate" method of valuation? What advantages do its advocates claim for it; and what, if any, are the objections to it?

Second Paper.

9. In valuing ordinary whole-life policies by the net premium method, how would you provide for the following conditions?

(*a*) Claims being payable immediately on proof of death and title.

(*b*) There being a large proportion of the policies at half-yearly or quarterly premiums.

(*c*) The business having been placed on the books in unequal proportions in the different months of the year.

Explain briefly the rationale of the methods you suggest.

10. A life office which has hitherto valued its ordinary whole-life policies by the combined H^M and $H^{M(5)}$ 3 per-cent Tables (H^M alone for the first five years) has decided to change its valuation basis, and is considering the desirability of substituting the O^M 3 per-cent Table.

Discuss the question from the point of view of

(1) The magnitude of the reserves:

(2) The probable effect on (*a*) the margin of loading, (*b*) the expected claims, and

(3) The cost of allotting a given rate of bonus.

11. Draft a memorandum of instructions as to the valuation of limited payment whole-life policies by Karup's (Altenburger's) method, giving effect to the actual incidence of the premium income during the year.

12. Describe briefly the method suggested by Mr. Ackland for utilizing the $O^{[M]}$ Table for the purpose of valuing whole-life policies with allowance for selection, giving a clear explanation of the principles involved.

13. A company distributes surpluses as a cash allotment at each valuation of a uniform proportion of the total amount of premiums paid since the last preceding investigation; and, under a discounted bonus scheme, it proposes to allow a reduction of premium in anticipation of bonus. How would you settle the percentage of reduction that could be allowed under the scheme? How would you deal with such policies in the books for the purposes of the annual revenue accounts and balance sheets, and how would you treat them in the periodical valuations?

14. A company distributes surplus quinquennially on the compound reversionary bonus plan, at the same percentage for whole-life and endowment assurances policies, the bonuses vesting immediately when declared. It is proposed to offer an alternative scheme, under which the bonuses are to be deferred for 10, 15, or 20 years, at the option of the assured declared when the policy is effected, the scales of premium being unaltered. How would you provide for these deferred bonuses at the successive distributions, so that they shall be forthcoming at the end of the selected periods, and how would you fix their amounts? Also, how would you deal with the liability of the office in respect of the deferred bonuses at the successive valuations?

15. Discuss the question whether it is desirable to allot the same rate of reversionary bonus to endowment assurances as to whole-life assurances. Mention any investigations into the subject with which you may be familiar, and indicate the conclusions to which they led.

16. In the case of a company declaring its bonuses on the compound reversionary bonus plan, how would you ascertain the percentage of bonus which the divisible surplus would provide, keeping in view policies at half-yearly or quarterly premiums?

What conditions must exist, so that in all reasonable probability the company may maintain the rate of bonus in the future? Give reasons for your answer.

Third Paper.

17. You are asked to quote an annual premium for a temporary assurance of £100 per annum, the first payment of £100 being due on the death of the life assured within 20 years from the date of issue of the policy, and the payments to continue during the residue of that term. What practical difficulty has to be borne in

mind, and how this could be overcome? Give a formula for the office premium which you would charge, and state upon what data you would base it.

18. How would you calculate the office premium uniform throughout life for a whole-life participating policy on the life of a child, the assurance and participation in profits to commence on the next anniversary of the policy after the child attains the age of 21? In the event of the previous death of the child all premiums to be returned without interest, and, in the event of the death of the father, the payment of premiums to be suspended until the child reaches the age of 21.

What return of premium would you make if the child should die before age 21, the father having died previously?

19. A company issues a whole-life participating policy for £1,000 in respect of which the sum assured is to be paid by twenty annual instalments of £50 each, the first falling due on proof of death and title. What annual premium would you charge, assuming interest at 3 per-cent in respect of the deferred instalments of the sum assured, and therefore that $v^{20} = \cdot 55368$; and assuming that the ordinary participating premium is £3. 5*s*. 5*d*. per-cent?

What reversionary bonus should be allotted as compared with the ordinary bonuses of the office

(*a*) If bonuses are to be paid in full at death;

(*b*) If bonuses are to be paid, like the sum assured, by twenty annual instalments?

How would you treat such a policy in the periodical valuations?

*20. A, aged 36, is entitled, contingently on his surviving his mother aged 72, to an undivided one-third share of trust funds, with benefit of survivorship in the event of the death of either or both of his sisters, aged 34 and 30, all of them taking equal shares. The funds are invested as follows:

£10,000 London Brighton & South Coast Railway $4\frac{1}{2}$ per-cent Debenture Stock;

£100 Great Indian Peninsula Railway B Annuities;

Freehold property, which may be taken as readily saleable for £4,000.

You are consulted by an intending lender as to the value of the reversion. What further information would you require, and

* In answering this question the Candidate is to set out his work as in an actual numerical valuation, but to exhibit the final results in terms of the actuarial functions involved, without inserting any numerical values, stating, however, the mortality tables and rates of interest which he would use.

how would you value A's interest? Mention any points to which you would think it well to call the attention of the lender.

*21. A testator left freehold ground rents of £320 per annum (the unexpired term of the lease being 40 years) in trust for his wife, now aged 58, for life, and on her death to be sold and the proceeds divided. The ground rent is secured upon City property of excellent construction, which, it may be assumed, will require no alteration or rebuilding for an indefinite period. The net rack rental may be taken as £2,280 per annum. Competent appraisers have valued the ground rents as being now worth £15,500, and at the end of the 40 years the value of the freehold in possession may be taken at £50,000.

A male aged 32 is entitled to a one-fourth share contingently on his surviving the life tenant.

How would you value the reversion?

*22. There are certain trust funds, well invested, to the amount of £20,000 in present value, and yielding an annual income of £775, and the fund is divisible in April 1927. Two sisters, A now aged 25, and B now aged 21, and their brother C of full age, are interested in the funds. A takes three-fifths of the income during her life to April, 1927, and three-fifths of the capital if she survive; and also the remainder of the income and capital on like conditions if B die. Similarly, B takes two-fifths of the income during her life to April, 1927, and two-fifths of the capital if she survive: and also the remainder of the income and capital if A die. If both A and B die before April 1927, then C takes the whole absolutely. The three beneficiaries have agreed on immediate partition. How would you apportion their respective shares?

23. A reversionary company has from time to time purchased a large number of policies in various offices, some of the policies being non-participating, but the great majority participating, and all being subject to renewal premiums. At a periodical valuation of the assets of the company, how would you value these policies? What in your opinion is the best way of treating the renewal premiums in the books of the company?

24. A life office, doing a high-class ordinary business, finds it advisable to alter its scale of surrender-values. It is suggested that all future surrender-values should be equivalent to the full $4\frac{1}{2}$ per-cent H^M net premium reserves, excluding the first year of the policies' duration. Criticize this proposal, paying special attention to the various classes of policies.

* In answering these questions the Candidate is to set out his work as in an actual numerical valuation, but to exhibit the final results in terms of the actuarial functions involved, without inserting any numerical values, stating, however, the mortality tables and rates of interest which he would use.

Examination for Admission to the Class of Fellow (Part IV).

First Paper.

1. Describe the features of an estate tail, and, in particular, show how such an estate can be barred. What is a *quasi*-entail?

2. What is a power of appointment? What are the main differences between general and special powers? What is an illusory appointment?

3. In conveyances executed since 1881, it is usual for the party conveying, if absolutely entitled to the interest conveyed, to convey as "beneficial owner." What do these words imply?

4. What is a voluntary settlement? In what circumstances can it be set aside (*a*) under the Bankruptcy Act, 1883; (*b*) under the Statute 13 Eliz., c. 5? What is the position of a *boná fide* purchaser for value under either Act?

5. Define novation. In what circumstances is it likely to arise in connection with a life assurance company? Give the provisions of the Life Assurance Companies Act, 1872, dealing with the subject.

6. What has hitherto been the rule as to damages under Lord Campbell's Act (Fatal Accidents Act, 1846) in the case of (*a*) accident policies: (*b*) ordinary whole-life policies? What alteration has been made in this rule by recent legislation?

7. Discuss the question of insurable interest in relation to life assurance contracts. If an assurance is effected without insurable interest, can the person who has paid the premiums recover them back?

8. It is proposed to form a new life assurance company under the Companies Acts, 1862 to 1907. What steps must be taken before the company can commence business?

Second Paper.

9. There is a widows' annuity fund connected with a very large bank, to which every employee of the bank must contribute, whether married or not. What statistics would you require for the valuation of the fund? What formula would you employ for estimating the liability in respect of the annuities, allowing for compulsory withdrawal from membership on leaving the service of the bank before age 65, and for the rule that, after compulsory

retirement on pension at age 65, the member may elect to continue or discontinue his contributions to the fund?

Assume that all the widows' annuities are of equal amount, and do not discuss the question of the contributions.

10. A friendly society is to be established in connection with a large colliery, and membership is to be compulsory. Contributions of present employees are to be regulated by ages now attained, and are to be paid partly by the employees and partly by the proprietors as their liability under the Shop Clubs Act, 1902.

On what data would you base the contributions for the following benefits:

Sickness and accident—

10/- per week for 26 weeks, and 5/- per week for the remainder of incapacity.

Death of member—

£20 if leaving a widow or children under 16 years of age; £5 if single, or a widower without children under 16 years of age.

No data as to past experience can be supplied, but particulars of the ages and family status of existing employees are available.

Would you give effect to the element of withdrawals from the service? Give reasons for your answer.

11. Show how you would proceed if, in valuing a friendly society providing sickness benefits throughout life, you desired to test the effect of various assumptions as to changes in the rate of sickness over particular groups of ages.

12. It is desired to convert a registered friendly society into a company registered under the Companies Acts, 1862 to 1907. What steps must be taken to effect this end? What are the advantages and disadvantages involved in such a change?

13. Prepare a report to a Board of Directors setting forth your views as to the desirability of introducing a scheme of life assurance without medical examination.

14. Draft a suitable form of register for keeping records of the freehold ground rents held as investments by an insurance company.

15. Draft a clause for insertion in a convertible-term policy, giving the assured the option, during a specified period, of enlarging the policy into a whole-life or endowment assurance policy, irrespective of the state of his health, subject to payment of an increased premium corresponding to the age at the date of conversion.

16. Prepare a statement in compliance with the requirements of the 5th Schedule of the Life Assurance Companies Act, 1870, giving information (but excluding the summaries of figures) relating to the valuation of a life assurance company.

Third Paper.

17. Give a specimen of the Bank of England weekly return, and explain its various items.

18. Write a short account of the Bankers' Clearing House system, and explain its connection with the Bank of England.

19. Describe briefly the various items constituting the British National Debt at the present time, and show in what manner its reduction is being effected.

20. Give a short history and description of Local Loans Stock. What method is now adopted in respect of any bad debts?

21. What are leasehold ground rents? Would you advise your company to invest in such securities? Discuss the question generally, and refer specially to the case where your company already holds the freehold ground rent which is amply secured, and is offered an improved temporary ground rent secured on the same property. Give some idea of probable prices.

22. Give a list of the principal securities included in the first group of the Stock Exchange daily official list. What is your opinion as to the advisability of an insurance company investing its funds in any of these? Refer to particular securities.

23. Explain the characteristics of the following securities:

East Indian Railway Annuity, Class A.
" " " " " B.
" " " " " C.
Great Indian Peninsula Railway Annuity, Class B.
" " " " 3 per-cent Guaranteed Stock.

24. Your company is asked to make advances secured by way of first mortgage on:

(*a*) Freehold of newly-constructed flats with shops under.
(*b*) Leasehold of premises let to a well-known bank on a long lease.
(*c*) Freehold of a large factory in the North of England.

Give your opinion of each one of these from a strictly financial point of view.

PROCEEDINGS OF THE INSTITUTE.—Session 1908-1909.

First Ordinary Meeting, 30 *November* 1908.

The first ordinary meeting of the Session 1908-1909 was held at the Hall of the Institute, on the 30th day of November 1908.

The President (Mr. G. F. Hardy) in the Chair.

The President delivered an Inaugural Address.

Second Ordinary Meeting, 14 *December* 1908.

The President (Mr. G. F. Hardy) in the Chair.

A paper entitled "On a new method of Constructing and of Graduating Mortality and other Tables", was read in abstract by the Author. Mr. George King.

The following gentlemen took part in the discussion :—Messrs. A. Levine. C. W. Kenchington, W. A. Workman, W. Palin Elderton, T. G. Ackland, and H. W. Manly. In the course of the debate the Joint Hon. Secretary, Mr. Faulks, introduced and read a communication on the subject of the paper from Dr. J. Buchanan.

Third Ordinary Meeting, 25 *January* 1909

The President (Mr. G. F. Hardy) in the Chair.

Mr. Alexander Smith Sellar, F.F.A., was duly elected an Associate of the Institute.

A paper entitled "On an Approximate Method of Valuation of Whole-Life Assurances, grouped according to attained ages, with allowance for Selection, on the basis of $O^{[M]}$ Mortality", by Mr. E. H. Brown, was read by the Author.

The following gentlemen took part in the discussion :—Messrs. E. C. Thomas, H. J. Rietschel, T. G. Ackland, and the President.

Fourth Ordinary Meeting, 22 *February* 1909.

The President (Mr. G. F. Hardy) in the Chair.

Messrs. William Davidson, F.F.A., Francis Moffat Hope, F.F.A., and Gordon William Thomson, F.F.A., were duly elected Associates of the Institute.

A paper entitled "Some Financial and Statistical considerations of the Old Age Pension Scheme", by Mr. V. Marr, was read by the Author.

The following gentlemen took part in the discussion :—Messrs. O. T. Falk, A. R. Barrand, E. Woods, S. G. Warner, E. A. Rusher, and A. W. Watson. A communication from Mr. J. W. Thomson, on the subject of the paper, was introduced and read by the Joint Hon. Secretary, Mr. J. E. Faulks, in the course of the debate.

Fifth Ordinary Meeting, 29 *March* 1909.

The President (Mr. G. F. Hardy) in the Chair.

Mr. John Howard Robertson, M.A., F.F.A., was duly elected an Associate of the Institute.

A paper entitled "On the Annuity Business of British Offices and the Valuation thereof", was read in abstract by the Author, Mr. H. J. P. Oakley.

The following gentlemen took part in the discussion:—Messrs. W. T. May, A. Hewat, H. J. Rietschel, E. A. Rusher, E. H. Brown, R. R. Tilt, and the President.

Sixth Ordinary Meeting, 26 *April* 1909.

The President (Mr. G. F. Hardy) in the Chair.

A paper entitled "Notes on Mortality and Life Assurance in India", by Mr. A. T. Winter, was read by the Author.

The following gentlemen took part in the discussion:—Messrs. A. W. Tarn, C. W. Kenchington, H. E. W. Lutt, F. J. Vincent, E. A. Rusher, and the President.

The Sixty-second Annual General Meeting, 3 *June* 1909.

The Vice-President (Mr. Thomas G. Ackland) in the Chair.

The proceedings of the Annual General Meeting will be found on page 451.

REPORT, 1908–1909.

The Council have the pleasure to report to the Members upon the progress of the Institute during the Session of 1908-1909, the sixty-first year of its existence.

There has been a *decrease* of 11 in the number of members, as compared with the previous year. At the end of the official year in which the Institute was incorporated by the Royal Charter the number of Members was 434, while ten years later, at 31 March 1895, it was 775. Since that time the numbers have been as follows:

On 31 March	1896,	788.
,,	1897,	826,
,,	1898,	860,
,,	1899,	834,
,,	1900,	822,
,,	1901,	818,
,,	1902,	842,
,,	1903,	828,
,,	1904,	856,
,,	1905,	881,
,,	1906,	922,
,,	1907,	956,
,,	1908,	1,009.
,,	1909,	998.

The following schedule shows the additions, changes, and losses in the membership, which have occurred during the year ending 31 March last:

Schedule of Membership, 31 *March* 1909.

	Honorary Members	Fellows	Associates	Students	Corresponding Members	Total
i. Number of Members in each class on 31 March 1908 .	1	253	313	421	21	1,009
ii. Withdrawals by						
(1) Death . . .	1	4	2	1	2	79
(2) Resignation or otherwise . .	...	2	18	49	...	
	...	247	293	371	19	930
iii. Additions to Membership						
(1) By Election . .	...	...	6	...	...	68
(2) By Order of Council	...	...	...	50	...	
(3) By Re-instatement	...	1	4	7	...	
	...	248	303	428	19	998
iv. Transfers						
(1) By Examination:						
from Associates	...	...	5	...	...	...
to Fellows .	...	5	...	...	...	...
	...	253	298	428	19	998
(2) By Examination:						
from Students	...	...	...	1	...	...
to Fellows .	...	1	...	...	...	...
	...	254	298	427	19	998
(3) By Examination:						
from Students	...	...	...	27	...	...
to Associates .	...	...	27	...	...	...
v. Number of Members in each class on 31 March 1909 .	...	254	325	400	19	998

There are also 128 candidates admitted as Probationers, and 39 as Students conditionally on their passing Part I of the Examination. These are not included in the above Schedule of Membership.

The Council have, with great regret, to report the loss by death, since the last Annual Meeting, of the Honorary Member, Lt.-Col. W. H. Oakes; two Corresponding Members, MM. L. Duboisdenghien and H. Laurent; six Fellows, Messrs. J. B. Cherriman, M.A., A. G. D. Court, Griffith Davies, F. Hendriks, J. Holliday, M.A., and Col. J. M. Templeton, C.M.G.; one Associate, Mr. I. C. Pierson; and one Student, Mr. A. M. Sprules.

The Annual Subscriptions, together with admission and other fees (including class fees for Parts I and II), amounted to £2,394. 0*s*. 0*d*., as compared with £2,481. 13*s*. 6*d*. received in the previous year. The total Income for the year was £3,264. 17*s*. 10*d*., and the total Expenditure £2,491. 11*s*. 8*d*. The Revenue Account and Balance Sheet are given herewith (pp. 444-5).

[*Continued on page* 446.]

Dr. *Revenue Account for the*

	£	s.	d.	£	s.	d.
Amount of Funds at the beginning of the year, made up as under—						
General Fund	8,108	1	4			
Messenger Legacy Fund	388	10	8			
Brown Prize Fund	285	14	7			
Subscriptions—				8,782	6	7
Fellows	730	16	0			
Associates	647	17	0			
Students	421	1	0			
Probationers	75	1	6			
	1,874	15	6			
Fines on Re-instatement	3	13	6			
Application Fees—				1,878	9	0
Associates	10	10	0			
Students	33	1	6			
Probationers	29	18	6			
				73	10	0
Examination Fees for year 1908				292	19	0
Class Fees for Parts I and II				149	2	0
Sales of Publications—						
Journal	208	18	7			
Text-Book, Part I	55	13	2			
Text-Book, Part II	142	18	11			
Government Annuity Tables	1	0	2			
Select Life Tables	2	2	2			
Frequency-Curves and Correlation	17	11	4			
Short Collection of Actuarial Tables	9	11	2			
Hardy's Friendly Societies	1	9	10			
Legal, Financial, and Statistical Lectures	12	0	8			
Transactions of Second International Congress	12	0	0			
British Offices Valuation Tables	56	14	0			
Barrand's Paper on Life Assurance Law	2	2	5			
Examination Questions	6	6	8			
British Offices Life Tables	48	10	7			
Dividends and Interest—				576	19	8
General Fund	273	13	8			
Messenger Legacy Fund	11	13	1			
Brown Prize Fund	8	11	5			
				293	18	2
Refunded by the Faculty of Actuaries, being one-half share of cost of "Synopsis of British Offices Life Tables"				16	14	3
				£12,063	18	8

Balance Sheet.

LIABILITIES.

	£	s.	d.	£	s.	d.	£	s.	d.
General Fund				8,861	3	0			
Messenger Legacy Fund	233	9	2						
Accumulated Dividends	166	14	7						
				400	3	9			
Brown Prize Fund	200	0	0						
Accumulated Dividends	94	6	0						
				294	6	0			
							9,555	12	9
Examination Fees for year 1909							113	8	0
Sundry unpaid accounts							48	15	7
							£9,717	16	4

year ending 31 *March* 1909. **Cr.**

	£	s.	d.	£	s.	d.
Journal—						
Printing of Nos. 232, 233, 234, and 235	558	7	0			
Clerical assistance	86	5	0			
				644	12	0
Library—Binding, Purchases, and Index Cards				78	3	7
Publications' Account—Binding, &c.				38	10	10
Meetings				54	13	11
Examination charges				92	0	2
Tutors for classes in Parts I and II				233	2	0
Office Expenditure—						
Rent	600	0	0			
Salaries	308	15	4			
House expenses	81	9	3			
Corporation Duty	13	16	5			
Fire and other Insurances	31	12	9			
Stationery and Printing	135	0	11			
Furniture and Fittings	10	3	9			
Postage and Telegrams	50	2	3			
Sundries	3	3	9			
				1,234	4	5
Mr. Burn's Lectures on Finance				99	10	6
Cost of "Synopsis of British Offices Life Tables"				33	8	6
Amount of Funds at the end of the year, as per Balance Sheet				9,555	12	9
				£12,063	18	8

Examined and found correct, 28 *April* 1909.

STANLEY HAZELL,
ALBERT G. SCOTT,
HAROLD DOUGHARTY, } *Auditors.*

31 *March* 1909.

ASSETS.

	£	s.	d.
£3,000 Natal 3 per-cent Inscribed Stock	2,460	0	0
£1,200 Metropolitan Railway 3½ per-cent Debenture Stock	1,128	0	0
£2,000 Great Eastern Railway 4 per-cent Debenture Stock	2,279	11	3
£1,000 Great Northern Railway Preferred Ordinary Stock	965	0	0
£1,350 Great Western Railway 4½ per-cent Debenture Stock	1,660	10	0
Cash on Deposit Account	500	0	0
Cash on Current Account	724	15	1
	£9,717	16	4

[The Institute also possesses certain copyrights and stocks of publications (see p. 446).]

Examined and found correct, 28 *April* 1909.

STANLEY HAZELL,
ALBERT G. SCOTT,
HAROLD DOUGHARTY, } *Auditors.*

The stock in hand of the Institute publications on 31 March was as follows:

No. of Copies	Description of Work
17,030	Parts of *Journal*.
786	Index to Vols. 1 to 40.
409	*Text-Book*, Part I (New Edition).
796	,, Part II (Second Edition).
665	Government Joint-Life Annuity Tables.
752	Select Life Tables.
96	A Short Collection of Actuarial Tables.
1,589	Frequency-Curves and Correlation (W. P. Elderton).
151	Messenger Prize Essay (Friendly Societies).
36 *in cloth* / 2,660 *in paper*	Lectures on Finance and Law (Clare and Wood Hill).
1,584	Lectures on the Companies Acts (A. C. Clauson).
1,418	Lectures on the Law of Mortgage (W. G. Hayter).
778	Lectures on the Measurement of Groups and Series (A. L. Bowley).
340	South African War Mortality (F. Schooling and E. A. Rusher).
385	Barrand's Paper on Life Assurance Law.
1,768	British Offices Valuation Tables.
676	Transactions of the Second International Congress of Actuaries.
1,092	Examination Questions, 1904–8.

The following papers were submitted at the sessional meetings of the Institute, namely:

30 *November* 1908.—Inaugural Address by the President, Mr. G. F. Hardy.

14 *December* 1908.—"On a New Method of Constructing and of Graduating Mortality and other Tables."—Mr. George King.

25 *January* 1909.—"On an Approximate Method of Valuation of Whole-Life Assurances, grouped according to attained Ages, with allowance for Selection, on the basis of $O^{[M]}$ Mortality."—Mr. E. H. Brown.

22 *February* 1909.—"Some Financial and Statistical considerations of the Old Age Pension Scheme."—Mr. Vyvyan Marr.

29 *March* 1909.—"On the Annuity Business of British Offices and the Valuation thereof."—Mr. H. J. P. Oakley.

26 *April* 1909.—"Notes on Mortality and Life Assurance in India."—Mr. A. T. Winter.

For the Examinations held in the United Kingdom and the Colonies on 19, 20, 21, 22, 23 and 24 April 1909, 318 entries were received, namely:

49	for Part	I.
74	,, ,,	I. (§) 3.
123	,, ,,	II.
55	,, ,,	III.
17	,, ,,	IV.

The results will be duly announced. The Examinations have on this occasion been conducted by the Board of Examiners recently constituted by the Council. The Council warmly acknowledge the valuable services of the Board, and also those of the Honorary Supervisors at centres other than London.

In their Report of May 1908, the Council referred to the then impending issue of a new Syllabus of the Institute Examinations. The Syllabus, with the necessary explanatory papers, was issued in June last, and the examination in Part I, in April 1909, was conducted under the new Syllabus, which will come into force as regards the other Parts of the Examination in 1910.

During the session there has been published, on behalf of the Institute jointly with the Faculty of Actuaries in Scotland, a "Synopsis of British Offices Life Tables (1893)," which it is hoped will facilitate reference to the various tables based on the British Offices Experience. Having regard to the increase in the size of recent numbers of the *Journal* and the consequent increase in cost, the Council decided, as from January 1909, to increase the price of each number to 2*s*. for Members of the Institute and 3*s*. for non-members.

A series of 12 Lectures on financial subjects has been delivered during 1908-9, under the auspices of the Institute, by Mr. Joseph Burn, F.I.A. The lectures have been well attended and, it is believed, generally appreciated by Members of the Institute. A reprint of the lectures, with some additions, will shortly be published, and the volume should prove of great assistance to students and to the profession generally. The Council desire to take this opportunity of expressing their appreciation of the admirable manner in which Mr. Burn performed the duty entrusted to him. In this connection, it may be mentioned that during the session several books on Finance and kindred subjects have been added to the Institute Library.

Shortly after the last Annual General Meeting, the Council were gratified to receive a request from the City of London College, that a Member of the Council should be nominated to serve on the governing body of the College as representing the Institute. The nomination was accepted by the President.

The Council have appointed as official delegates on behalf of the Institute to the Sixth International Congress of Actuaries, to be held in Vienna in June next, the President of the Institute (Mr. G. F. Hardy), the Honorary Treasurer (Mr. Ernest Woods), and Mr. W. P. Phelps, one of the Honorary Secretaries.

EXAMINATIONS, 1909.

Examinations were held on the 19th, 20th, 21st, 22nd, 23rd and 24th of April 1909, in the United Kingdom, the Colonies, and India, at London, Liverpool, Edinburgh, Dublin, Adelaide, Melbourne, Sydney, Montreal, Toronto, Ottawa, Winnipeg, Calcutta, with the following results.

As regards Part I. the successful candidates are placed in two classes only, while in Parts II, III and IV they are placed in three classes; the names being printed in alphabetical order in each class.

PART I (1908 *Syllabus*).

Forty-nine candidates sent in their names, of whom forty-two presented themselves (thirty-five in the United Kingdom, and seven in the Colonies), and four passed, namely:—

Class I:

None.

Class II:

Cook, H. M.
Golden, C. A.
Hurley, J. C.
Rossetti, G. A. M.

PART II (1906 *Syllabus*).

One hundred and twenty-three candidates sent in their names, of whom one hundred and seventeen presented themselves (ninety in the United Kingdom, and twenty-seven in the Colonies), and thirty-two passed, namely:—

Class I:

King, A. E.
Wolfenden, H. H.

Class II:

Bullwinkle, L. A.
Cowdy, H. L.
Epps, G. S. W.
Harley, B.
Harrington, E. W.
Holgate, B.
Lafford, H. G.
Marples, P. M.
Mol, W. J. B.
Moore, H. F.
Nathan, E. B.
Rowland, S. J.
Savory, D. S.
Sen, J. C.
Sharp, H. G.
Stocks, J.
Watson, A. D.
Woodall, E. A.

Class III:

Baker, S. H.
Doyle, J. P.
Gilliland, W. H.
Hall, A. F.
Harvey, P. N.
Hustwitt, W. E.
Lohan, J. J.
Pollard, E. C.
Strong, W. B.
Wenn, A. E.
Wenyon, H. J.
White, W. C.

PART III (1906 *Syllabus*).

Fifty-five candidates sent in their names, of whom fifty-two presented themselves (forty-four in the United Kingdom, and eight in the Colonies), and eighteen passed, namely:—

Class I:

None.

Class II:

Farmer, E. C.
Fielder, W. C.
Sturt, H. R.
Vaughan, H.

Class III:

Ball, S. R.
Burrows, V. A.
Cameron, F. J.
Carter, G. S.
†Crump, P. C.
Edwards, H. H.
Gunningham, S. J.
Leigh, S. G.
Levey, R.
Makepeace, F. L.
†Maudling, R. G.
Nicholl, C. C.
Reeve, G. M.
Thomson, F. R. T.

PART IV (1906 *Syllabus*).

Seventeen candidates sent in their names, of whom fifteen presented themselves (thirteen in the United Kingdom, and two in the Colonies), and twelve passed, namely :—

Class I:

†Langstaff, J. M.

Class II:

†Atkins, L. G.
†Ellis, R. G. G.
†File, L. K.
†Hallett, W. S.
†Hancock, E. J.
†Maudling, R. G.

Class III:

†Ashton, W. R.
†Downes, E. G.
†Goodman, G.
†Raynes, H. E.
†Turner, S.

PART I, § 3 (1908 *Syllabus*) and PART II, § 1 (1906 *Syllabus*).

COMPOUND INTEREST AND ANNUITIES.

Seventy-four candidates, who had already passed, or been exempted from, Part I of a Syllabus prior to 1908, entered for this section alone, of whom seventy-two presented themselves (sixty-six in the United Kingdom, and six in the Colonies), and thirty-four passed, whose names are printed, in Roman type, in the following list. Of the one hundred and seventeen candidates who presented themselves for Part II (1906 *Syllabus*),

† Those marked (†) have now completed the Examination for the Class of Fellow.

forty-four passed in the paper on Compound Interest and Annuities only, whose names are printed, in *italics*, in the following list:—

Allen, S.
Askwith, T. N.
Bazell, H.
Blake, L. S.
Bowles, F. M.
Brenton, W. P.
Brown, B. G. H.
Brown, P. G.
Cammack, E. E.
Capon, G. W.
Carey, N. L.
Carpmael, C.
Chandler, F. J.
Chandler, F. P.
Chase, H. P.
Clarke, H. T.
Cooper, J. L.
Drake, C. C. H.
Eames, G. S.
Edwards, H. A.
Emmerson, W. H. R.
Fairlie, J.
Fielder, T. L.
Forbes, J.
Foster, W. J.
Frost, C. F.
Gawler, O.
Goodall, E. V.
Hamley, E. F.
Hammant, F. C.
Handford, J. J. W.
Hawes, E. E.
Hodge, C. W.
Humphreys, H. L.
Johns, A. H.
Johnston, A. E.
Johnstone, W. D.
Jones, R. McN.
Keable, H. B.
Keevil, N. A. C.
Kime, V. M.
Ledger, R. J.
LeRossignol, L. F.
Lever, E. H.
McCulloch, J. A.
MacTavish, A. N.
Manly, G. W.
Marlin, J. H.
Marshall, A. W.
Martin, F. C.
Monilaws, S. H.
Morton, F.
Morton, F. W.
Needell, B.
Newland, E. A.
Phillips, E. W.
Preston, J. E.
Priestman, B.
Robinson, A.
Ruddle, F.
Sanders, B. G. T.
Shurrock, C. W.
Sloan, J. J. E.
Smith, F. J.
Spiegel, E. W. R.
Stephenson, H. R.
Sturgeon, R. W.
Stutfield, M.
Tayler, H. H.
Tutill, H. L.
Van Homrigh, G. M.
Vineberg, H. E.
Warner, A. J.
Welsh, W.
Wood, R. S.
Woodhouse, D. A.
Wright, A. W.
Yeldham, W. J.

By Order of the Council,

THOMAS G. ACKLAND,
Chairman of Board of Examiners.

J. E. FAULKS,

W. P. PHELPS,
Joint Honorary Secretaries.

PROCEEDINGS AT THE ANNUAL GENERAL MEETING.

The Sixty-second Annual General Meeting of the Institute of Actuaries was held at Staple Inn Hall, Holborn, on Monday evening, 3 June 1909, Mr. Thomas Gans Ackland, in the Chair.

The Report of the Council (given on p. 442) having been taken as read,

The CHAIRMAN, in moving the adoption of the report, said his first duty was to mention that their President, Mr. George Francis Hardy, was not able to be present that evening. He was necessarily desirous of attending the Congress in Vienna, and had had to set out, unfortunately, before the annual meeting, which long since had been fixed on the very eve of the day on which those who were visiting the Congress had to start, in order to get there at the last moment. Mr. Hardy was desirous of arriving at Vienna in ample time to have some rest before the meeting, and he regretted and apologised for not being present. Therefore he (Mr. Ackland), as senior Vice-President, had to take his place, and to crave the indulgence of the members while he submitted to them some remarks in moving the adoption of the report. The report was for the sixty-first year of the history of the Institute, which had, therefore, entered on the seventh decade of its existence.

With regard to the membership, it would be noticed that, for the first time for several years, there was a very slight diminution in the number of members. Last year the number was 1,009, for the first time touching the figure of a thousand; this year the number was 998. Examining the analytical statement on the second page of the report, it would be found that the small diminution arose almost entirely from the reduction in the number of students, an unduly large number having resigned during the year, whilst on the other hand those elected by the Council had not reached quite the usual figure. The result was a somewhat material drop of about 5 per-cent in the number of students, and he thought that, both in respect of resignations and in respect of reduced elections, the new syllabus could be looked at as the main reason for the very small reduction in total membership, a reduction which he thought need not cause any anxiety. The figure given was, as stated in the report, exclusive of the Probationers, and of the number of students elected, mainly in the Colonies, conditional on their passing Part I of the Examination.

The report then dealt with the list of deaths, and he was sorry to state that the number of those who had passed away during the year was considerably in excess of the average of past years, and also that the list contained the names of some unusually distinguished and eminent men. Mr. Frederick Hendriks and Mr. Griffith Davies were both of great age when they died. They were both admitted in 1884 under the Charter, not having been previously members of the Institute, but having joined the old Actuaries Club many years ago, and, consequently, not having had direct relations with the work of the Institute prior to the grant of the Royal Charter. Mr. Hendriks was a contributor to the *Journal*, and in the first ten volumes his contributions were fairly numerous, the first being set out on page 1 of volume I, sixty-one years ago, when Mr. Hendriks could not have much more than attained his majority. To show his

continued interest in the *Journal*, it might be news to the members that, so recently as in volume xxxiv, Mr. Hendriks contributed an interesting review, running over three or four pages, on a book published in Holland dealing with actuarial science. Mr. Griffith Davies bore an honoured name. He was a nephew of the actuary of the same name, who wrote a book on annuities many years ago, and whose name was so familiar to many of the members. Colonel Oakes was well known as having computed and published useful tables for the work of the actuary and others, and, of these, "Oakes' Tables on Interest", and "Oakes' Tables on Reciprocals" were on the desk of almost every actuary who was in practical work. He had been the only Honorary Member of the Institute for many years past.

Colonel Templeton, a Fellow of the Institute, was eminent on the practical side of life assurance. He founded, and administered very successfully, a large insurance institution in Australasia, and attained great eminence in that colony. Mr. Israel Pierson was well known, not only as an Associate of the Institute, but as an American actuary of repute, and as largely concerned in the founding and maintenance of the Actuarial Society of America, of which he was ultimately President. Those who attended the Congress in New York would remember his gracious reception, and the able manner in which he conducted the proceedings as President of the Congress in 1903. Mr. J. B. Cherriman, a Fellow of the Institute, was a former Insurance Commissioner for Canada, and had latterly lived a somewhat retired life. Then, coming to the younger men in the long list, Mr. Holliday and Mr. Court were both promising Fellows, who had obtained their diplomas and done useful work in the profession; they were called away just when it seemed they were likely to serve the profession most usefully. Finally, there was Mr. A. M. Sprules, a young man who had qualified in the first examination. He also wished to add to the list one name personally known to himself, Mr. J. T. Smith, of Accrington, who sat for the examination in Part III. Almost immediately afterwards he was seized with a severe illness, and, within a week or a little more, was called home, and, in his case, they had again to regret a promising career suddenly cut short. There were two foreign names, both of some considerable importance; M. Duboisdenghien, of Belgium, who was prominently connected with the Permanent Committee of International Congresses, and M. Laurent, who was well known and eminent as an actuary in France. The Institute deplored the loss of these members, and desired to express its sympathy with those who were left behind sorrowing for them.

Turning now to the practical consideration of the accounts, it was satisfactory to find that the assets had increased by about £700. He would, however, call attention to the fact, as had been done at recent annual meetings, that the revenue account included items on the receipt side which were not always to be depended upon, and which were not, perhaps, strictly items of revenue, such as sales of publications, which amounted in the past year to £576. 19*s*. 8*d*. It was not possible altogether to rely on an increase in the funds, especially in view of the fact that the expenditure for the current year in the matter of examinations would be considerably in excess of that of previous years. The Council were considering the form of the account in order to see whether it would be desirable to submit to the members in future years a statement which would more clearly discriminate between

the items of revenue and of capital, and also bring in such property of the Institute as was indicated under the heading of "Assets." Apart from that, he thought there was reason to be satisfied with the progress of the accounts. There was also a long list of publications, which illustrated sufficiently, without any remarks of his own, the scope of useful work that the Institute was endeavouring to do in the way of publications for the benefit of the profession, and especially for the benefit of their own students and members.

The Papers read during the session had been of singular interest, and covered a very wide field, dealing, as they did, not only with theoretical demonstrations of actuarial science, but also with its practical applications, and with questions of finance and legislation, in interesting papers on Old-Age Pensions and the like. The discussions on those essays had very largely added to the value of the papers themselves.

Turning to the burning question of the Examinations, it would be seen that the entries this year reached the large number of 318, which he thought must be a record, and was certainly the highest for several years past. It would be seen, however, that there was a very considerable falling off in the number of entries in Part I, arising again, he thought, out of the revised syllabus. The results were not yet published, for the simple reason that the Colonial papers had only quite recently come to hand, and those from the farthest regions of Australasia had not even yet been received. No statement would be published until the whole of the papers had been examined, and the results laid before the Board of Examiners. The results had, however, been ascertained for the United Kingdom, and, in order that the candidates should not have to wait unduly, the decisions had been made known to both successful and unsuccessful candidates in this country. It was a matter of common knowledge that the results of Part I in the United Kingdom had been singularly disastrous this year. Those who were able to judge agreed that the papers were at about the usual level, but they included under the revised syllabus matters relating to the calculus, to compound interest, and to annuities-certain, that had not heretofore been included in Part I, and, no doubt, that would account for the fact that an unusually large number failed to pass, out of the 35 candidates in the United Kingdom. They were sorry for these young men, but it was clear that they were not sufficiently prepared in their work, and that they would require at least a further year's study, before they presented themselves again in Part I.

It was perhaps not fitting, having regard to the position he held, that he should say much respecting the Board of Examiners, but he thought that the absent President would concur in the remark that the Council were very hopeful as to the result of the work of the new Board, in arriving at broad views of the whole question, and bringing about greater consistency in the examinations, both as to the papers set and as to the results.

The Lectures given by Mr. Burn had been well attended and much appreciated, and the members were greatly indebted to Mr. Burn, who, in the midst of a busy life, and at some little sacrifice of health, was able to deliver those addresses to the students who eagerly listened to them. They were all looking forward to the publication of the lectures, and it was hoped that they would be produced at an early date. In that connection, he

might be allowed to add that the lectures on Construction and Graduation of Life Tables, by the President, delivered as far back as 1905, were now in an advanced state, and it was hoped that ere long they would be in the hands of the members. The President was doing everything in his power, and the printer either had already received, or would immediately receive, orders for the final production of the work.*

The report referred, in its closing paragraph, to the Congress of Actuaries to be held in Vienna. They were all looking forward to that Congress, and those who were able to attend were anticipating with peculiar interest meetings not only with many of their British friends, but also with honoured actuaries whom they had learnt to know and respect in foreign countries. They were quite sure that their Austro-Hungarian friends would give them as hearty a welcome as had been done on previous occasions in Paris, Brussels, Berlin and New York, and that they would all have an exceedingly happy time, and return profited professionally, and by having made new friendships, as well as by having participated in the many social enjoyments provided.

He wished to refer to one other work that had been done by the Council during the year, in connection with the Assurance Companies Bill. At the time the report was prepared, the Bill was not before the Houses of Parliament, and there was nothing definite upon which to make a statement, but the Council had had under consideration, through their Legislation Committee, the several sections and schedules of this important Bill, and had made, in common with other bodies dealing with the practical and theoretical sides of life assurance, lengthy suggestions to the Board of Trade, which, he thought he might say, had received, and were receiving, most careful consideration. The Bill was read in the House of Lords early in the previous week, and copies could now be obtained, and, no doubt the Council would be taking up the Bill in its proposed form, and making such renewed recommendations to the authorities as might appear to be desirable. He had much pleasure in moving—"That the Annual Report, and Statement of Accounts appended, copies of which have been duly circulated, be received and adopted."

Mr. G. Todd, in seconding the motion, said the Chairman had so amply covered all the ground that the seconding of the motion was reduced to a pure formality. There was, however, one point to which he wished to refer, namely, the penultimate paragraph of the report, which mentioned the recognition of the Institute by the City of London College. He thought that was a matter for congratulation, and that any recognition that the Institute or its members could gain from outside bodies was most satisfactory, in the interest, principally, of the younger members. The profession was now becoming a large body, and the scope for younger members was being somewhat restricted by amalgamation and other causes, and if some outside field could be found in which the services of the young members could be utilized it would be a matter for very considerable congratulation.

The report and statement of accounts were unanimously adopted.

Messrs. Owen Kentish and W. J. Harriss were appointed scrutineers in connection with the ballot for Council and Officers, and the ballot was then opened.

* *See* Review, p. 471, *infra.*

The scrutineers reported that the following list, submitted by the Council, had been voted upon with unanimity:—

President.

GEORGE FRANCIS HARDY.

Vice-Presidents.

GEORGE TODD, M.A.
FRANCIS ERNEST COLENSO, M.A.
SAMUEL GEORGE WARNER.
ERNEST COLQUHOUN.

Council.

THOMAS GANS ACKLAND.
ARTHUR DIGBY BESANT, B.A.
THOMAS G. C. BROWNE.
HENRY COCKBURN.
FRANCIS ERNEST COLENSO, M.A.
ERNEST COLQUHOUN.
*WILLIAM PALIN ELDERTON.
JOSEPH ERNEST FAULKS, B.A.
GEORGE FRANCIS HARDY.
CHARLES DANIEL HIGHAM.
*LEWIS FREDERICK HOVIL.
GEORGE KING.
ABRAHAM LEVINE, M.A.
*GEORGE JAMES LIDSTONE.
HENRY WILLIAM MANLY.
WILLIAM PEYTON PHELPS, M.A.
EDWARD ARTHUR RUSHER.
GERALD HEMMINGTON RYAN.
*FREDERICK SCHOOLING.
JOHN SPENCER.
EDWARD ROBERT STRAKER.
ROBERT RUTHVEN TILT.
GEORGE TODD, M.A.
RALPH TODHUNTER, M.A.
HAROLD MOLTKE TROUNCER, M.A.
SAMUEL GEORGE WARNER.
JAMES DOUGLAS WATSON.
ERNEST WOODS.
FRANK BERTRAND WYATT.
THOMAS EMLEY YOUNG, B.A.

Treasurer.

THOMAS GANS ACKLAND.

Honorary Secretaries.

JOSEPH ERNEST FAULKS, B.A. | WILLIAM PEYTON PHELPS, M.A.

* New Members of the Council.

Mr. McDOUGALD proposed the re-election of Messrs. A. G. Scott and H. Dougharty, and the election of Mr. J. C. Wardrop, as Auditors for the ensuing year.

Mr. SHEARER seconded the motion, which was unanimously carried.

Mr. W. O. NASH proposed a vote of thanks to the President, Vice-Presidents, Council, Officers, Examiners and Honorary Supervisors, for their services during the past year. The most important member of the profession was the President for the time being. He was sure it was a source of congratulation that Mr. Hardy, whose absence was so much regretted that evening, had been called upon to fill the Presidential chair. It had been quite apparent from Mr. Ackland's remarks, what a variety of duties fell on the President and Officers. The duties in connection with the Congress were very extensive, and a great deal of work had been done in the House of Commons, both with reference to the Assurance Companies Bill and the Finance Bill. The Honorary Secretaries had a great deal of routine work. Then there were the Lecturers, and particularly Mr. Burn, who should be thanked for their services. Also the Board of Examiners, who did

a great deal of conscientious labour. Last, but not least, there was the Editor of the *Journal*, who was to be thanked for the good work he had done.

Mr. Molyneux seconded the motion, which was carried with acclamation.

The Chairman, in reply, said he was sure if Mr. Hardy were present he would say how much he was indebted to the members for their cordial vote. Speaking for all those who had served the Institute in the various capacities mentioned, he might say that the work was undoubtedly hard, and those who appeared in any form on the official list were men who laboured with vigour in the different departments they took up. The members were particularly indebted to the Honorary Secretaries for the remarkable amount of work they did for the Institute. Those who merely attended the sessional meetings had little idea of the number of times the Hall was open during the week for various Committees, all of which involved great labour on the part of the Honorary Secretaries. He should like to mention especially the skill and diligence which Mr. Faulks devoted to the work of the Legislation Committee, particularly in preparing, under circumstances of great urgency, a very able Memorandum, dealing with the points to which the Institute desired to call attention, on consideration of the Assurances Companies Bill.

Mr. Hodgson moved, and Mr. Oakley seconded, a vote of thanks to Messrs. Stanley Hazell, A. G. Scott and H. Dougharty for their services as Auditors during the past year.

The Chairman, in putting the vote, said that the auditors undoubtedly gave a great deal of attention to the accounts, which was evidenced by the full and careful report which they presented to the Council.

The resolution was carried unanimously.

Mr. Hazell, replying on behalf of the auditors, said the interest of the work much more than compensated for any time taken in doing it. He felt quite sorry that his term of office had come to an end, but in retiring to the seclusion from which he had been summoned, he felt he should take away with him a better knowledge of the scope of the work of the Institute, and certainly a very pleasant recollection of his official connection with it.

Additions to the Library.

The following works have been added to the Library since the publication of the *Journal* for October 1908:

By whom presented (when not purchased).

Accountants and Auditors, Society of Incorporated
List of Members, &c., 1908–9. *The Society.*

Accountants, Institute of Chartered, in England and Wales.
List of Members, 1909. *The Institute.*

By whom presented (when not purchased).

Actuarial Society of America.

Transactions, 1908–9. — *The Society.*

Containing *inter alia*—

"The Mortality Experience of the Mutual Benefit Life Insurance Company under Extended Insurance", by E. E. Rhodes.

"On a System of Valuation by Movement and Recurrence", by G. Bohlman.

"The Incontestable clause in Life Insurance Policies", by W. M. Strong.

"The Nature of the Contribution Principle involved in insurance contracts", by W. S. Nichols.

"Is Human Life Lengthening"? by J. K. Gore.

"A Note on the Original Contribution Plan", by P. C. H. Papps.

"An Important Factor in the Interest Rate", by W. M. Strong.

"On the Redemption of Bonds by a Special Form of Cumulative Sinking Fund", by D. A. Walker.

Actuaries, Faculty of

Transactions, 1908–9. — *The Faculty.*

Containing *inter alia*—

"The Fundamental Principles of Pension Funds", by J. J. McLauchlan.

"Some Remarks upon the Curves of Policy Values", by R. R. Brodie.

"The eligibility of Aged lives for Assurance", by G. W. Richmond.

Adams (H. C.).

The Science of Finance. An investigation of Public Expenditures and Public Revenues. 8vo. New York, 1908. — *Purchased.*

American Mathematical Society.

Transactions, 1908–9. — *The Society.*

American Statistical Association.

Transactions, 1908–9. — *The Association.*

Andréadès (A.).

History of the Bank of England (1640–1903). 8vo. 1909. — *Purchased.*

Australian Mutual Provident Society.

Sixtieth Annual Report, 1909. — *The Society.*

Austria-Hungary.

Absterbe-Ordnungen aus Beobachtungen an österreichischen und ungarischen Versicherten. 8vo. Wien, 1909. — *Anonymous.*

"Anker" Gesellschaft für Lebens-und Rentenversicherungen. Die Ermittlung einer Sterblichkeits-Tafel. Wien, 1909. — *The Company.*

Bericht der Arbeiter-Unfall-Versicherungs-Anstalt für das Königreich Böhmen, 1907.

Die Privaten Versicherungsunternehmungen in den im Reichsrathe vertretenen Königreichen und Ländern in Jahre 1903, 1904. — *The Austrian Government.*

By whom presented (when not purchased).

Austria-Hungary—(*continued*).

Denkschrift über die Berechnung des durchschnittlichen jährlichen Beitragserfordernisses in der Invaliden- und Altersrentenversicherung. Wien, 1909.	*Ministry of the Interior.*
Versicherungswissenschaftliche Mitteilungen der Mathematisch - Statistische Vereinigung des Österreichisch-ungarischen Verbandes der Privat-Versicherungs-Anstalten, 1909.	*The Society.*

Avebury (Lord).

A Short History of Coins and Currency. Sm. 8vo. 1903.	*Purchased.*

Bachelier (M. L.).

Étude sur les Probabilités des causes. Paris, 1909.	*The Author.*

Bankers.

Journal of the Institute of Bankers. List of Members, 1909. Catalogue of the Library. 3rd Edit. 8vo. 1906.	*The Institute.*

Bastable (C. F.).

Public Finance. 3rd Edit. 8vo. 1903.	*Purchased.*

Belgium.

Bulletin de l'Association des Actuaires Belges.	*The Association.*
Bulletin du Comité central du travail industriel, 1909.	*Le Comité.*
Bulletin du Comité permanent des Congrès internationaux d'Actuaires, 1908.	*Le Comité.*
Bulletin du Syndicat des Compagnies d'Assurances-vie populaires opérent en Belgique, 1906–9.	*Le Syndicat.*
Caisse Générale d'Épargne et de Retraite. Manuel des Sociétés d'habitations ouvrières. Documents, lois, et arrêtés relatifs à l'organisation, et au fonctionnement des sociétés agréés. Avec annexe. Brussels. 1908. Compte Rendu des Opérations et de la Situation de la Caisse Générale d'Epargne et de Retraite, 1908.	*The Belgian Government.*

Besso (M.).

Tavole de Vitalita composte da D. Giussepe Toaldo. Rome, 1909.	*The Author.*

"Biometrika."

Volume VI, Part IV.	*Purchased.*

Containing, *inter alia*—

"Frequency Constants of a Variable $z=f(x_1, x_2)$", by R. Pearl.

"The Correlation between a Variable and the Deviation of a Dependent Variable from its probable value", by J. A. Harris.

Bolt (Dr. J. C.).

Het Engelsche Verzekeringswesen. 1909.	*The Author.*

Borel (E.).

Éléments de la Théorie des Probabilités. 8vo. Paris, 1909.	*The Author.*

Bourne's Insurance Directory, 1909.

(With Supplement).	*Purchased.*

By whom presented (when not purchased).

Branson (G. A. H.).
The Stock Exchange and its Machinery. 8vo. N.D. — *Purchased.*

Brett (Thos.).
The Law of Mutual Life Assurance. 2nd Edit. 8vo. 1879. — *H. Cockburn.*

British Trade Year Book.
Covering the years 1880–1907. By J. Holt Schooling. 8vo. 1908. — *The Author.*

Bullock (C. J.).
Selected readings in Public Finance. 8vo. N.D. — *Purchased.*

Burnside (W. N.) and A. W. Panton.
The Theory of Equations: With an introduction to the theory of Binary Algebraic Forms. 5th Edit. 2 volumes. 8vo. 1904. — *Purchased.*

Canada.
Canadian Annual Financial Review, compiled by W. R. Houston. With Appendix. Ob. 8vo. Toronto, 1908, 1909. — *Purchased.*

Chalmers (Sir M. D.).
The Bills of Exchange Act, 1882. With Explanatory Notes and Index. 10th Edit. 8vo. 1904.
A Digest of the Law of Bills of Exchange, Promissory Notes, Cheques, and Negotiable Securities. 7th Edit. 8vo. 1909. — *Purchased.*

Conant (C. A.).
A History of Modern Banks of Issue, with an account of the economic crises of the present century. 5th Edit. 8vo. 1902. — *Purchased.*

Curtis (C. E.).
The Valuation of Land and Houses. 3rd Edit. 8vo. 1908. — *Purchased.*

Cutforth (A. E.)
Audits. 8vo. 1908. — *Purchased.*

Dawson (M. M.).
Comparative Reserve Tables. 2nd Edit. 8vo. New York. 1908. — *The Author.*

Deutsch (H.).
Arbitrage in Bullion, Coins, Bills, Stocks, Shares and Options, containing a summary of the relations between the London Money Market and the other Money Markets of the World. 8vo. 1904. — *Purchased.*

Denmark.
Beretning fra Forsikringsraadet for aaret, 1907. Copenhagen, 1909. — *Danish Government.*

Deutscher Verein für Versicherungs-Wissenschaft.
Veroffentlichungen, 1908–9.
Zeitschrift für die gesamte Versicherungs-Wissenschaft, 1908–9. — *The Society.*

Dicksee (L. R.).
Advanced Accounting; with an Appendix on the Law relating to Accounts, by J. E. G. DeMontmorency. 3rd Edit. 4to. 1907. — *Purchased.*

	By whom presented (when not purchased).
Duboisdenghien (L.).	
De l'organisation technique et de la comptablité rationnelle des Societes de Secours Mutuels. 8vo. Brussels. 1898.	*E. Olifiers.*
Duguid (C.).	
The Stock Exchange. 2nd Edit. 8vo. 1904.	*Purchased.*
Easton (H. T.).	
Money, Exchange, and Banking, in their practical, theoretical, and legal aspects. 2nd Edit. 8vo. 1907.	*Purchased.*
Economic Society (Royal).	
Journal of the, 1908–9.	*Purchased.*
Fisher (Irving).	
A Brief Introduction to the Infinitesimal Calculus, designed especially to aid in reading Mathematical Economics and Statistics. 3rd Edit. 8vo. New York, 1906. The Nature of Capital and Income. 8vo. New York, 1906. The Rate of Interest, its Nature, Determination and relation to Economic Phenomena. 8vo. New York, 1907.	*Purchased.*
Fisk (Dr. E. L.).	
The force of adverse Selection among Entrants at the extremes of Life (Ages 15 to 25 and 60 or over). New York, 1907.	*The Author.*
Foot (A.).	
The Practice of Insurance against Accidents and Employers' Liability. 2nd Edit. 8vo. 1908.	*The Author.*
France.	
Bulletin de l'Institut des Actuaires Français.	*The Institute.*
Fry (T. H.).	
The Recovery and Adjustment of Income Tax. 8vo. 1905. The Finance Act, 1907, in its relation to Income Tax. 2nd Edit. 8vo. 1909.	*Purchased.*
Germany.	
Sammlung von Versicherungsbedingungen deutscher Versicherungsanstalten. Lebensversicherung.	*Deutscher Verein für Versicherungs-Wissenschaft.*
Berlinischen Lebensversicherungs-Gesellschaft. Sterblichkeits-untersuchungen, 1870–1907. Berlin, 1909.	*The Company.*
Giffen (Sir R.).	
Economic Enquiries and Studies. 2 volumes. 8vo. 1904.	*Purchased.*
Gilbart (J. W.).	
The History, Principles, and Practice of Banking. New edition, revised by E. Sykes. 2 Volumes. 8vo. 1907.	*Purchased.*

By whom presented (when not purchased).

Goschen (Rt. Hon. Viscount.).

The Theory of the Foreign Exchanges. 11th Edit. 8vo. 1908. — *Purchased.*

Hamilton (Sir E. W.).

An Account of the operations under the National Debt Conversion Act, 1888, and the National Debt Redemption Act, 1889. 8vo. 1889. — *Purchased*

Hardy (G. F.).

The Theory of the Construction of Tables of Mortality and of similar statistical tables in use by the Actuary. 8vo. 1909. — *Official.*

Henderson (C. R.).

Industrial Insurance in the United States. 8vo. 1909. — *Purchased.*

Holland.

Archief voor de Verzekerings-Wetenschap. 1908–9. — *The Society.*

Catalogue de la Bibliothèque de l'Utrecht compagnie d'assurances sur la vie. 6th Edit. 2 volumes. 8vo. 1908. — *The Utrecht Company.*

Mededeelingen der Vereeniging voor Levensverzekering, 1908–9. — *The Society.*

Howarth (W.).

Our Banking Clearing System and Clearing Houses. 4th Edit. 8vo. 1907. — *Purchased.*

Insurance and Actuarial Society of Glasgow.

Transactions, Second, Third, Fourth and Fifth Series, 1886–1905. — *The Society.*

Containing, *inter alia*—

"Life Branch Work", by A. H. Morgan.
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By whom presented (when not purchased).

Insurance and Actuarial Society of Glasgow—(*continued*).

Transactions, Second, Third, Fourth and Fifth Series, 1886–1905—(*continued*).

"The Race for Records: Some remarks on competition and its influence on Life Assurance practice", by A. K. Rodger.
"Daily Questions of Legal title", by W. Crosbie.
"Annuities", by A. W. Watson.
"Life Assurance Investments", by N. B. Gunn.
"1801–1901, A Contrast", by D. Paulin.
"(1) Tubercular disease and heredity in relation to Life Assurance. (2) Intermittent albuminuria in relation to Life Assurance", by Dr. J. Finlayson.
"A Life Office Prospectus", by J. Stirling.
"The use of Alcohol as affecting Life Assurance risk", by Dr. N. Carmichael.
"Employers' Liability", by J. R. Gow.

Insurance Register, 1909. — *C. & E. Layton.*

Italy.

Bollettino della Associazione Italiana per l'incremento della scienza degli Attuari, 1908–9. — *The Association.*

Jevons (W. S.).

Investigations in Currency and Finance. New edition, abridged. 8vo. 1909. — *Purchased.*

Johnson (J. F.).

Money and Currency in relation to industry, prices and the rate of interest. 8vo. 1906. — *Purchased.*

Journal of the Federation of Insurance Institutes.

Volume XI. — *The Federation.*

Containing, *inter alia*—

"Double Endowment Assurance", by W. Penman, Jr.
"Fluctuations in Security Values", by W. S. Nicol.
"The Claims department of a Life Office", by G. W. Richmond.
"Tuberculosis and Insurance", by Dr. J. G. Emanuel.
"Valuations under the Employers' Liability Insurance Companies' Act 1907", by R. M. M. Roddick.

Another copy. — *Purchased.*

Knibbs (G. H.)

On the influence of Infantile Mortality on Birthrate. 1908. — *The Author.*

Lawson (W. R.).

American Finance—Domestic. 3rd Edit. 8vo. 1908. — *Purchased.*

Lewis (F. W.).

State Insurance: A social and industrial need. 8vo. 1909. — *Purchased.*

By whom presented (when not purchased).

Life Assurance Medical Officers' Association.

Transactions, 1894-97, 1904-5, and 1906-7. — *Purchased.*

Containing *inter alia*—

"Some medical points of difference between Life Assurance in the United States of America and in England", by Dr. T. G. Lyon.

"The assurance of Impaired Lives, chiefly with reference to special forms of assurance", by Dr. H. Fox.

"Syphilis in relation to Life Assurance", by Dr. T. C. Fox.

"The Registration of Completed Lives by means of Card Series", by Dr. C. E. Hoar.

"On Pleurisy with effusion in relation to Life Assurance", by Dr. H. Mackenzie.

"The influence of Heart Disease on Life Assurance", by Dr. C. T. Williams.

"The Relation of Cancer to Life Assurance", by Dr. J. F. Payne.

"Rupture in relation to Life Assurance", Dr. F. de H. Hall.

"Alcoholic Beverages and Life Assurance. The Comparative Mortality of Abstainers and Non-Abstainers", by Sir T. P. Whittaker.

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"Syphilis in relation to Life Assurance", by Dr. A. Shillitoe.

"The relative mortality of Females, considered in reference to Life Assurance", by Dr. A. L. Galabin.

"Albuminuria in relation to Life Assurance, and especially Haematogenous Albuminuria", by Dr. R. H. Fox.

"The influence of surgical operations upon the expectation of Life", by Dr. P. Gould.

"The Examination of the Urine for purposes of Life Assurance", by Dr. F. J. Wethered.

"Diseases of the upper respiratory passages in relation to Life Assurance", by Dr. W. H. Kelson.

"The occasional examiner", by Dr. O. Ward.

"Statistics relative to the causes of death in the Rock Life Assurance Office", by Dr. de H. Hall.

Life Offices' Association.

Memoranda as to the Meetings of the Standing Committee, 1908-9. — *The Association.*

London Mathematical Society.

Proceedings, 1908-9. — *The Society.*

Lowenfeld (H.).

The investment of Trust Funds in the safest and most productive manner. 8vo. N.D. — *Purchased.*

By whom presented (when not purchased).

Mackenzie (V. St. Clair).

The modern Balance Sheet. An analysis of Company Finance. With historical chapters by Anthony Pulbrook. 8vo. 1908. — *Purchased.*

Macleod (H. D.).

Indian Currency. 2nd Edit. 8vo. 1898. — *Purchased.*

Manes (Dr. A.—Editor).

Versicherungs-Lexikon. Ein Nachschlagewerk für alle Wissengebiete der Privat und der Sozial Versicherung insbesondere in Deutschland, Oesterreich, und der Schweiz. 2 Volumes. 4to. Tübingen, 1909. — *The Publishers.*

Marshall (A.).

Elements of Economics of Industry. 4th Edit. 8vo. 1907.
Principles of Economics. 5th Edit. 8vo. 1907. — *Purchased.*

Marton (E.).

Études sur les Institutions de Prévoyance. Sm. 8vo. Brussels, N.D. — *E. Olifiers.*

Mignon (F. A. C.).

Every Man his own Financier. A practical Handbook. 8vo. N.D. — *Purchased.*

Mill (J. S.).

Principles of Political Economy, with some of their applications to social philosophy. New Edit. 8vo. 1904. — *Purchased.*

Moody (J.).

Analyses of Railroad Investments. La. 8vo. New York, 1909.
Manual of Railroads and Corporation Securities. 1908, 1909. New York. — *Purchased.*

Morris (R. D.).

Life Assurance from the national and personal standpoint. 1909. — *Purchased.*

New South Wales, Insurance Institute of

Transactions, 1907, 1908. — *The Insurance Institute.*

Containing, *inter alia*—
"Idealism in Life Assurance", by B. R. Gelling.
"Proof of Death", by F. T. Bridges.
"Suicide in relation to Life Assurance", by W. J. Bloomfield.

Nicholson (J. S.).

Bankers' Money. A supplement to a treatise on Money. 8vo. 1902.
Elements of Political Economy. 2nd Edit. 8vo. 1906. — *Purchased.*

Palgrave (R. H. I.).

Dictionary of Political Economy. 3 volumes and Appendix. 8vo. 1901-8. — *Purchased.*

By whom presented (when not purchased)

Parliamentary Papers.

Acts of Parliament.

Cardiff Corporation, 1894. Commercial Union Assurance Company Limited, 1908. Companies, 1908. Companies (Consolidation), 1908. Consolidated Fund (Nos. 1 and 2), 1909. Finance, 1908. Friendly Societies, 1908. Great Western Railway (Superannuation Scheme), 1908. Kensington Borough Council (Superannuation), 1907. Madras Railway Annuities, 1908. Married Women's Property, 1908. North British and Mercantile Insurance Company, 1908. Northern Assurance Company, 1908. Old Age Pensions, 1908. Police (Superannuation), 1908. Swansea Corporation, 1902. Tees Conservancy, 1907.	*Purchased.*

Censuses.

1901, Western Australia. 3 volumes. Fol. Perth, 1903-4.	*The Government of W.A.*

Colonies.

Australia, Commonwealth of Official Statistics, 1908–9. Official Year Book, 1909.	*The Commonwealth Govt.*
Canada.	
Proceedings of the Banking and Commerce Committee in regard to the Insurance Bill, 1909. 8vo. Ottawa, 1909.	*T. B. Macaulay.*
Report of the Royal Commission on Life Insurance. 8vo. Ottawa, 1907.	*G. H. Ryan.*
Report of the Superintendent of Insurance for the year 1907.	*The Government Insurance Dept.*
New South Wales.	
Friendly Societies. Report of the Registrar for 1907. First quinquennial Valuation, 1900–1904. Fol. 1908.	*The Government of N.S.W.*
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Official Year Book, 1907-8. Report on the Vital Statistics of Sydney and suburbs for 1908. Statistical Register for 1907 and previous years. Vital Statistics, Report for 1907 and previous years.	*The Government of N.S.W.*
New Zealand.	
Official Year Book, 1908. Statistics of the Dominion for the year 1907.	*The Government of N.Z.*
Victoria. Statistical Register for 1907.	*The Government of Victoria.*

By whom presented (when not purchased).

Parliamentary Papers—(*continued*).

Colonies—(*continued*).

Western Australia.
- Friendly Societies. Report of the Proceedings by the Registrar, 1907.
- Seventh Annual Return and Report of Proceedings under the Industrial Conciliation and Arbitration Act, 1902-1907.
- Statistical Register for 1906 and previous years.

The Government of W. Australia.

Friendly Societies.
- Building Societies.
  - Thirteenth Annual Report of the Chief Registrar for 1907.
- Chief Registrar's Report for 1907. Part A, Part A, Appendix (O), Parts B and C.
- Guide Book of the Registry Office, 1908.

Purchased.

Government Insurance and Annuities.
- Accounts to 31 December 1907 and 31 December 1908.

Life Assurance.
- Report of the Departmental Committee on the Post Office System; with Minutes of Evidence. Fol. 1908.

National Debt.
- Annuities. Account to 5 January 1909.
- Return showing Liabilities and Assets at the close of each Financial year from 1835–36 to 1908–9, both inclusive.
- Savings Banks and Friendly Societies. Account to 20 November 1908.

Old Age Pensions.
- Memorandum on the Scheme in force in New Zealand and the Scheme of Insurance against Invalidity and Old Age in force in Germany. 1908.

Poor Laws and Relief of Distress.
- Report of the Royal Commission on, Fol. 1909.

Public Trustee.
- First General Annual Report, 1909.

Registrar-General. England.
- Seventieth Annual Report of Births, Deaths and Marriages, 1907.

Registrar-General. Ireland.
- Forty-fourth detailed Annual Report of Births, Deaths and Marriages, 1907.

Registrar-General. Scotland.
- Fifty-second detailed Annual Report of Births, Deaths and Marriages. (Abstracts of 1906.)
- Fifty-fourth Annual Report on Births, Deaths and Marriages, 1908, and Forty-fourth Annual Report on Vaccination.

Workmen's Compensation.
- Statistics of Proceedings under the Acts of 1897 and 1900, and the Employers' Liability Act, 1880, during the year 1907.

Purchased.

	By whom presented (when not purchased).
Periodicals.	
Accountants' Magazine. American Bankers' Magazine. Bankers' Magazine. Economist.	*Purchased.*
Insurance Record.	*The Editor.*
Post Magazine. Post Magazine Almanack.	*The Editor.*
Statesman's Year Book.	*Purchased.*
Phelps (E. B.).	
A Statistical Study of Infant Mortality. 8vo New York, 1908.	*The Author.*
Poley (A. P.) and F. H. C. Gould.	
The History, Law and Practice of the Stock Exchange. 8vo. 1907.	*Purchased.*
Porter (J. B.).	
The Laws of Insurance; Fire, Life, Accident and Guarantee. 5th Edit. 8vo. 1908.	*Purchased.*
Price (L. L.).	
Money and its relation to Prices. Being an enquiry into the causes, measurement, and effects of changes in general prices. 8vo. 1900.	*Purchased.*
Quetelet (L. A. J.).	
Théorie des Probabilités. Sm. 8vo. Brussels, 1853.	*E. Olifiers.*
Richmond (G. W.).	
British and Foreign Insurance Methods. Edinb. 1908.	*The Author.*
Ringwood (R.).	
Outlines of Banking Law: with an Appendix containing the Bills of Exchange Act 1882, The Bills of Exchange (Crossed Cheques) Act 1906, The Bankers' Books Evidence Act 1879. 8vo. 1906.	*Purchased.*
Roche (J. F.).	
A method of handling impaired life risks. 1908.	*H. Cockburn.*
Rollins (M.).	
Convertible Securities. Tables showing the prices of the various Convertible issues which are equivalent to the market quotations of the Securities Exchangeable therefor, &c. 8vo. Boston, U.S.A.	*Purchased.*
Root (J. W.).	
British National Finance. 8vo. 1909.	*Purchased.*
Royal Astronomical Society, Memoirs of the	
Volume LVII, Parts 3, 4, and Appendix 2. Volume LVIII, Volume LIX, Parts 1, 2, and 3. 1908–9.	*The Society.*
Scientific and Learned Societies of Great Britain and Ireland.	
Official Year Book, 1908.	*Purchased.*
Secretaries, Chartered Institute of	
List of Members, &c. 1907–8. 8vo.	*The Institute.*
Sheppard (W. F.).	
On the accuracy of Interpolation by Finite Differences. 1906.	*The Author.*

	By whom presented (when not purchased).
Smith (J. W.).	
A Handy Book on the Law of Banker and Customer. 8vo. 1907.	*Purchased.*
Snyder (C.).	
American Railways as Investments. A detailed and comparative analysis of all the leading Railways, from the Investor's point of view; with an introductory chapter on the method of estimating Railway Values. 8vo. New York, 1907.	*Purchased.*
Spicer (E. E.).	
An outline of the Money Market. 8vo. 1908.	*The Author.*
Statistical Society (Royal).	
Journal of the, 1908–9. General Index to Volumes 51 to 71. (1888–1908). 8vo. 1909.	*The Society.*
Stock Exchange Official Intelligence, 1909.	*Purchased.*
Stock Exchange Year Book, 1909.	*Purchased.*
Straker (F.).	
The Money Market. 8vo. 1904.	*Purchased.*
Switzerland.	
Rapport du Bureau Fédéral des Assurances sur les Entreprises Privées en Matière d'Assurances en Suisse, 1907.	*The Swiss Government.*
Tables	
Of Life Annuities at 3½ per-cent, based on Emory McClintock's Annuity Table. Mean and Terminal Values for Single Lives, Male and Female, and for two Joint Lives, distinguished according to sex. Fol. New York, 1903.	*T. G. Ackland, on behalf of the Board of Trade.*
Tate (W.).	
The Modern Cambist. Forming a manual of the Foreign Exchanges. 24th Edit. 8vo. 1908.	*Purchased.*
Templeman (F.).	
Marine Insurance: its principles and practice. 2nd Edit. 8vo. 1909. Two copies.	*Alliance Assur. Company.*
Thiele (T. N.).	
Sur la valeur propre des séries divergentes. 1908.	*The Author.*
Thorpe (C. H.).	
How to invest and how to speculate. Explanatory of the details of Stock Exchange business, and the main classes of Securities dealt in, together with a glossary of terms in common use. 8vo. 1901.	*Purchased.*
Toronto, Insurance Institute of	
Proceedings, 1907–8. Containing, *inter alia*— "Industrial Insurance", by H. P. Evans. "Classification of Risks in Personal Accident Insurance", by J. Lydiatt. "Some Sanitary conditions affecting a Life risk", by Lt.-Col. Dr. W. Nattress.	*The Insurance Institute.*

	By whom presented (when not purchased).
Toronto, Insurance Institute of—(*continued*).	
Proceedings, 1907–8—(*continued*).	
"Moral hazard in insurance; with special reference to Accident Business", by Dr. J. E. Elliott.	
"The Development, Organization and Management of the Agencies of Life Insurance Companies", by R. Junkin.	
"Surrender and Loan Values", by L. K. File.	
Tracts—	
Miscellaneous, principally astronomical and mathematical. Two volumes. 8vo.	*H. Boag.*
Tyler (F. W.).	
Premium and Interest Calculator. (Mechanical.)	*E. A. Tyler.*
United States of America.	
Official Publications.	
Eighth Annual Report of the Bureau of the Census. Mortality Statistics 1907. La. 4to. Washington, 1909.	*The U.S. Government.*
Connecticut. Forty-fourth Annual Report of the Insurance Commissioner, 1908.	*The Commissioner.*
Massachusetts. Fiftieth Annual Report of the Insurance Commissioner, 1905.	*W. Hughes.*
New York State. Forty-seventh, Forty-eighth, Forty-ninth Annual Reports of the Superintendent of Insurance. 1906–7–8. Tables compiled from Annual Statements for 1907, as filed by Life and other Companies. La. 4to. 1908.	*The Superintendent of Insurance.*
Victoria, Insurance Institute of	
Proceedings, 1909.	*The Insurance Institute.*
Violeine (P. A.).	
Tables pour faciliter les calculs des Probabilités sur la vie humaine, telsque rentes viagères, assurances, &c. 4to. Paris, 1859.	*E. Olifiers.*
Wansbrough (T. P.).	
The Permanent Way. (Being a method of Branch Office Organization.) 8vo. 1909.	*The Author.*
Willis (W.).	
The Law of Negotiable Securities. 2nd Edit. 8vo. 1901.	*Purchased.*
Withers (H.).	
The Meaning of Money. 8vo. 1909.	*Purchased.*
Yorkshire, Insurance Institute of	
Proceedings, 1907–8.	*The Insurance Institute.*
Containing, *inter alia*— "The working of the Compensation Act", by H. C. Bowling.	

	By whom presented (when not purchased).
Young (T. E.).	
A plain guide to Investment and Finance. 8vo. 1908.	*Purchased.*
Young (T. E.) and R. Masters.	
Insurance Office Organization, Management and Accounts. 2nd Edit. 8vo. 1908.	*Purchased.*

The following additional copies of works already in the Library have also been added:—

Actuarial Society of America.	
Transactions. Volumes 6, 7, 8, 9, and 10. 1900–1908.	*W. Hughes.*
Actuarial Society of Edinburgh.	
Transactions. Volumes 1 to 4.	*Purchased.*
Nicol (W. S.).	
Forms of Life Office Books, 1897.	*The Author.*
Nicoll (J.).	
A description of certain of the principal Stock Exchange Securities. 1897.	*The Author.*
Rye (R. A.).	
The Libraries of London: a guide for students. 8vo. 1908.	*The University of London.*
United States of America.	
Official Publications.	
New York.	
Acts of the Legislature passed at the Session of 1906, upon the recommendation of the Joint Committee of the Senate and Assembly. 8vo. Albany, 1906.	*The Superintendent of Insurance.*

REVIEW.

The Theory of the Construction of Tables of Mortality and of similar Statistical Tables in use by the Actuary. A Course of Lectures by GEORGE FRANCIS HARDY, F.I.A.

Delivered at the Institute of Actuaries during the Session 1904–5.
London: C. & E. Layton. 1909.

IN the interval between the delivery of these Lectures and their recent appearance in book form, several important additions have been made to graduation literature. Mr. Elderton's treatise on "Frequency Curves", Mr. King's series of papers on "Summation and Osculatory Interpolation Formulas", and Mr. Lidstone's "Notes on the Rationale of the Summation Method", will be at once recalled, and mention may also be made of the contributions to the Berlin Congress Transactions. It would not, therefore, have been very surprising if the lectures had now been found to be to a certain extent out of date. Yet they have, in fact, lost little or nothing of the freshness and interest which impressed those who had the good fortune to hear them, although certain of the processes which then seemed new and somewhat mysterious have since become comparatively familiar. This may be attributed partly to the fact that they give in the main Mr. Hardy's own ideas about graduation, and the results of his original researches, rather than a formal *resumé* of the subject, and partly to that classic quality which renders the best work suggestive and stimulating long after its results have become part of the common stock of scientific knowledge.

Mr. Hardy's outlook on the subject of graduation is at once comprehensive and impartial. His theoretical knowledge and practical experience give him a certain detachment from anything of the nature of partisanship, and it is characteristic that although the modern developments of the Summation method may be traced to a note which fortunately found its way from his pen into vol. xxxii of the *Journal*, he has no special partiality for that method. Mr. Hardy, in fact, is in the enviable position of being able to recognise the merits and possibilities alike of graphic, summation, interpolation and frequency-curve methods. In Lecture II, for example, an interesting method is suggested of deducing a practical summation-formula such that the mean error in Δ^2 or Δ^3 may be a minimum, and some useful remarks will be found in an earlier part of the same lecture on the best ways of applying the graphic method. At the same time Mr. Hardy's opinion as to the relative merits of the various methods is clearly indicated in the statement at the beginning of Lecture III that "when . . . by giving suitable values "to the constants a frequency-curve can be made to fit the "observations within the limits of the errors of observation . . . the "graduated curve thus produced is probably a better representation "of the original than any that would result from a graphic or

"finite difference method of graduation." Two facts, however, should be noted; firstly, that the expression "frequency-curve" is used by Mr. Hardy in the wide sense of "any curve which exhibits the law of variation in a particular function", so that it includes Makeham, and in fact any mathematical formula, and is not restricted to the special family of curves with which it is more particularly associated; and secondly, that no specified frequency-curve, as such, has, in Mr. Hardy's view, any inherent validity or *a priori* claim to represent any given distribution or series of rates, so that its applicability must be judged by the result, that is to say by considering "whether the observed differences between the graduated and ungraduated values fall within what may be properly considered to be the limits of error." This view of the limited function of frequency-curves appears again and again in the course of the lectures, and in fact underlies the entire treatment of the subject. "Systematic differences" (between the observed values and those given by a frequency-curve graduation) "are generally to be expected in dealing with age statistics . . . they are not incompatible with a close agreement in the general features of the two curves, but they serve as a warning that in statistics of this nature formulæ representing the distribution of deviations from the mean must be regarded as approximations only." Again, "Makeham's formula cannot be treated as a 'law of mortality' to which all tables may be expected to conform . . . its suitability must be tested as that of any other frequency curve but with rather more latitude owing to its practical advantages." Even the 'normal curve' "must not be regarded as representing a law of nature, but rather an extremely convenient and often very close approximation to observation"—although, it may be added, Mr. Hardy proceeds to give such a simple and seductive demonstration of the formula that the student may feel some difficulty in recognising that "the formula is empirical and not to be established by *a priori* reasoning."

The general principle indicated in the last paragraph admits of somewhat greater freedom in the determination of constants than would be permissible on the assumption that a given distribution or series of rates must necessarily find its representation in a particular type of frequency-curve. The constants, when found, may be modified to secure a "better general agreement", or for adequate practical reasons (as in the $\mathrm{O}^{[am]}$ Table), or an additional term may be introduced to represent a particular feature of the data (as in the determination of the form of A_t in the $\mathrm{O}^{[\mathrm{M}]}$ Table). It may be noticed that the effect of a small change in the constants can, in many cases, be readily calculated. Thus, in the case of the normal curve, it is proportional to $2x^2ydc/c^3$, or in that of the curve $kx^me^{-x/a}$ to $(mdm/x+xda/a^2)y$, so that, when a rough fit has been obtained, improved values may be determined by one or more summations of the deviations. In this way the difficulties caused by the incompleteness of a distribution, or the absence of high contact, might perhaps be practically dealt with.

Since the graduation of absolute frequencies is of less importance to the actuary than that of rates, Mr. Hardy is inclined to consider the Pearson family of curves as of somewhat restricted application to actuarial statistics. There are, however, many actuarial distributions which can be well represented by some member of the family, and it may occasionally be necessary to graduate such distributions (whether, as in the case of exposures, as a step in the determination of graduated rates, or for other purposes). The indications given by Mr. Hardy of the classes of statistics to which the various types may be found applicable constitute, therefore, a very useful feature of his discussion of these curves in Lecture III. Mr. Hardy practically rules out the types corresponding to a positive finite value of the criterion, on account of the fact that y diminishes with increasing slowness for large values of x. In view of the examples given by Mr. Elderton (one of experiences, one of deaths, and one of entrants), it seems doubtful whether this objection—which would apparently apply to any asymptotic curve—is entirely valid. There can, however, be little question that the limited skew curve is the most generally useful for actuarial distributions, and the simple method given on p. 50 for determining the applicability of this and other K-negative curves by examination of $\Delta^2 \log y$ instead of by calculation of K is most valuable. In reproducing Mr. Henderson's table of frequency-curves, Mr. Hardy has somewhat improved it and has inserted the values of the third and fourth moments, so that it now gives in a concise form all necessary particulars (except the areas of the curves) for purposes of calculation.* The numbering of the types differs from both Mr. Henderson's and Mr. Elderton's, which is not very convenient, but unless an arrangement based strictly on the values of K from $-\infty$ to $+\infty$ be adopted, Mr. Hardy's classification by reference to the character of the curve seems preferable. In connection with the Pearson family, Mr. Hardy mentions several other curves which may be used to represent single series distributions—the symmetrical and skew binomial being particularly interesting, although they seem less adaptable and less easily applicable than the family types which they respectively resemble—but the most important alternative suggestion comes in Lecture VI, where the remarkable adaptability of the curve $\frac{1}{\sqrt{\pi}} e^{-[f(t)]^2} f'(t)$ to a skew distribution is illustrated by graduations of the $\mathrm{O^M}$ data and of an occupation-group from the Census Report. The numerical work in the application of this curve is reduced to comparatively small proportions by the use of the calculated values of $\frac{2}{\sqrt{\pi}} \int_0^z e^{-t^2} dt$, of which a table is appended to the Lectures.

So far as the graduation of rates, as distinguished from distributions, is concerned, the case for frequency-curves rests

* The column giving the distance of the mean from the origin is inaccurately headed "Mean $= \mu_1$."

mainly on Makeham's formula. One other curve mentioned by Mr. Hardy—$ma^x + nb^x$—appears, from graduations of the O^M data and the general population table (pp. 70 and 90), to have considerable merits, and it possesses a uniform seniority property of some interest but not of a sufficiently simple form to enable it to compete seriously with Makeham. In Mr. Hardy's discussion of the Makeham formula, the most interesting point is the investigation of methods of determining the constant c. The plan of calculating c direct from the data by representing E by some practicable frequency-curve and recomputing the deaths is very attractive, and since the object is not to graduate E, but the ratios of θ to E, the fact that it may not be possible to fit E very closely with a suitable curve does not seem a serious objection. Moreover, it is shown, by some very interesting analysis in Note G, that the scope for this method can be extended considerably beyond the normal curve process so successfully exemplified by Mr. Elderton. Mr. Hardy, however, expresses a preference for the determination of c by trial, in which case the other constants may be calculated by the now well-established method of equating the first and second sums of $E_{x+\frac{1}{2}}\mu_{x+\frac{1}{2}}$ and the observed deaths. Apparently Mr. Hardy considers that the error due to the assumption that $\mu_{x+\frac{1}{2}} = m_x$ may, in most cases, be neglected. Since, however (as is pointed out in a footnote), colog ${}_e p_x$ is very nearly equal to

$$\theta_x/(E_x - \frac{1}{2}\theta_x - \frac{1}{12}m_x\theta_x)\text{—or alternatively to } m_x(1 + \frac{1}{12}m^2{}_x)\text{—}$$

it seems that better values of the constants might be obtained, without the trouble of recomputation (as in the $O^{[NM]}$ Table), by equating $E_{x+\frac{1}{2}}$ colog ${}_e p_x$, *i.e.*, $E_{x+\frac{1}{2}}(a + \beta c^x)/M$, and $\theta_x(1 + \frac{1}{12}m^2{}_x)$, where the values of m could be taken, either from a preliminary simple adjustment of the data, or from an existing table showing similar mortality.

The application of Makeham's formula to Select Tables is illustrated in Lecture V by reference to the $O^{[M]}$ Table, but, unfortunately, the process by which the expression for $\log l_{[x]+t}$ was arrived at, although explained in rather more detail than in "Principles and Methods", is rendered somewhat obscure by one or two apparent discrepancies on p. 77.* The general method of application might perhaps be more instructively illustrated by the graduation of the $O^{[NM]}$ Table, where the nature of the selection rendered it possible to employ simpler and more normal expressions for $f(t)$ and $\psi(t)$. It may be mentioned that, although the function colog p_x is convenient to work with in the ordinary application of Makeham's formula,

* The difference between μ_{x+t} and $\mu_{[x]+t}$ so far as it depends on the change in β, leads to $B_t = B - 2n(10-t)\log_\epsilon 10 . \beta c^{-t}$, and the relation between β_t and β would apparently be $\beta_t = \beta[1 - n(19-2t)c^{-t}]$ where t is <10. With regard to the determination of the constants, it would appear that if the value of m had been ascertained as stated at the foot of the page there would have remained three, not four, unknown quantities.

the constants a and β do not lend themselves very well to the construction of select values, owing to the discontinuity at the point of junction, where the value of colog p depends partly on ultimate and partly on select values. The form of either $\log \frac{l_{x+t}}{l_{[x]+t}}$ (the function actually used by Mr. Hardy in the $O^{[M]}$ table) or $\mu_{x+t} - \mu_{[x]+t}$ is more easily determined, because both must vanish at the point of junction, as should also (to secure a smooth junction) the first differential co-efficient of $\mu_{x+t} - \mu_{[x]+t}$ or the second of $\log \frac{l_{x+t}}{l_{[x]+t}}$. This at once suggests for $A - A_t$ and $B - B_t$ an expression of the term $k(r-t)^n$ in the case of a junction after r years where n is >1. A relatively steep rise in μ during the first year or two can be secured by increasing the value of n. Thus, in the $O^{[NM]}$ Table, Mr. Hardy found a curve of the form $m(5-t)^3$ sufficient to represent the difference between A and A_t. The introduction in the $O^{[M]}$ Table of the term $m'(c')^t$ to represent the rapid rise in A_t affords an interesting example of the way in which a practical difficulty may be met in apparent disregard of the requirements of theory, since the value of the term became so insignificant after the first two or three years as in no way to affect the smoothness of junction. In some remarks on the construction of select annuity tables—in the course of which some additional light is thrown on the principle underlying the method of weighting employed in the calculation of the $O^{[am]}$ and $O^{[af]}$ constants—Mr. Hardy mentions that, for the purpose of forming a hypothetical table, the values of $\log l_{[x]+5}$ might be graduated by a curve of the fourth degree. It may be remembered also that Mr. Steffensen has shown* that the reciprocal of e_x may be fairly well represented by a formula of the Makeham type. Neither of these methods, however, gives a Makeham curve for μ, and when (as would usually be the case) this is desirable, it might be practicable to adopt a method similar to that employed in determining the constants A and B from exposures and deaths, but based on the approximate relation $\mu\dot{e} = 1 - \frac{1}{2}(\dot{e}_{-1} - \dot{e}_1)$ with suitable weights.

The conclusion to which the study of Mr. Hardy's Lectures inevitably leads is that there is no royal road to the graduation of actuarial statistics. With improved methods and increased knowledge has come a more exacting standard of what a good graduation should be, so that an apparently perfect result may fall under the condemnation that the deviations from the observations are too small, as compared with the probable errors. The scope for individual judgment, and for the skill that comes of practical experience and extensive knowledge, has increased rather than diminished, and in the application of

* Transactions of the Berlin Congress, vol. ii, pp. 247–266.

frequency-curves analogies may even be found to that process of "hand-polishing," which, in the graphic method, affords an exercise for the "taste and fancy of the operator." If, however, the problems of graduation have become more complex, every student of the subject must be the more grateful for the help which Mr. Hardy's Lectures will give in successfully dealing with them.

R. T.

THE LIFE COMPANIES OF THE UNITED KINGDOM.

Summary of the Life Assurance and Annuity Revenue Accounts.

[Extracted from the Parliamentary Returns for 1908, published in 1909.]

INCOME	Ordinary Companies	Industrial Companies	TOTAL
	£	£	£
Balance at the beginning of the Year	309,991,863	35,155,688	345,147,551
Adjustment chiefly in connection with the transfer of Sinking Fund and other Assurances from Life Funds made by certain Companies at the beginning of the year	−150,827	−71,123	−221,950
	309,841,036	35,084,565	344,925,601
Premiums	26,862,882	13,097,109	39,959,991
Consideration for Annuities . .	1,988,937	3,877	1,992,814
Interest and Dividends (less Tax)	11,861,220	1,215,055	13,076,275
Increase in value of Investments .	17,805	...	17,805
Fines, Fees, &c.	15,546	1,419	16,965
Capital Paid-up	18,731	26,204	44,935
Customs Timber Measuring, &c. .	3,731	...	3,731
Transfers from other Accounts .	153,380	92,057	245,437
	£350,763,268	£49,520,286	£400,283,554

OUTGO	Ordinary Companies	Industrial Companies	TOTAL
	£	£	£
Claims	19,147,918	5,177,337	24,325,255
Cash Bonuses and Reduction of Premiums	1,120,892	2,412	1,123,304
Surrenders	1,991,464	159,436	2,150,900
Annuities	2,282,377	5,693	2,288,070
Commission	1,418,917	3,312,788	4,731,705
Expenses of Management . .	2,185,824	2,362,919	4,548,743
Bad Debts	538	299	837
Decrease in value of Investments .	923,190	10,633	933,823
Interest on Capital and Dividends and Bonuses to Shareholders .	316,483	605,136	921,619
Transfers to other Accounts, &c. .	838,390	99,874	938,264
Balance* at the end of the Year .	320,537,275	37,783,759	358,321,034
	£350,763,268	£49,520,286	£400,283,554

* This Balance includes the whole of the Life and Annuity Funds (£352,584,617), and, in addition, the Capital, &c., of Companies whose business is limited to Life Assurance only.

Summary of the Balance Sheets.

LIABILITIES	Ordinary Companies	Industrial Companies*	Total
	£	£	£
Paid-up Capital (including sundry Shareholders' Balances)	12,866,691	2,369,354	15,236,045
Life and Annuity Funds	315,416,839	37,167,778	352,584,617
Fire Funds of Companies transacting Life Business	14,713,508	...	14,713,508
Marine Funds of Companies transacting Life Business	1,477,924	...	1,477,924
Reserve Funds	4,201,584	1,392,043	5,593,627
Other Funds	7,554,131	159,195	7,713,326
Profit and Loss Balances	4,720,457	6,683	4,727,140
Depreciation and Investment Balances	4,164,295	4,042	4,168,337
Outstanding Claims	5,603,651	9,933	5,613,584
Outstanding Accounts	978,277	30,656	1,008,933
Temporary Loans	822,594	95,009	917,603
	£372,519,951	£41,234,693	£413,754,644

ASSETS	Ordinary Companies	Industrial Companies*	Total
	£	£	£
Mortgages	97,216,141	4,696,172	101,912,313
Loans on Policies	21,191,163	190,920	21,382,083
,, Rates	35,524,194	11,428,848	46,953,042
British Government Securities	6,028,947	2,161,348	8,190,295
Indian and Colonial Government Securities	18,588,673	1,647,969	20,236,642
Foreign Government Securities	12,801,562	734,290	13,535,852
Debentures	79,431,573	5,528,612	84,960,185
Shares and Stocks	41,250,368	1,598,846	42,849,214
Companies' own Shares	501,443	...	501,443
Land and House Property and Ground Rents	30,494,004	10,470,226	40,964,230
Life Interests and Reversions	10,638,659	6,637	10,645,296
Loans on Personal Security	1,811,344	4,422	1,815,766
Agents' Balances and Outstanding Premiums	7,363,319	786,401	8,149,720
Outstanding Interest	3,529,477	345,815	3,875,292
Cash, Deposits, Stamps, &c.	5,702,283	764,648	6,466,931
Deficiencies, Establishment Expenses, &c.	434,569	869,539	1,304,108
Miscellaneous	12,232	...	12,232
	£372,519,951	£41,234,693	£413,754,644

* In the case of one or two Companies transacting both Ordinary and Industrial business, but not returning separate Balance Sheets, the Liabilities and Assets given in the above columns, under the heading "Industrial Companies", are those appertaining to the *combined* operations of such companies.—[Ed. *J.I.A.*]

INCREASE (+) *or* DECREASE (—) *in the Chief Items of this Year's* SUMMARY *as compared with the corresponding Items for the previous Year.*

	Ordinary Companies		Industrial Companies	
INCOME.		£		£
Premiums	+	848,650	+	656,241
Consideration for Annuities	—	70,647	—	2,869
Interest and Dividends (less Tax) . . .	+	512,168	+	88,506
OUTGO.				
Claims	+	710,368	+	440,056
Annuities	−	63,145	+	295
Surrenders	+	97,899	+	30,564
Commission	+	36,151	+	129,248
Expenses of Management	—	64,511	+	145,171
Net Decrease in value of Investments .		905,385		10,633
LIABILITIES.				
Paid-up Capital (including sundry Shareholders' Balances)	+	1,057,998	+	66,070
Life and Annuity Funds	+	8,972,892	+	2,816,466
ASSETS.				
Mortgages (including Loans on Rates) .	+	3,478,407	+	1,002,772
Life Interests and Reversions	+	367,506		...
Loans on Policies	+	1,795,091	+	20,024
British Government Securities	+	241,702	+	8,777
Indian and Colonial Government Securities	—	221,598	+	125,415
Foreign Government Securities	+	1,079,359	+	73,699
Debentures	+	6,446,070	+	699,480
Shares and Stocks	+	308,726	+	138,877
Land and House Property and Ground Rents	+	1,314,150	+	442,285
Loans on Personal Security	—	323,990	+	1,348

NUMBER OF COMPANIES.

The total number of Companies appearing in the above Summary is 89, of which 72 are classed as Ordinary, 8 as Industrial, and 9 appear in both Classes, the returns of these Companies showing the Ordinary and Industrial business separately.

During the year three names have been added to the Official List of Companies, viz.: China Mutual Life Insurance Company, Limited; Norwich Union Fire Insurance Society, Limited; and Royal London Mutual Insurance Society, Limited; in which cases the Board of Trade have issued their Warrant under the provisions of Section 1 of "The Life Assurance Companies Act, 1872."

Summary of the Assurances in Force, *as shown by the last Returns of the Companies.*

ORDINARY BUSINESS.

	With Profits		Without Profits		Total		Re-assurances	Net Amount
	No.	Amount	No.	Amount	No.	Amount	Amount	
Assurances.		£		£		£	£	£
Whole Term of Life	802,050	379,162,102	159,611	73,106,116	961,661	452,268,218	28,740,600	423,527,618
Limited number of Premiums	62,515	38,065,008	16,308	7,416,016	78,823	45,481,024	2,399,905	43,081,119
	864,565	417,227,110	175,919	80,522,132	1,040,484	497,749,242	31,140,505	466,608,737
Endowments	1,506	365,208	26,008	6,109,106	27,514	6,474,314	20,010	6,454,304
Endowment Assurances	1,396,605	222,506,094	143,906	30,516,096	1,540,511	253,022,190	3,705,046	249,317,144
Joint Lives	16,246	3,205,406	2,690	958,600	18,936	4,164,006	246,006	3,918,000
Last Survivor	761	654,108	1,198	1,595,660	1,959	2,249,768	315,606	1,934,162
Contingent	26	48,276	6,196	8,288,600	6,222	8,336,876	2,281,610	6,055,266
Issue	35	86,055	2,303	7,408,601	2,338	7,494,656	2,306,740	5,187,916
Miscellaneous	6,714	2,666,550	25,606	17,506,190	32,320	20,172,740	2,600,102	17,572,638
	2,286,458	646,758,807	383,826	152,904,985	2,670,284	799,663,792	42,615,625	757,048,167
Annuities.								
Immediate	...	...	...	...	42,800	2,270,670	56,966	2,213,704
Deferred	...	...	...	...	18,205	470,620	26,674	443,946
	...	...	...	...	61,005	2,741,290	83,640	2,657,650

INDUSTRIAL BUSINESS—(Sickness and Friendly Society Contracts not included).

	With Profits		Without Profits		Total		Re-assurances	Net Amount
	No.	Amount	No.	Amount	No.	Amount	Amount	
Assurances.		£		£		£	£	£
Whole Term of Life	59	5,638	24,510,672	243,507,207	24,510,731	243,512,845	1,670	243,511,175
Limited number of Premiums	...	...	294	5,793	294	5,793	...	5,793
	59	5,638	24,510,966	243,513,000	24,511,025	243,518,638	1,670	243,516,968
Endowments	...	...	2,120,512	20,233,809	2,120,512	20,233,809	...	20,233,809
Endowment Assurances	23	2,312	741,043	7,733,193	744,066	7,735,505	88	7,735,417
Joint Lives, &c.	...	...	438,236	6,958,707	438,236	6,958,707	400	6,958,307
	82	7,950	27,813,757	278,438,709	27,813,839	278,446,659	2,158	278,444,501
Annuities.								
Immediate	...	...	...	...	51	1,761	...	1,761
Deferred	...	...	...	...	7	132	...	132
	...	...	...	...	58	1,893	...	1,893

The above figures are based on Returns deposited, for the most part, during the past five years, and are, therefore, merely an approximation to the amount of contracts in force at the present time. The figures of the Colonial and Foreign Companies have been excluded, as their Returns do not separately show the extent of business in the United Kingdom.

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Printed by C. & E. Layton, Farringdon Street.